Homework Assignment 1
Due on Tuesday 3/11

Writing Problems:

The textbook Numerical Linear Algebra has an electronic version which can be found on the library website.

You can also download onestep_domain.m and multistep_domain.m for your reference for plotting the stability domain.

1. Do the following exercise problems in the text book by Bradie,
Sec 7.9: 3(a), 5, 7, 8(d), 11

2. Do the following exercise problems in the text book by Trefethen and Bau,
Exercise 2: 2.1, 2.5, 2.6
Exercise 3: 3.2, 3.3, 3.4, 3.6

3. Write down the condition of the region of absolute stability for the second and third order of backward differentiation formulas. Use Matlab to plot the regions.

4. (a) Let $\mathbf{A}$ be a nonsingular matrix. Prove that the solution of $\mathbf{y}' = \mathbf{A}\mathbf{y} + \mathbf{b}$, $\mathbf{y}(t_0) = \mathbf{y}_0$ is

$$\mathbf{y}(t) = e^{(t-t_0)\mathbf{A}}\mathbf{y}_0 + \mathbf{A}^{-1}(e^{(t-t_0)\mathbf{A}} - \mathbf{I})\mathbf{b}$$

Deduce that if all eigenvalues of $\mathbf{A}$ have negative real part, then $\lim_{t\to\infty} \mathbf{y}(t) = -\mathbf{A}^{-1}\mathbf{b}$

(b) Let $\mathbf{A} = \begin{bmatrix} 7991 & -11988 \\ 5994 & -8992 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{y}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Use Euler method and trapezoidal method to solve

$$\mathbf{y}' = \mathbf{A}\mathbf{y} + \mathbf{b} \quad \mathbf{y}(0) = \mathbf{y}_0.$$

Use time steps $h = 0.01, 0.001, 0.0001, 0.00001$ to solve the solution until $t = 10$. Plot the error for different time steps. Which time step can Euler generate good numerical solution? Is trapezoidal method better for this problem? Why?