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Repdigits As Sums Of Two Associated Pell Numbers*

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Abstract

In this paper, we show that $2, 4, 6, 8$ and 44 are the only repdigits that are sums of two associated Pell numbers, which again confirms that the sum of two Lucas-balancing numbers can never be a repdigit with at least two digits. As a consequence of our main result, we show that $4, 8$ and 88 are the only repdigits which are sums of two Pell-Lucas numbers.

1 Introduction

The Lucas sequence $(U_n(r,s))_{n\geq 0}$ and companion Lucas sequence $(V_n(r,s))_{n\geq 0}$ are defined by

$$U_{n+1}(r,s) = rU_n(r,s) - sU_{n-1}(r,s), \quad V_{n+1}(r,s) = rV_n(r,s) - sV_{n-1}(r,s), \tag{1}$$

where r, s are integers such that $\Delta = r^2 - 4s > 0$ and the initial terms are given by $(U_0, U_1) = (0, 1)$ and $(V_0, V_1) = (2, r)$ respectively. The Binet formulas for these sequences are given by

$$U_n = \frac{\alpha^n - \beta^n}{\alpha - \beta},\ V_n = \alpha^n + \beta^n \tag{2}$$

where $(\alpha, \beta) = (\frac{r+\sqrt{r^2-4s}}{2}, \frac{r-\sqrt{r^2-4s}}{2})$ are roots of the characteristic equation $X^2 - rX + s = 0$. Clearly, $\alpha + \beta = r,\ \alpha\beta = s,\ \alpha - \beta = \sqrt{\Delta}$. These sequences can be extended to negative indices n as $U_{-n} = -s^{-n}U_n$ and $V_{-n} = s^{-n}V_n$ respectively. The sequences of Fibonacci (F_n), Lucas (L_n), Pell (P_n) and Pell-Lucas (Q_n) numbers satisfy the above recurrences with particular values of r and s. In particular, $F_n = U_n(1, -1)$, $L_n = V_n(1, -1)$, $P_n = U_n(2, -1)$, $Q_n = V_n(2, -1)$.

The solution n, of the Diophantine equation $1 + 2 + \cdots + (n - 1) = (n + 1) + (n + 2) + \cdots + (n + r)$ is called a balancing number (see [4]) with corresponding balancer r. The sequence of balancing numbers is denoted by $(B_n)_{n\geq 1}$ and can be viewed as a particular Lucas sequence since $B_n = U_n(6, 1)$. If B_n is the n^{th} balancing number, then $\sqrt{8B_n^2 + 1}$ is called the n^{th} Lucas-balancing number denoted by C_n (see [18]). The Lucas-balancing numbers satisfy the same recurrence as that of balancing numbers. The sequence of balancing and Lucas-balancing numbers can be expressed in terms of Pell and Pell-Lucas numbers. In particular, $B_n = P_nQ_n/2 = P_nq_n$, $C_n = q_{2n}$, where $q_n = Q_n/2$ is called as the associated Pell number. The associated Pell numbers satisfy the same recurrence as that of Pell numbers. It can be seen that Q_n are all even and q_n are all odd.

Let $g \geq 2$ be any positive integer. A natural number N is called a *base g repdigit* if it is of the form

$$N = a\left(\frac{g^m - 1}{g - 1}\right), \quad \text{for some } m \geq 1,\ \text{where } a \in \{1, 2, \cdots, g - 1\}.$$

The base 10 repdigits are simply called repdigits.

The existence of repdigits in Fibonacci, Lucas, Pell, Pell-Lucas, balancing and Lucas-balancing sequences has been studied in [6, 9, 19]. Similar study have been carried out by replacing Fibonacci, Lucas, balancing

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and Lucas-balancing numbers by their respective consecutive products (see [7, 12, 19]). In [20], the authors studied the existence of repdigits that are expressible as products of balancing and Lucas-balancing numbers with their indices in arithmetic progressions. Fibonacci, Lucas, Pell and Pell-Lucas numbers which are expressible as sum of two repdigits have been studied in [1, 2, 3]. Repdigits which are sums of three Fibonacci or Lucas numbers have been investigated in [10, 16]. Subsequently, repdigits that are sums of four Fibonacci or Lucas or Pell numbers have been investigated in [11, 15]. Repdigits in the base b expansion as sum of four balancing numbers can be found in [8]. Repdigits that are sum of two Fibonacci and two Lucas numbers appear in [17], where as those which are product of two Pell or Pell-Lucas numbers can be seen in [21]. In [22], Şiar and Keskin searched for the repdigits that are sum of two Lucas numbers.

In this paper, we use the method similar to that of [22] to prove the following result:

Theorem 1 *The only repdigits which are sum of two associated Pell numbers are* $2, 4, 6, 8$ *and* 44.

2 Preliminaries

To solve the Diophantine equations involving repdigits and the terms of binary recurrence sequences, many authors have used Baker's theory to reduce lower bounds concerning linear forms in logarithms of algebraic numbers. These lower bounds play an important role while solving such Diophantine equation. We start with recalling some basic definitions and results from algebraic number theory.

A modified version of a result of Matveev [14] appears in [5, Theorem 9.4]. Let $\mathbb{L}$ be an algebraic number field of degree D. Let $\eta_1, \eta_2, \dots, \eta_l \in \mathbb{L}$ not 0 or 1 and $b_1, b_2, \dots, b_l$ be nonzero integers. Put

$$B = \max\{|b_1|, \dots, |b_l|\}, \quad \text{and} \quad \Gamma = \prod_{i=1}^{l} \eta_i^{b_i} - 1.$$

Let $A_1, \dots, A_l$ be positive integers such that

$$A_j \geq h^{'}(\eta_j) := \max\{Dh(\eta_j), |\log\eta_j|, 0.16\}, \quad j = 1, \dots, l,$$

where η is an algebraic number having the minimal polynomial

$$f(X) = a_0(X - \eta^{(1)}) \cdots (X - \eta^{(k)}) \in \mathbb{Z}[X]$$

over the integers with $a_0 > 0$. The logarithmic height of η is given by

$$h(\eta) = \frac{1}{k}\left(\log\, a_0 + \sum_{j=1}^{k} \max\{1, \log|\eta^{(j)}|\}\right).$$

In particular, if $\eta = a/b$ is a rational number with $\gcd(a, b) = 1$ and $b > 1$, then $h(\eta) = \log(\max\{|a|, b\})$. The following properties of logarithmic height holds.

$$h(\eta \pm \gamma) \leq h(\eta) + h(\gamma) + \log 2,$$

$$h(\eta\gamma^{\pm 1}) \leq h(\eta) + h(\gamma),$$

$$h(\eta^m) = |m|h(\eta).$$

Theorem 2 ([5, Theorem 9.4]) *If* $\Gamma \neq 0$ *and* $\mathbb{L} \subseteq \mathbb{R}$, *then*

$$\log|\Gamma| > -1.4 \cdot 30^{l+3} l^{4.5} D^2 (1 + \log D)(1 + \log B) A_1, \cdots, A_l.$$

Another main tool for the proof of our main results is a variant of Baker and Davenport reduction method due to de Weger [13].

Let $\vartheta_1, \vartheta_2, \epsilon \in \mathbb{R}$ and let $x_1, x_2 \in \mathbb{Z}$ be unknowns. Let

$$\Gamma = \epsilon + x_1\vartheta_1 + x_2\vartheta_2. \tag{3}$$

Let c, δ be positive constants. Set $X = \max\{|x_1|, |x_2|\}$. Let X_0, Y be positive. Assume that

$$|\Gamma| < c \cdot \exp(-\delta \cdot Y), \tag{4}$$

$$Y \leq X \leq X_0. \tag{5}$$

When $\epsilon = 0$ in (3), we get

$$\Gamma = x_1\vartheta_1 + x_2\vartheta_2.$$

Put $\vartheta = -\vartheta_1/\vartheta_2$. We assume that x_1 and x_2 are coprime. Let the continued fraction expansion of ϑ be given by

$$[a_0, a_1, a_2, \ldots],$$

and let the k^{th} convergent of ϑ be p_k/q_k for $k = 0, 1, 2, \ldots$. We may assume without loss of generality that $|\vartheta_1| < |\vartheta_2|$ and that $x_1 > 0$. We have the following results.

Lemma 3 ([13, Lemma 3.2]) *Let*

$$A = \max_{0 \leq k \leq Y_0} a_{k+1}.$$

If (4) *and* (5) *hold for* x_1, x_2 *and* $\epsilon = 0$, *then*

$$Y < \frac{1}{\delta}\log\left(\frac{c(A+2)X_0}{|\vartheta_2|}\right).$$

When $\epsilon \neq 0$ in (3), put $\vartheta = -\vartheta_1/\vartheta_2$ and $\psi = \epsilon/\vartheta_2$. Then, we have

$$\frac{\Gamma}{\vartheta_2} = \psi - x_1\vartheta + x_2.$$

Let p/q be a convergent of ϑ with $q > X_0$. For a real number x, we let $||x|| = \min\{|x - n|, n \in \mathbb{Z}\}$ be the distance from x to the nearest integer. We have the following result.

Lemma 4 ([13, Lemma 3.3]) *Suppose that*

$$||q\psi|| > \frac{2X_0}{q}.$$

Then, the solutions of (4) *and* (5) *satisfy*

$$Y < \frac{1}{\delta}\log\left(\frac{q^2 c}{|\vartheta_2| X_0}\right).$$

3 Proof of Theorem 1

Proof of Theorem 1. Assume that

$$N = q_{m_1} + q_{m_2} = d\left(\frac{10^k - 1}{9}\right) \tag{6}$$

for some positive integers $0 \le m_2 \le m_1$, $k > 0$ and $d \in \{1, 2, \ldots, 9\}$. A quick computer search reveals the solutions in the range $0 \le m_2 \le m_1 \le 100$. Precisely, the solutions to (6) are given by

$$(N, m_1, m_2, k) \in \left\{ \begin{array}{l} (2,0,0,1), (2,0,1,1), (2,1,1,1), \\ (4,0,2,1), (4,1,2,1), (6,2,2,1), \\ (8,0,3,1), (8,1,3,1), (44,2,5,2) \end{array} \right\}.$$

From now on, we assume that $m_1 > 100$. Further, since associated Pell numbers are all odd, (6) has no solution for odd d. Thus, $d \in \{2, 4, 6, 8\}$.

Since the associated Pell numbers satisfy $\alpha^{n-1} \le 2q_n < \alpha^{n+1}$ for all $n > 1$, where $\alpha = 1 + \sqrt{2}$ and from (6), we have $q_{101} \le q_{m_1} + q_{m_2} < 10^k - 1$. It follows that

$$10^{k-1} \le d\left(\frac{10^k - 1}{9}\right) = q_{m_1} + q_{m_2} \le 2q_{m_1} < \alpha^{m_1+1}.$$

Taking logarithm on both sides of the last inequality, we get

$$(k-1)\frac{\log 10}{\log \alpha} \le m_1 + 1$$

yielding

$$2.61k - 3.61 < m_1.$$

Since $q_{101} < 10^k - 1$, the above inequality implies that $37 \le k < m_1$. Using the Binet formula of associated Pell numbers in (6), we get

$$\frac{\alpha^{m_1} + \beta^{m_1}}{2} + \frac{\alpha^{m_2} + \beta^{m_2}}{2} = d\left(\frac{10^k - 1}{9}\right), \quad (\text{where } \beta = 1 - \sqrt{2})$$

i.e.,

$$\alpha^{m_1} + \alpha^{m_2} - 2d\frac{10^k}{9} = -\left(\beta^{m_1} + \beta^{m_2} + \frac{2d}{9}\right),$$

which implies

$$\left|\alpha^{m_2}(1 + \alpha^{m_1 - m_2}) - 2d\frac{10^k}{9}\right| \le |\beta|^{m_1} + |\beta|^{m_2} + \frac{2d}{9} \le 4.$$

Dividing both sides by $\alpha^{m_2}(1 + \alpha^{m_1 - m_2})$, we get

$$\left|1 - 10^k \alpha^{-m_2} \frac{2d}{9(1 + \alpha^{m_1 - m_2})}\right| \le \frac{4}{\alpha^{m_2}(1 + \alpha^{m_1 - m_2})} < \frac{4}{\alpha^{m_1}} < \alpha^{1.58 - m_1}. \tag{7}$$

Put

$$\Gamma := 1 - 10^k \alpha^{-m_2} \frac{2d}{9(1 + \alpha^{m_1 - m_2})}. \tag{8}$$

If $\Gamma = 0$, then $\alpha^{m_1} + \alpha^{m_2} = 2d\frac{10^k}{9}$ and hence, $\alpha^{m_1} + \alpha^{m_2} \in \mathbb{Q}$, which is not possible for any $m_1, m_2 > 0$. Therefore $\Gamma \ne 0$. Take

$$\eta_1 = \alpha,\ \eta_2 = 10,\ \eta_3 = \frac{2d}{9(1 + \alpha^{m_1 - m_2})},\ b_1 = -m_2,\ b_2 = k,\ b_3 = 1.$$

Using the properties of logarithmic height, we get $h(\eta_1) = \frac{\log\alpha}{2}$, $h(\eta_2) = \log 10$ and

$$\begin{aligned} h(\eta_3) &\le h(2d) + h(9) + h(\alpha^{m_1 - m_2}) + \log 2 \\ &\le \log 18 + \log 9 + (m_1 - m_2)\frac{\log\alpha}{2} + \log 2 \\ &< 5.8 + (m_1 - m_2)\frac{\log\alpha}{2}. \end{aligned}$$

The degree of $\mathbb{L} := \mathbb{Q}(\sqrt{2})$ is $D = 2$. Since $1 \le |\log\alpha| \le 2h(\alpha)$, $|\log 10| \le 2h(10)$ and $\left|\log\frac{2d}{9(1+\alpha^{m_1-m_2})}\right| \le 2h(\eta_3)$, we take

$$A_1 := \log\alpha,\ A_2 := 4.61,\ A_3 := 11.6 + (m_1 - m_2)\log\alpha.$$

Also $B = m_1 \ge \max\{\mathrm{m}_2, \mathrm{k}, 1\}$. In view of Theorem 2 and (7), we have

$$m_1\log\alpha - 1.58\log\alpha < 3.97 \cdot 10^{12}(1 + \log m_1)(11.6 + (m_1 - m_2)\log\alpha). \tag{9}$$

Once again, using the Binet formula of associated Pell numbers in (6), we get

$$\alpha^{m_1} - 2d\frac{10^k}{9} = -\Big(\beta^{m_1} + \beta^{m_2} + \alpha^{m_2} + \frac{2d}{9}\Big).$$

Taking the absolute on both sides of the above equation, we have

$$\left|\alpha^{m_1} - 2d\frac{10^k}{9}\right| = \left|\beta^{m_1} + \beta^{m_2} + \alpha^{m_2} + \frac{2d}{9}\right|$$

which implies

$$\left|\alpha^{m_1} - 2d\frac{10^k}{9}\right| \le |\beta|^{m_1} + |\beta|^{m_2} + \alpha^{m_2} + \frac{2d}{9} \le \alpha^{m_2} + 4 < \alpha^{m_2+1.83}.$$

Dividing both sides by α^{m_1}, we get

$$\left|1 - \alpha^{-m_1}10^k\frac{2d}{9}\right| < \alpha^{m_2-m_1+1.83}. \tag{10}$$

Put

$$\Gamma' = 1 - \alpha^{-m_1}10^k\frac{2d}{9}. \tag{11}$$

As above, one can justify that $\Gamma' \ne 0$. Take

$$\eta_1 = \alpha,\ \eta_2 = 10,\ \eta_3 = \frac{2d}{9},\ b_1 = -m_1,\ b_2 = k,\ b_3 = 1.$$

A similar argument to the above gives

$$A_1 := \log\alpha,\ A_2 := 4.61,\ A_3 := 10.2,\ B = m_1.$$

Again applying Theorem 2, we have

$$(m_1 - m_2)\log\alpha - 1.83\log\alpha < 4.02 \cdot 10^{13}(1 + \log m_1). \tag{12}$$

Substituting (12) into (9) gives $m_1 < 8.88 \cdot 10^{29}$. Put $X_0 = 8.88 \cdot 10^{29}$. Let

$$\Lambda = \log\Big(\frac{2d}{9}\Big) - m_1\log\alpha + k\log 10.$$

Using (6), we obtain that

$$\alpha^{m_1} - 2d\frac{10^k}{9} = -\frac{2d}{9} - \beta^{m_1} - 2q_{m_2} \le -\frac{2d}{9} - \beta^{m_1} - 2 < 0.$$

So $\Lambda > 0$. From (10), we see that

$$0 < \Lambda < e^{\Lambda} - 1 < \alpha^{m_2-m_1+1.83},$$

which implies that

$$|\Lambda| < \alpha^{1.83}\,\alpha^{m_2-m_1} < \alpha^{1.85}\exp(-0.88\cdot(m_1 - m_2)).$$

Thus, $\Lambda < \alpha^{1.85}\exp(-0.88\cdot Y)$ holds with $Y := m_1 - m_2$. We also have

$$\frac{\Lambda}{\log 10} = \frac{\log(2d/9)}{\log 10} - m_1\frac{\log\alpha}{\log 10} + k.$$

Thus, we take

$$c = \alpha^{1.85},\ \delta = 0.88,\ x_1 = m_1,\ x_2 = k,\ \psi = \frac{\log(2d/9)}{\log 10},$$

$$\vartheta = -\frac{\log\alpha}{\log 10},\ \vartheta_1 = \log\alpha,\ \vartheta_2 = \log 10,\ \epsilon = \log(2d/9).$$

Clearly, $\max\{|x_1|, |x_2|\} = m_1 \le X_0$. The smallest value of $q > X_0$ is

$$q_{68} = 2512046602227734280329853086909.$$

We find that

$$q_{70} = 144803942540586860757348134097483$$

satisfies the hypothesis of Lemma 4. Applying it, we get $m_1 - m_2 = Y < 90.85$. Now, we take $0 \le m_1 - m_2 \le 90$. Put

$$\Lambda' = \log\left(\frac{2d}{9(1+\alpha^{m_1-m_2})}\right) - m_2\log\alpha + k\log 10.$$

It can be easily seen that $\Lambda' > 0$. Using (7), it follows that

$$0 < \Lambda' < e^{\Lambda'} - 1 = |\Gamma'| < \alpha^{1.58-m_1},$$

which implies that

$$|\Lambda'| = \alpha^{1.58-m_1}\exp(-0.88\cdot m_1).$$

We consider

$$c = \alpha^{1.58},\ \delta = 0.88,\ x_1 = m_2,\ x_2 = k,\ \psi = \log\left(\frac{2d}{9(1+\alpha^{m_1-m_2})}\right)/\log 10,$$

$$Y = m_1,\ \vartheta = -\frac{\log\alpha}{\log 10},\ \vartheta_1 = \log\alpha,\ \vartheta_2 = \log 10,\ \epsilon = \log\left(\frac{2d}{9(1+\alpha^{m_1-m_2})}\right).$$

The smallest value of $q > X_0$ is $q = q_{69}$. For $\epsilon \ne 0$, we find that $q = q_{75}$ satisfies the hypothesis of Lemma 4. Applying it, we obtain $m_1 \le 100.842$ i.e., $m_1 \le 100$, which is a contradiction to our assumption that $m_1 > 100$.

Now, when $\epsilon = 0$. The largest partial quotient a_k for $0 \le k \le 144$ is $a_{120} = 561$. Applying Lemma 3, we get

$$n < \frac{1}{0.88}\cdot\log\left(\frac{\alpha^{1.58}(561+2)\cdot 8.88\cdot 10^{29}}{|\log 10|}\right).$$

We obtain $n < 86.2$ i.e. $n \le 86$, which again contradicts the assumption that $n > 100$. This completes the proof. ■

The following two corollaries are consequences of Theorem 1.

Corollary 5 *The only repdigits which are sum of two Pell-Lucas numbers are* $4, 8$ *and* 88.

Proof. Since Pell-Lucas numbers are all even,

$$N = Q_{m_1} + Q_{m_2} = d\left(\frac{10^k - 1}{9}\right) \tag{13}$$

has no solution for odd d. Thus, $d \in \{2, 4, 6, 8\}$. Hence, the solutions in the statement follows directly from Theorem 1. Precisely,

$$(N, m_1, m_2, k) \in \left\{ \begin{array}{l} (4,0,0,1), (4,0,1,1), (4,1,1,1), \\ (8,0,2,1), (8,1,2,1), (88,2,5,2) \end{array} \right\}$$

are the only solutions to (13). ■

Corollary 6 *The only repdigits which are sum of two Lucas-balancing numbers are* $2, 4$ *and* 6.

Proof. Since the Lucas-balancing numbers are all even indexed associated Pell numbers, Theorem 1 assures that

$$N = C_{m_1} + C_{m_2} = d\left(\frac{10^k - 1}{9}\right)$$

has only solutions $(N, m_1, m_2, k) \in \{(2,0,0,1), (4,0,1,1), (6,1,1,1)\}$. ■

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