Real Analysis Homework 2, due 2007-9-25 in class

Show Your Work to Each Problem

1. (10 points) Use Lemma 3.16 to prove Lemma 3.15. Note that in proving Lemma 3.16 we do not have to use Lemma 3.15.

2. (10 points)

   (a) (5 points) Assuming the validity of Theorem 3.30 and the existence of non-measurable sets in $\mathbb{R}^n$ at this moment. Show that there exist two nonempty sets E_1 and E_2 in $\mathbb{R}^n$ such that $E_1 \cap E_2 = \varnothing$, but

   $$|E_1 \cup E_2|_e < |E_1|_e + |E_2|_e .$$

   Hence the condition $d(E_1, E_2) > 0$ in Lemma 3.16 can not be replaced by just $E_1 \cap E_2 = \varnothing$.

   (b) (5 points) Construct a sequence of nonempty sets $E_k \subset [0,1]$, $k = 1, 2, 3...$, so that

   $$\limsup E_k = [0,1], \qquad \liminf E_k = \varnothing.$$

3. (10 points) Assuming there exists a non-measurable set contained in $[0,1]$, do Exercise 17 in p. 48.

4. (10 points) Do Exercise 18 in p. 48.