Homework Assignment 14
Due on Thursday 6/6

Do the following exercise problems in the text book by Salas, Hille and Etgen,
Sec 8.5: 6, 10, 16, 20, 24, 26, 29, 35, 40, 45, 50
Sec 10.1: 4, 12, 25, 28, 30,
Sec 10.2: 10, 22, 25, 31, 34, 41, 45, 46, 49, 54, 63,
Do the following problems.

Exercise I. Let $I_n(x) = \displaystyle\int_0^x \frac{1}{(t^2+1)^n}\,dt$ for all positive integer n.

1. Show that for $n \geq 2$, we have

$$\frac{2n-2}{(x^2+1)^n} = \frac{d}{dx}\left(\frac{x}{(x^2+1)^{n-1}}\right) + \frac{2n-3}{(x^2+1)^{n-1}}$$

2. Use 1 to show that for $n \geq 2$, we have

$$I_n(x) = \frac{1}{2n-2}\left(\frac{x}{(x^2+1)^{n-1}} + (2n-3)I_{n-1}(x)\right)$$

Use the formula to show that

$$\int_0^x \frac{1}{(t^2+1)^2}\,dt = \frac{1}{2}\left(\frac{x}{x^2+1} + \arctan x\right)$$

and

$$\int_0^x \frac{1}{(t^2+1)^3}\,dt = \frac{1}{8}\left(\frac{2x}{(x^2+1)^2} + \frac{3x}{x^2+1} + 3\arctan x\right)$$

Exercise II. Consider the curve Γ defined by $ax^2 + 2bxy + cy^2 = 1$. Suppose (x_0, y_0) is a point on Γ. Show that the tangent line to Γ at point (x_0, y_0) is given by

$$ax_0x + by_0x + bx_0y + cy_0y = 1.$$

Hint: Use implicit derivative to fine y' at (x_0, y_0),