## Practice Exam 2

1. Let $A$ be the following 4 by 4 matrix.

$$
A=\left[\begin{array}{rrrr}
1 & 1 & 0 & 0 \\
2 & 4 & -4 & 0 \\
0 & -1 & -1 & -9 \\
0 & 0 & 5 & 14
\end{array}\right]
$$

(i) Find $L U$ decomposition of $A$ by using Crout decomposition.
(ii) Find $\operatorname{det}(A)$.
2. Let

$$
L=\left[\begin{array}{lll}
1 & 0 & 0 \\
4 & 1 & 0 \\
5 & 1 & 1
\end{array}\right], \quad U=\left[\begin{array}{rrr}
-2 & -3 & 6 \\
0 & 10 & -13 \\
0 & 0 & -3
\end{array}\right], \quad \mathbf{b}=\left[\begin{array}{r}
4 \\
-11 \\
-4
\end{array}\right]
$$

Define $A=L U$. Solve the linear system $A \mathbf{x}=\mathbf{b}$ by forward and backward substitutions.
3. Consider the matrix

$$
A=\left[\begin{array}{rrr}
b & -1 & a \\
-1 & 3 & 0 \\
a & 0 & 4
\end{array}\right]
$$

(i) What necessary and sufficient conditions must $a$ and $b$ satisfy for $A$ to be strictly diagonally dominant?
(ii) What necessary and sufficient conditions must $a$ and $b$ satisfy for $A$ to be symmetric positive definite?
4. (20 points) Let $x_{0}=0, x_{1}=1, x_{2}=2$ and $f(x)=x^{3}-3 x+4$. Let $P_{2}(x)$ be the unique interpolating polynomial of degree at most 2 interpolates $f$ at $x_{0}, x_{1}$ and $x_{2}$.
(i) (10 points) Construct the Lagrange form of $P_{2}(x)$.
(ii) (10 points) Construct the Newton form of $P_{2}(x)$.
5. Let $s(x)$ be a cubic spline interpolant of $f$ relative to the partition $\left\{x_{i}\right\}_{i=0}^{n}$. On each interval $\left[x_{j}, x_{j+1}\right], s$ is written as

$$
s(x)=s_{j}(x)=a_{j}+b_{j}\left(x-x_{j}\right)+c_{j}\left(x-x_{j}\right)^{2}+d_{j}\left(x-x_{j}\right)^{3}
$$

and $s\left(x_{j}\right)=a_{j}=f\left(x_{j}\right)$. Let $h_{j}=x_{j+1}-x_{j}$.
(i) What equations can you get from continuity of $s(x), s^{\prime}(x)$ and $s^{\prime \prime}(x)$ ?
(ii) Derive the equation that involves $c_{j-1}, c_{j}, c_{j+1}, a_{j-1}, a_{j}, a_{j+1}$ only by using equations obtained in (i).
(iii) What is the not-a-knot boundary condition? What equation does the boundary condition translate to in terms of the coefficients?
(iv) What is the clamped boundary condition? What equation does the boundary condition translate to in terms of the coefficients?
6. Consider the linear system

$$
\begin{array}{rr}
x_{1}+3 x_{2}+2 x_{3}=-4 \\
-3 x_{1}-4 x_{2}+9 x_{3}= & 3 \\
2 x_{1}-4 x_{2}-2 x_{3}=10
\end{array}
$$

(i) What is the augmented matrix?
(ii) Perform the first pass of the Gaussian elimination with no pivoting.
(iii) Perform the first pass of the Gaussian elimination with partial pivoting (use explicit row change).
(iv) Perform the first pass of the Gaussian elimination with scaled partial pivoting (use explicit row exchange).

