Real Analysis Homework 6, due 2007-10-24 in class

1. (10 points) Do Exercise 7 in p. 62.
2. (10 points) Show that the limit of a decreasing sequence of functions (with common domain E) usc at $x_0 \in E$ is also usc at x_0. Give an example of a decreasing sequence of functions continuous at $x_0 \in E$ but its limit is not continuous at x_0 (by the first part of the problem we know that the limit is at least usc at x_0).
3. (10 points) Do Exercise 11 in p. 62.
4. (10 points) Do Exercise 12 in p. 62.

Remark 1 (be careful) If $g(x)$ is a continuous function on $[a,b]$ and $f(x) = g(x)$ a.e. on $[a,b]$, it does not, in general, imply that $f(x)$ is continuous a.e. on $[a,b]$. For example, take $g(x) = 1$ and let

$$f(x) = \begin{cases} 1, & x \text{ is irrational in } [0,1] \\ 0, & x \text{ is rational in } [0,1]. \end{cases}$$

We see that $f(x) = g(x)$ a.e. on $[0,1]$, but $f(x)$ is discontinuous everywhere on $[0,1]$.