Homework Assignment 2
Due on Tuesday 3/25

Programming Problems:

1. Write a matlab function that computes QR factorization using Gram-Schmidt method (Algorithm 7.1). You can assume the input matrix A is a square nonsingular matrix. Your code should take a square matrix A as input and output two matrices Q, R such that $A = QR$.

2. Write a matlab function that computes QR factorization using Modified-Schmidt method (Algorithm 8.1). You can assume the input matrix A is a square nonsingular matrix. Your code should take a square matrix A as input and output two matrices Q, R such that $A = QR$.

3. Do Exrercise 4.3.

Submit your codes through iLMS.

4. In matlab, type "load clown". The matrix "X" is an image matrix of a clown. You can use the commends "image(X)" and "colormap(gray)" to see the picture in gray scale. Use SVD as a compression method for the image. Use 10, 20, 30, 50 singular vectors and values to reconstruct image. Print out the approximated picture and compute the compression ratio. Do the same experiment for your own picture and choose suitable number of singular values in your compression.

5. Selecting "Hilbert matrix" tests your codes in 1 and 2. The $n \times n$ Hilbert matrix is given by

$$\mathbf{H} = \begin{bmatrix} 1 & 1/2 & 1/3 & \dots & 1/n \\ 1/2 & 1/3 & 1/4 & \dots & 1/(n+1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1/n & 1/(n+1) & 1/(n+2) & \dots & 1/(2n-1) \end{bmatrix}$$

That is, $\mathbf{H}_{i,j} = 1/(i+j-1)$. You can use 'hilb(n)' to generate the $n \times n$ Hilbert matrix. Use 100×100 Hilbert matrix as input to compute QR factorization by codes in 1 and 2. Compute the corresponding error $\|Q * Q - I\|_F$ for orthogonality.

Writing Problems:

Do the following exercise problems in the text book by Trefethen and Bau,

Exercise 4: 4.1, 4.2, 4.5

Exercise 5: 5.2, 5.3, 5.4

Exercise 6: 6.1, 6.3, 6.5

Exercise 7: 7.2, 7.3, 7.4

Exercise 8: 8.1, 8.3