

Real Analysis Homework 14, due 2008-1-2 in class

1. (10 points) Let $E \subset \mathbf{R}^n$ be a measurable set ($|E| < \infty$ or not). If for any $0 < p < \infty$ we have $f \in L^p(E)$ and $\|f\|_p \leq K$, where K is a constant independent of p . Show that $f \in L^\infty(E)$ and $\|f\|_\infty \leq K$ also.
2. (10 points) Do Exercise 5 in P. 143.
3. (10 points) Prove the converse of Hölder inequality (Theorem 8.8) for the case $p = 1$ and $p = \infty$.
4. (10 points) Assume $1 \leq p < \infty$. Do Exercise 12 in P. 144.