

Practice Exam 1

1. Let $p_n = \frac{3n^2 - 1}{7n^2 + n + 2}$. Compute the limit $\lim_{n \rightarrow \infty} p_n$ and determine the (best) rate of convergence.
2. Let $p_n = \frac{1}{3^{(5^n)}}$. Compute the limit $\lim_{n \rightarrow \infty} p_n$ and determine the order of convergence.
3. Let $g(x) = x(2 - ax)$ for some positive real number a .
 - (i) Find the positive fixed point p of the function $g(x)$.
 - (ii) Determine the order of convergence and the asymptotic error constant of the sequence $p_n = g(p_{n-1})$ toward p .
4. $f(x) = x^2 + 2x - 1$ has a simple root in $[0, 1]$. Use the following methods to find the approximations for the root. For each method, compute the approximation until p_2 .
 - (i) Bisection method with $a_1 = 0$ and $b_1 = 1$.
 - (ii) Newton's method with $p_0 = 1$.
 - (iii) Secant method with $p_0 = 0$ and $p_1 = 1$.
 - (iv) Which of above methods is best for finding the root of $f(x)$ and why?