Homework Assignment 8
Due on Thursday 4/25

Do the following exercise problems in the text book by Salas, Hille and Etgen,
Sec 5.5: 2, 6, 8, 18, 20, 24, 30, 34
Sec 5.6: 6, 10, 26, 29, 33, 36
Sec 5.7: 4, 12, 14, 21, 24, 30, 35, 36, 37, 44, 47, 48, 52, 62, 72, 82, 84
Sec 5.8: 1-15, 24, 26, 33, 36
Sec 5.9: 5, 14, 16, 21, 37

Do the following problems.

Exercise 1. Let $L(x) = \int_1^x \frac{1}{t}\,dt$ for $x > 0$. Show that

1. $L(a) + L(b) = L(ab)$.
2. $L(a^n) = nL(a)$.

Exercise 2. Find the following integrals.

1. $\int_{-1}^{1} x\sqrt{1-x^2}\,dx$
2. $\int_0^{\pi/3} (\cos^2 x - \sin^2 x)\,dx$ (Hint: $\cos^2 x - \sin^2 x = (\cos x + \sin x)(\cos x - \sin x)$)
3. $\int_{-1}^{1} \frac{\sin x}{1+x^2}\,dx$

Exercise 3. Let f be a continuous function on $[a, b]$.

1. Suppose $f(c) \geq 0$ for some $c \in [a, b]$. Show that there exists an interval I in $[a, b]$ such that $f(x) > f(c)/2 > 0$ on I.

2. Use 1. to show that if $f(x) \geq 0$ and $\int_b^a f(x)\,dx = 0$, then $f(x) = 0$ on $[a, b]$.