Real Analysis Homework 13, due 2007-12-26 in class

1. (10 points) Do Exercise 21 in P. 144.

2. (10 points) Prove the following more general version of the Tchebyshev inequality: Assume $f \geq 0$ is measurable on E satisfying $\int_E f^p dx < \infty$, where $0 < p < \infty$ is a constant. Then for any $\alpha > 0$ we have
$$|\{x \in E : f(x) > \alpha\}| \leq \frac{1}{\alpha^p} \cdot \int_E f^p dx.$$

3. (10 points) Let $H(x)$ be the Heaviside function given by
$$H(x) = \begin{cases} 1 & \text{if} \quad x > 0 \\ \frac{1}{2} & \text{if} \quad x = 0 \\ 0 & \text{if} \quad x < 0. \end{cases}$$
Find the set of those $x \in \mathbf{R}$ such that
$$\lim_{h \to 0^+} \frac{1}{2h} \int_{x-h}^{x+h} f(\theta)\, d\theta = f(x).$$
Determine if the point $x = 0$ is in the Lebesgue set of f or not.

4. (10 points) Let $g(x)$ be the function given by
$$g(x) = \begin{cases} \sin \frac{1}{x} & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0. \end{cases}$$
Determine if the point $x = 0$ is in the Lebesgue set of g or not. Also let $G(x) = \int_0^x g(s)\, ds$, $x \in \mathbf{R}$. Do we have $G'(0) = g(0)$ or not.