Homework Assignment 1
Due on Tuesday 10/1

Writing Problems:

1. Do the following exercise problems in the text book by Bradie,
Sec 1.2: 1(b, c), 2(c, d), 3, 4, 7, 11, 15
Sec 1.3: 1(c), 2, 3, 12
Sec 1.4: 1(a), 7, 13

2. Suppose $f(x)$ has the continuous $n-th$ derivative and $f^n(x)$ is uniformly bounded, i.e, $|f^n(x)| < M$ for some positive constant M. State and prove the Taylor's expansion of $f(x)$ at a fixed point a and give an upper bound for the remaining term.

3. Write down the Taylor's expansion of $\arctan(x)$.

4. Search the IEEE standard for floating point number systems. Write a brief introduction of binary32 (single precision) and binary64 (Double precision).

Coding Problems:

I. Write a code to compute the Fibonacci sequence. The Fibonacci sequence is given by

$$f_1 = 1,\ f_2 = 1 \quad \text{and} \quad f_{n+2} = f_{n+1} + f_n \forall n \in \mathbb{N}.$$

(i) Find f_{24}, f_{44}.

(ii) Compute the value of $f_{n+2}f_n - f_{n+1}^2$. What do you observe? Just state the relationship you see. Prove it if you can but not required.

(iii) Compute $\frac{f_{n+1}}{f_n}$. What do you observe? Just state the relationship you see. Prove it if you can but not required.

II. Use the fact $\pi = 4\arctan(1)$ to estimate the value of π.

(i) Use 10 terms in Taylor's expansion of $\arctan(1)$ (See 3) to estimate the value of π. What will be the error's bound?

(ii) If you want to achieve 8 significant digits of π, how many terms you need in the Taylor's expansion. What is the approximate number you obtain?