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Ranking Of Trapezoidal Bipolar Fuzzy Information System Based On Total Ordering*

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Abstract

In this research article, the concepts of (α, β)-cut, strong (α, β)-cut and level set of a bipolar fuzzy number are introduced. First and second decomposition theorems for bipolar fuzzy numbers are established. By using the concept of double upper lower dense sequence, the concept of total ordering of bipolar fuzzy numbers is developed. Finally, trapezoidal bipolar fuzzy information system is presented.

1 Introduction

Fuzzy numbers proposed by Zadeh and Chang [13] and intuitionistic fuzzy numbers by Burillo et al. [11] are special cases of fuzzy sets [24] and intuitionistic fuzzy sets [8], respectively. These are the two powerful tools to characterize uncertainty, imprecision and vagueness. Fuzzy numbers give the degree of acceptance whereas intuitionistic fuzzy numbers give membership and nonmembership, but sometimes the information available considers two poles: one for satisfaction and the other for dissatisfaction. In Chinese medicine, Yin and Yang are the two sides. *Yin* is the negative side of a system and *Yang* is the positive side of a system. The notion of bipolar fuzzy sets (YinYang bipolar fuzzy sets) was introduced by Zhang [25, 26] in the space $\{\forall\ (x, y) \mid (x, y) \in [-1, 0] \times [0, 1]\}$. In various domains of data processing, bipolarity is a base feature in considerations where positive information shows what is preferred or possible whereas negative information shows what is negated or surely excluded. If the given information is moreover enriched with vagueness, hesitation and imprecision, as in the case of spatial data processing, then BFSs constitute a convenient information representation structure. Akram et al. [2, 3, 4, 5, 6, 7] considered several concepts, including bipolar fuzzy graphs, bipolar fuzzy numbers and multi-criteria decision-making (MCDM) methods based on bipolar fuzzy information. In any MCDM technique, the important steps include assessment of importance weights of criteria, calculation of aggregate scores and ranking of alternatives. Following this procedural steps, the most complicated step is computing the ranking order of alternatives. A number of rank measures for the ranking of fuzzy numbers and intuitionistic fuzzy numbers have been

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proposed [1, 9, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 23]. A rank measure for fuzzy numbers based on a dense sequence was introduced in [23] in which Wang el al. defined an upper dense sequence in the interval $[0, 1]$ to identify any given fuzzy number using countably many real valued parameters. Later, Lakhshman et al. [20] extended this concept to upper lower dense sequence contained in $[0, 1]^2$ and used extended lexicography to rank intuitionistic fuzzy numbers.

In this paper, a new double upper lower dense sequence in $[-1, 0] \times [0, 1]$ is introduced. There are many double upper lower dense sequences contained in $[-1, 0] \times [0, 1]$. A new decomposition theorem for bipolar fuzzy sets is established using any one of the double upper lower dense sequence defined in $[-1, 0] \times [0, 1]$. Then by the use of a chosen double upper lower dense sequence, infinitely many total orderings on the set of all bipolar fuzzy numbers can be defined.

The paper is organized as follows: some basic concepts such as bipolar fuzzy numbers (BFNs), trapezoidal bipolar fuzzy numbers (TrBFNs) and triangular bipolar fuzzy numbers (TBFNs) are introduced in section 2. The concept of (α, β)-cut of bipolar fuzzy sets as well as BFNs using intersection property is introduced in section 3. First, second and third decomposition theorems are developed and by adopting the concept of double upper lower dense sequence (DULDS), which is a key factor in the development of third decomposition theorem, total ordering on the set of BFNs is defined in section 4. In section 5, the application of ranking based on total ordering in trapezoidal bipolar fuzzy information system (TrBFIS) is presented. Finally comes the conclusion in section 6.

A bipolar fuzzy set P on the universal set Q is characterized by a satisfaction degree of a certain property σ_P^+ and a satisfaction of its counter property σ_P^-, where

$$\sigma_P^+(x) : Q \to [0, 1] \quad \text{and} \quad \sigma_P^-(x) : Q \to [-1, 0].$$

A bipolar fuzzy set can be represented as:

$$P = \{< x, \sigma_P^+(x), \sigma_P^-(x) >| \; x \in Q\}.$$

2 (α, β)-Cut of Bipolar Fuzzy Numbers

DEFINITION 1. A bipolar fuzzy number (BFN)

$$\zeta = \prec I, K \succ = \prec [o_1, o_2, o_3, o_4], [\tau_1, \tau_2, \tau_3, \tau_4] \succ$$

is a bipolar fuzzy subset of a real line R with satisfaction degree λ_I and dissatisfaction degree λ_K given as:

$$\lambda_I(x) = \begin{cases} \lambda_I^L(x), & \text{if } x \in [o_1, o_2], \\ 1, & \text{if } x \in [o_2, o_3], \\ \lambda_I^R(x), & \text{if } x \in [o_3, o_4], \\ 0, & \text{otherwise}, \end{cases}$$

and

$$\lambda_K(x) = \begin{cases} \lambda_K^L(x), & \text{if } x \in [\tau_1, \tau_2], \\ -1, & \text{if } x \in [\tau_2, \tau_3], \\ \lambda_K^R(x), & \text{if } x \in [\tau_3, \tau_4], \\ 0, & \text{otherwise,} \end{cases}$$

with

$$\lambda_I^L(x) : [o_1, o_2] \to [0,1], \quad \lambda_I^R(x) : [o_3, o_4] \to [0,1],$$
$$\lambda_K^L(x) : [\tau_1, \tau_2] \to [-1,0], \quad \lambda_K^R(x) : [\tau_3, \tau_4] \to [-1,0],$$

where $\lambda_I^L(x)$ and $\lambda_K^L(x)$ denote left membership functions for $\lambda_I(x)$ and $\lambda_K(x)$, respectively. Similarly $\lambda_I^R(x)$ and $\lambda_K^R(x)$ denote right membership functions for $\lambda_I(x)$ and $\lambda_K(x)$, respectively.

DEFINITION 2. A BFN $\zeta = \prec I, K \succ = \prec [o_1, o_2, o_3, o_4], [\tau_1, \tau_2, \tau_3, \tau_4] \succ$ is a TrBFN, denoted by $\prec (o_1, o_2, o_3, o_4), (\tau_1, \tau_2, \tau_3, \tau_4)$, if its satisfaction degree λ_I and dissatisfaction degree λ_K are given as:

$$\lambda_I(x) = \begin{cases} \frac{x-o_1}{o_2-o_1}, & \text{if } x \in [o_1, o_2], \\ 1, & \text{if } x \in [o_2, o_3], \\ \frac{o_4-x}{o_4-a_3}, & \text{if } x \in [o_3, o_4], \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\lambda_K(x) = \begin{cases} \frac{\tau_1-x}{\tau_2-\tau_1}, & \text{if } x \in [\tau_1, \tau_2], \\ -1, & \text{if } x \in [\tau_2, \tau_3], \\ \frac{x-\tau_4}{\tau_4-\tau_3}, & \text{if } x \in [\tau_3, \tau_4], \\ 0, & \text{otherwise.} \end{cases}$$

DEFINITION 3. A BFN is a TBFN, if its satisfaction degree λ_I and dissatisfaction degree λ_K is given as:

$$\lambda_I(x) = \begin{cases} \frac{x-o_1}{o_2-o_1}, & \text{if } x \in [o_1, o_2], \\ \frac{o_3-x}{o_3-o_2}, & \text{if } x \in [o_2, o_3], \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\lambda_K(x) = \begin{cases} \frac{\tau_1-x}{\tau_2-\tau_1}, & \text{if } x \in [\tau_1, \tau_2], \\ \frac{x-\tau_3}{\tau_3-\tau_2}, & \text{if } x \in [\tau_2, \tau_3], \\ 0, & \text{otherwise.} \end{cases}$$

DEFINITION 4. A bipolar fuzzy variable (BFV) is characterized by a triplet $(X, U, R(X, u))$, where X is the name of variable, U is a continuous domain for the variable X, u is the generic name for the elements of U and $R(X, u)$ is a bipolar fuzzy number defined on U which represents the restriction on the values of u imposed by X.

DEFINITION 5. A bipolar fuzzy linguistic variable (BFLV) is characterized by a quintuple $(x, T\ (x), U, P(X), N(X))$, where

(a) x is the name of the variables,

(b) $T(x)$ is the set of linguistic values taken by X,

(c) X represents the generic name of $T(x)$,

(d) U is a subset of $\mathbb{R}$ representing the numerical domain of x,

(e) $R(X) : U \to [0,1] \times [-1,0]$ is a semantic rule which associates every linguistic value in T with a bipolar fuzzy number defined on U.

From Definition 5, it is clear that bipolar linguistic variables are the variables whose linguistic values are bipolar fuzzy numbers defined on U.

DEFINITION 6. Let P be a BFS on the universal set Q. For any $\alpha \in [0,1]$ and $\beta \in [-1,0]$, (α,β)-cut of P denoted as $P^{(\alpha,\beta)}$, is defined as:

$$P^{(\alpha,\beta)} = \{x \in Q \mid \sigma_P^+(x) \geq \alpha, \sigma_P^-(x) \leq \beta\}.$$

EXAMPLE 1. Let $Q = \{q_1, q_2, q_3, q_4, q_5\}$ be a universal set and $E = \{(q_1, 0.5, -0.6), (q_2, 0.3, -0.5), (q_3, 0.7, -0.8), (q_4, 0.4, -1), (q_5, 1, 0)\}$. For $\alpha = 0.4$ and $\beta = -0.5$, $P^{(\alpha,\beta)} = \{q_1, q_3, q_4\}$.

DEFINITION 7. Let P be a BFS in the universe of discourse Q. For any $\alpha \in [0,1]$ and $\beta \in [-1,0]$, the strong (α,β)-cut of P denoted as $P^{(\alpha,\beta)+}$, is defined as:

$$P^{(\alpha,\beta)+} = \{x \in Q \mid \sigma_P^+(x) > \alpha, \sigma_P^-(x) < \beta\}.$$

EXAMPLE 2. For Example 1, we have $P^{(\alpha,\beta)+} = \{q_1, q_3\}$.

DEFINITION 8. Let P be a bipolar fuzzy set on the universal set Q. The level set of P denoted as $L(P)$, is defined as:

$$L(P) = \{(\alpha,\beta) \mid \sigma_P^+(x) = \alpha,\ \sigma_P^-(x) = \beta\}.$$

Since BFNs are specific case of BFSs, so (α,β)-cut of BFNs can be defined on a similar pattern as defined below.

DEFINITION 9. Let $\zeta = \prec I, K \succ = \prec [o_1, o_2, o_3, o_4], [\tau_1, \tau_2, \tau_3, \tau_4] \succ$ be a bipolar fuzzy number. For $\alpha \in [0,1]$ and $\beta \in [-1,0]$, (α,β)-cut of ζ, denoted as $\zeta^{(\alpha,\beta)}$, is defined as:

$$\zeta^{(\alpha,\beta)} = \{x \in \mathbb{R} \mid \lambda_I(x) \geq \alpha, \lambda_K(x) \leq \beta\}.$$

It is clear that for $\alpha, \beta \neq 0$, (α,β)-cut of a BFN is a closed interval which is obtained by taking intersection of two closed intervals, one is the α-cut of I and the other is the $\beta - cut$ of K. The graphical representation for (α,β)-cut is given as in Fig. 1.

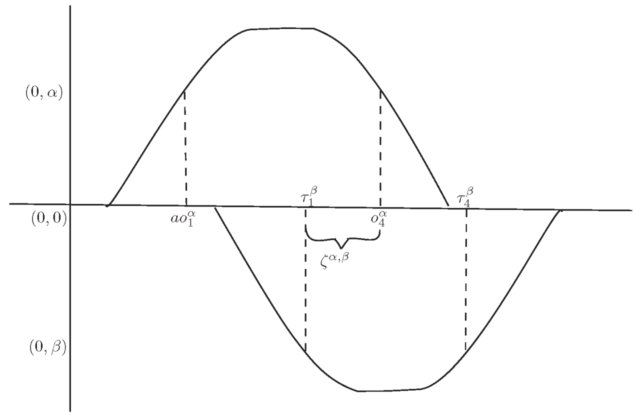

Figure 1: (α, β)-cut of Bipolar Fuzzy Number.

In case of TrBFNs (α, β)-cut can be represented as:

$$\zeta^{(\alpha,\beta)} = [o_1^\alpha, o_4^\alpha] \cap [\tau_1^\beta, \tau_4^\beta],$$

where

$$\begin{aligned} \iota_1^\alpha &= o_1 + \alpha(o_2 - o_1), \quad \iota_4^\alpha = o_4 - \alpha(o_4 - o_3), \\ b_1^\beta &= \tau_1 - \beta(\tau_2 - \tau_1), \quad \kappa_4^\beta = \tau_4 + \beta(\tau_4 - \tau_3). \end{aligned}$$

Similarly, for TBFNs, (α, β)-cut can be represented as:

$$\zeta^{(\alpha,\beta)} = [o_1^\alpha, o_3^\alpha] \cap [\tau_1^\beta, \tau_3^\beta],$$

where

$$\begin{aligned} o_1^\alpha &= o_1 + \alpha(o_2 - o_1), \quad o_4^\alpha = o_3 - \alpha(o_3 - o_2), \\ \tau_1^\beta &= \tau_1 - \beta(\tau_2 - \tau_1), \quad \tau_3^\beta = \tau_3 + \beta(\tau_3 - \tau_2). \end{aligned}$$

EXAMPLE 3. Let $\zeta = \prec (2, 4, 5, 6), (-3, 4, 5, 7) \succ$ be a TrBFN. For $\alpha = 0.5$ and $\beta = -0.7$.

$$\begin{aligned} \zeta^{(0.5,-0.7)} &= [o_1^{0.5}, o_4^{0.5}] \cap [\tau_1^{-0.7}, \tau_4^{-0.7}] \\ &= [3, 5.5] \cap [1.9, 5.6] \\ &= [3, 5.5]. \end{aligned}$$

DEFINITION 10. Let $\zeta = \prec I, K \succ = \prec [o_1, o_2, o_3, o_4], [\tau_1, \tau_2, \tau_3, \tau_4] \succ$ be a BFN. For $\alpha \in [0, 1]$ and $\beta \in [-1, 0]$, the strong (α, β)-cut of ζ, denoted as $\zeta^{(\alpha,\beta)+}$, is defined as:

$$\zeta^{(\alpha,\beta)+} = \{x \in \mathbb{R} \mid \lambda_I(x) > \alpha, \lambda_K(x) < \beta\}.$$

DEFINITION 11. Let P be a BFS on the universal set Q. A special BFS in Q related to $P^{(\alpha,\beta)}$ denoted as ${}_{(\alpha,\beta)}P$, is defined as:

$$ {}_{(\alpha,\beta)}P = \begin{cases} (\alpha,\beta), & x \in P^{(\alpha,\beta)}; \\ (0,0), & x \notin P^{(\alpha,\beta)}. \end{cases} $$

DEFINITION 12. Let P be a bipolar fuzzy set in the universal set Q. A special BFS on K related to $P^{(\alpha,\beta)+}$ denoted as ${}_{(\alpha,\beta)+}P$, is defined as:

$$ {}_{(\alpha,\beta)+}P = \begin{cases} (\alpha,\beta), & x \in P^{(\alpha,\beta)+}; \\ (0,0), & x \notin P^{(\alpha,\beta)+}. \end{cases} $$

3 Decomposition Theorems for Bipolar Fuzzy Numbers

In this section, we will represent an arbitrary BFS using special BFSs by establishing decomposition theorems on BFS. For $(\alpha,\beta) \in D = [-1,0] \times [0,1]$:

THEOREM 1. First Decomposition Theorem: For any BFS P

$$ P = \bigcup_{(\alpha,\beta)\in D} {}_{(\alpha,\beta)}P. $$

PROOF. For each particular $x \in Q$, let us denote $(\sigma_P^+(x), \sigma_P^-(x)) = (p,q) = P(x)$, where p is satisfaction degree for the belongingness of x in P and q is dissatisfaction degree for the belongingness of x in P. Then

$$ \begin{aligned} &\left(\bigcup_{(\alpha,\beta)\in[0,1]\times[-1,0]} {}_{(\alpha,\beta)}P \right)(x) \\ &= \left(\sup_{\alpha\in[0,1]}, \inf_{\beta\in[-1,0]} {}_{(\alpha,\beta)}P(x) \right) \\ &= \max\left[\left(\sup_{\alpha\in[0,p]}, \inf_{\beta\in[-1,0]} {}_{(\alpha,\beta)}P(x) \right), \left(\sup_{\alpha\in[p,1]}, \inf_{\beta\in[-1,0]} {}_{(\alpha,\beta)}P(x) \right) \right] \\ &= \sup_{\alpha\in[0,p]}, \inf_{\beta\in[-1,0]} {}_{(\alpha,\beta)}P(x) \\ &= \max\left[\left(\sup_{\alpha\in[0,p]}, \inf_{\beta\in[q,0]} {}_{(\alpha,\beta)}P(x) \right), \left(\sup_{\alpha\in[0,p]}, \inf_{\beta\in[-1,q]} {}_{(\alpha,\beta)}P(x) \right) \right] \\ &= \sup_{\alpha\in[0,p]}, \inf_{\beta\in[q,0]} {}_{(\alpha,\beta)}P(x) = (p,q). \end{aligned} $$

THEOREM 2. Second Decomposition Theorem: For any BFS P

$$P = \bigcup_{(\alpha,\beta)\in L(P)} {}_{(\alpha,\beta)}P.$$

PROOF. The proof is similar to the Theorem 1.

EXAMPLE 4. Let $Q = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$ be a universe of discourse. Let

$$E = \left\{\frac{(0.5,-0.7)}{q_1}, \frac{(0.3,-0.8)}{q_2}, \frac{(0.2,-0.5)}{q_3}, \frac{(0.7,-0.1)}{q_4}, \frac{(0.5,-0.5)}{q_5}, \frac{(0,-0.8)}{q_6}, \frac{(1,0)}{q_7}\right\}$$

be bipolar fuzzy set on X. Then

$$\begin{aligned}
{}_{(0.5,-0.7)}P =& \left\{\frac{(0.5,-0.7)}{q_1}, \frac{(0,0)}{q_2}, \frac{(0,0)}{q_3}, \frac{(0,0)}{q_4}, \frac{(0,0)}{q_5}, \frac{(0,0)}{q_6}, \frac{(0,0)}{q_7}\right\}\\
{}_{(0.3,-0.8)}P =& \left\{\frac{(0,0)}{q_1}, \frac{(0.3,-0.8)}{q_2}, \frac{(0,0)}{q_3}, \frac{(0,0)}{q_4}, \frac{(0,0)}{q_5}, \frac{(0,0)}{q_6}, \frac{(0,0)}{q_7}\right\}\\
{}_{(0.2,-0.5)}P =& \left\{\frac{(0.2,-0.5)}{q_1}, \frac{(0.2,-0.5)}{q_2}, \frac{(0.2,-0.5)}{q_3}, \frac{(0,0)}{q_4}, \frac{(0.2,-0.5)}{q_5}, \frac{(0,0)}{q_6}, \frac{(0,0)}{q_7}\right\}\\
{}_{(0.7,-0.1)}P =& \left\{\frac{(0,0)}{q_1}, \frac{(0,0)}{q_2}, \frac{(0,0)}{q_3}, \frac{(0.7,-0.1)}{q_4}, \frac{(0,0)}{q_5}, \frac{(0,0)}{q_6}, \frac{(0,0)}{q_7}\right\}\\
{}_{(0.5,-0.5)}P =& \left\{\frac{(0.5,-0.5)}{q_1}, \frac{(0,0)}{q_2}, \frac{(0,0)}{q_3}, \frac{(0,0)}{q_4}, \frac{(0.5,-0.5)}{q_5}, \frac{(0,0)}{q_6}, \frac{(0,0)}{q_7}\right\}\\
{}_{(0,-0.8)}P =& \left\{\frac{(0,0)}{q_1}, \frac{(0,-0.8)}{q_2}, \frac{(0,0)}{q_3}, \frac{(0,0)}{q_4}, \frac{(0,0)}{q_5}, \frac{(0,-0.8)}{q_6}, \frac{(0,0)}{q_7}\right\}\\
{}_{(1,0)}P =& \left\{\frac{(0,0)}{q_1}, \frac{(0,0)}{q_2}, \frac{(0,0)}{q_3}, \frac{(0,0)}{q_4}, \frac{(0,0)}{q_5}, \frac{(0,0)}{q_6}, \frac{(1,0)}{q_7}\right\}.
\end{aligned}$$

Thus, we conclude that

$$P = \bigcup_{(\alpha,\beta)\in D} {}_{(\alpha,\beta)}P = \bigcup_{(\alpha,\beta)\in L(P)} {}_{(\alpha,\beta)}P.$$

3.1 Double Upper Lower Dense Sequence

Though, the decomposition theorems proved above, identify a BFS as well as a BFN (regarding as a special case of BFSs), but they do not provide an approach of countable parameters. In this subsection, the concept of double upper lower dense sequence (DULDS) on I is introduced for establishing a new decomposition theorem for bipolar fuzzy sets. The already defined UDS [23] on interval $(0, 1]$ is sufficient for the value of

α in calculating α-cut of FN. But, in case of bipolar fuzzy numbers two sequences are required for values of α and β. In this subsection, a LDS in $[0,1]$ and an UDS in $[-1,0]$ is introduced and some traits of defined sequences are investigated.

DEFINITION 13. Let S be a subset of interval $[0,1]$. It is said to be a dense set in $[0,1]$ if for each point $x_\circ$ in $[0,1]$ and any $\epsilon > 0$, $\exists\ \delta \in S$ such that $|x_\circ - \delta| < \epsilon$.

DEFINITION 14. Let S be a subset of interval $[0,1]$. It is said to be upper dense in $[0,1]$ if for each point $x_\circ$ in $[0,1]$ and any $\epsilon > 0$, $\exists\ \delta \in S$ such that $\delta \in [x_\circ, x_\circ + \epsilon)$. S is lower dense in $[0,1]$ if for each point $x_\circ$ in $[0,1]$ and any $\epsilon > 0$, $\exists\ \delta \in S$ such that $\delta \in (x_\circ - \epsilon, x_\circ]$.

THEOREM 3. If S is a dense set in the interval $[0,1]$ and $0 \in S$, then S is lower dense in $[0,1]$.

THEOREM 4. If S is a dense set in the interval $[0,1]$ and $1 \in S$, then S is upper dense in $[0,1]$.

DEFINITION 15. Let S be a subset of interval $[-1,0]$. It is said to be a dense set in $[-1,0]$ if for each point $x_\circ$ in $[-1,0]$ and any $\epsilon > 0$, $\exists\ \delta \in S$ such that $|x_\circ - \delta| < \epsilon$.

DEFINITION 16. Let S be a subset of interval $[-1,0]$. It is said to be an upper dense set in $[-1,0]$ if for each $x_\circ$ in $[-1,0]$ and any $\epsilon > 0$, $\exists\ \delta \in S$ such that $\delta \in [x_\circ, x_\circ + \epsilon)$. S is lower dense in $[-1,0]$ if for each point $[-1,0]$ and any $\epsilon > 0$, $\exists\ \delta \in S$ such that $\delta \in (x_\circ - \epsilon, x_\circ]$.

THEOREM 5. If S is a dense set in interval $[-1,0]$ and $-1 \in S$, then S is lower dense in $[-1,0]$.

THEOREM 6. If S is a dense set in interval $[-1,0]$ and $0 \in S$, then S is upper dense in $[-1,0]$.

From Theorems 3 and 4, it is observed that, any upper lower dense sequence in $[0,1]$ is actually a dense sequence in interval $[0,1]$ with real numbers 0 and 1. Similarly from theorems 5 and 6, any upper lower dense sequence in $[-1,0]$ is a dense sequence in interval $[-1,0]$ with real numbers -1 and 0.

DEFINITION 17. Let $S_\alpha = \{\alpha_i\}$ be given ULDS in $[0,1]$ and let $S_\beta = \{\beta_j\}$ be ULDS in $[-1,0]$. Then the double upper lower dense sequence is of the form

$$S_{(\alpha,\beta)} = \{(\alpha_i, \beta_j) : i, j = 1, 2, ...\} \subseteq D.$$

Examples for upper lower dense sequences in $[0,1]$ and $[-1,0]$ are given as follows:

EXAMPLE 5. Let

$$S_1 = \left\{ \frac{p}{q} \ \text{ where } \ p \leq q \quad p, q \in R^+ \cup \{0\} \right\}$$

be the set of rational numbers in $[0,1]$. Since rational numbers are countable and a result from calculus states that, “for every countable set X, there exists a sequence in which every element of X appears". So, the sequence $S_\alpha = \{\alpha_i : i = 1,2,...\}$ in which every element of S_1 appears (many times) can be obtained by sweeping out the Fig. 2.Clearly, $S_\alpha = \{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{1}{5}, ...\}$ is an ULDS in $[0,1]$.

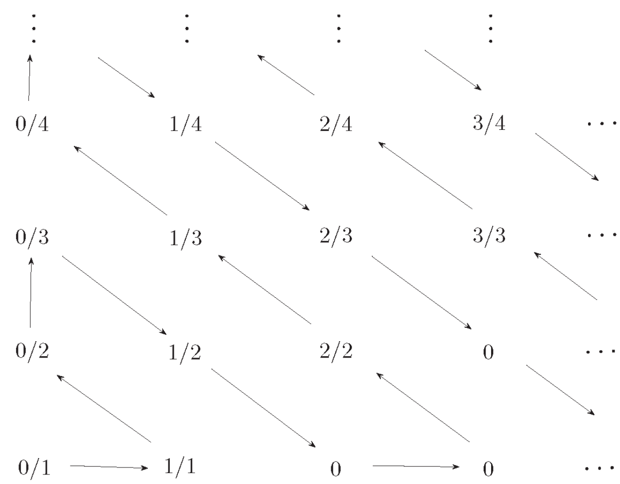

Figure 2: Using diagonal sweeping to list the rational numbers contained in $[0,1]$.

EXAMPLE 6. Let

$$S_2 = \left\{ \frac{-p}{q} \text{ where } p \le q \quad p, q \in R^+ \cup \{0\} \right\}$$

be the set of rational numbers in $[-1,0]$. The sequence $S_\beta = \{\beta_i : i = 1,2,...\}$ in which every element of S_2 appears (many times) can be obtained by sweeping out the Fig. 3.

Clearly, $S_\beta = \{0, -1, -\frac{1}{2}, \frac{-1}{3}, -\frac{1}{4}, -\frac{2}{3}, -\frac{1}{5}, ...\}$ is an ULDS in $[-1,0]$.

EXAMPLE 7. In Examples 5 and 6, let $S_{(\alpha,\beta)} = \{(\alpha_i, \beta_j) \mid \alpha_i \in S_\alpha, \beta_j \in S_\beta\}$. Then $S_{(\alpha_i,\beta_j)}$ being the cartesian product of two countable sets is countable and forms DULDS in D as illustrated in Fig 4.

$$S_{(\alpha,\beta)} = \{(0,0), (1,0), (0,-1), (0,-1/2), (1,-1), (1/2,0), (1/3,0), ...\}.$$

Since an element with membership $(0,0)$ can not be contained in a bipolar fuzzy set so we can consider $S_{(\alpha,\beta)} = \{(1,0), (0,-1), (0,-1/2), (1,-1), (1/2,0), (1/3,0), ...\}$ excluding $(0,0)$ as a double upper lower dense sequence which can be used to identify any given bipolar fuzzy set/number as described in the following theorem:

THEOREM 7. Third decomposition theorem for bipolar fuzzy sets: Let P be a bipolar fuzzy subset of Q and S be a given DULDS in D. Then

$$P = \bigcup_{(\alpha,\beta)\in S} {}_{(\alpha,\beta)}P.$$

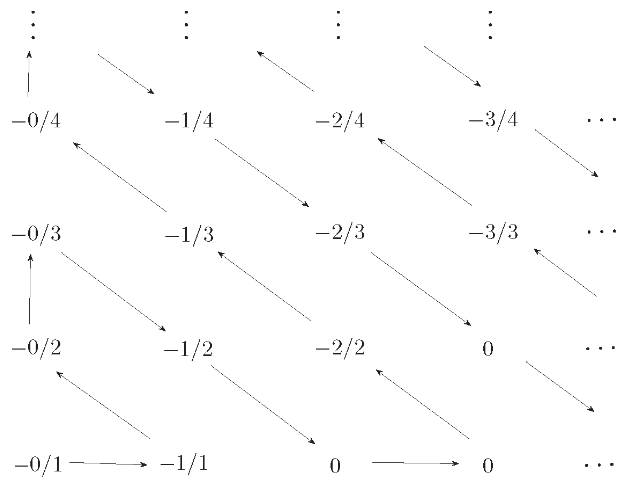

Figure 3: Using diagonal sweeping to list the rational numbers contained in $[-1,0]$

PROOF. Let P be a bipolar fuzzy set and S be given DULDS in D.
Claim: $P = \bigcup_{(\alpha,\beta)\in S} {}_{(\alpha.\beta)}P$.

Since $S \subseteq D$, we see that

$$\bigcup_{(\alpha,\beta)\in S} {}_{(\alpha.\beta)}P \subseteq \bigcup_{(\alpha,\beta)\in D} {}_{(\alpha.\beta)}P = P.$$

On the other hand, we have to show that

$$\sigma_P^+(x) \leq \sup_{(\alpha,\beta)\in S} \sigma^+_{{}_{(\alpha,\beta)}P}(x) \quad \text{and} \quad \sigma_P^-(x) \geq \inf_{(\alpha,\beta)\in S} \sigma^-_{{}_{(\alpha,\beta)}P}(x)$$

for each $x \in Q$. For each $x \in Q$, let $(\sigma_P^+(x), \sigma_P^-(x)) = (p,q)$

$$\sigma_P^+(x) = \sup_{(\alpha,\beta)\in[0,1]\times[-1,0]} \sigma^+_{{}_{(\alpha,\beta)}P}(x) = \sup_{(\alpha,\beta)\in[0,p]\times[-1,0]} \sigma^+_{{}_{(\alpha,\beta)}P}(x),$$
$$\sigma_P^-(x) = \inf_{(\alpha,\beta)\in[0,1]\times[-1,0]} \sigma^+_{{}_{(\alpha,\beta)}P}(x) = \inf_{(\alpha,\beta)\in[0,1]\times[b,0]} \sigma^+_{{}_{(\alpha,\beta)}P}(x).$$

For each $\alpha \in [0,p]$ and $\beta \in [q,0]$, (since S_α is upper dense sequence in $[0,1]$ and S_β is lower dense sequence in $[-1,0]$) there exists $(l_\alpha, l_\beta) \in S$ such that

$$\sigma^+_{{}_{(\alpha,\beta)}P}(x) \leq \sigma^+_{{}_{(l_\alpha,l_\beta)}P}(x) \leq \sup_{(\alpha,\beta)\in S} \sigma^+_{{}_{(\alpha,\beta)}P}(x),$$
$$\sigma^-_{{}_{(\alpha,\beta)}P}(x) \geq \sigma^-_{{}_{(l_\alpha,l_\beta)}P}(x) \geq \inf_{(\alpha,\beta)\in S} \sigma^-_{{}_{(\alpha,\beta)}P}(x).$$

By taking supremum with respect to $\alpha \in [0,a]$ and infemum with respect to $\beta \in [b,0]$, we have

$$\sigma_P^+(x) = \sup_{(\alpha,\beta)\in[0,p]\times[-1,0]} \sigma^+_{{}_{(\alpha,\beta)}P}(x) \leq \sup_{(\alpha,\beta)\in S} \sigma^+_{{}_{(\alpha,\beta)}P}(x),$$

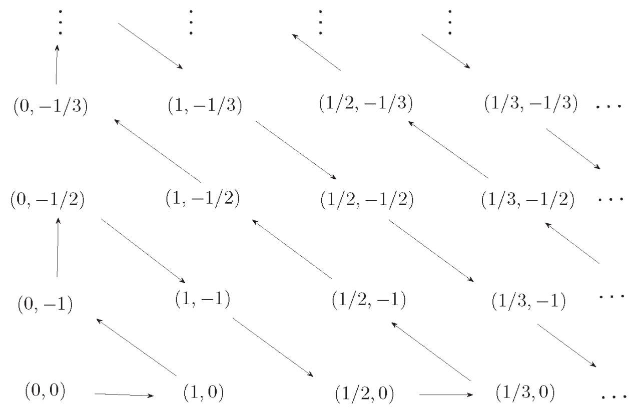

Figure 4: Using diagonal sweeping to list the elements in S_α.

$$\sigma^-_P(x) = \inf_{(\alpha,\beta)\in[0,1]\times[q,0]} \sigma^-_{(\alpha,\beta)}{}_P(x) \geq \inf_{(\alpha,\beta)\in S} \sigma^-_{(\alpha,\beta)}{}_P(x).$$

The proof is complete.

3.2 Total Ordering on the Set of All Bipolar Fuzzy Numbers

Let $S = \{(\alpha_i, \beta_j) \mid i, j = 1, 2, ...\}$ be given double upper lower dense sequence in D for any given bipolar fuzzy number ζ. Since S is countable and so has 1-1 correspondence with set of natural numbers. Let $\mu^k(\alpha_i, \beta_j)$ represents the kth pair in the sequence for this 1-1 correspondence. Since the (α, β)-cut of a bipolar fuzzy number is acquired by taking the intersection of α-cut of satisfaction degree of ζ and $\beta - cut$ of dissatisfaction degree of ζ. We denote it by $[a_k, b_k]$. Let $J_k = a_k + b_k$, where $k = 1, 2, \ldots$. Using these countably many parameters, we define a relation on the set of all BFNs as follows:

DEFINITION 18. Let ζ_1 and ζ_2 be two BFNs. For a given DULDS S in D, we use $J_k(\zeta_1)$ and $J_k(\zeta_2)$ to signify above mentioned J for ζ_1 and ζ_2 respectively. $\zeta_1 = \zeta_2$ iff their $(\alpha, \beta) - cuts$ for each pair (α_i, β_j) are equal, i.e. $\zeta_1^{(\alpha_i,\beta_j)} = \zeta_2^{(\alpha_i,\beta_j)}$ $\forall$ $i, j = 1, 2, ...$ and $\zeta_1 \prec \zeta_2$ iff there exists an integer $k_\circ > 0$ such that $J_{k_\circ}(\zeta_1) < J_{k_\circ}(\zeta_2)$ and $J_k(\zeta_1) = J_k(\zeta_2)$ for all integers $k < k_\circ$, where $k > 0$.

THEOREM 8. Relation $\preceq$ is a total ordering on the set of all BFNs.

PROOF. The proof is similar as the proof of Theorem 3.8 in [20].

3.3 Numerical Examples

We can see from the following examples that how the ranking of bipolar fuzzy numbers can be achieved through total orderings.

EXAMPLE 8. Let $\zeta_1 = \prec (0.6, 0.65, 0.67, 0.7), (0.6, 0.7, 0.75, 0.8) \succ$ and $\zeta_2 = \prec (0.6, 0.65, 0.67, 0.75), (0.2, 0.3, 0.35, 0.4) \succ$ be two BFNs. The total ordering $\preceq$ defined by using DULDS $S_{(\alpha,\beta)}$ given in Example 7 and the way shown in Definition 18 is adopted. For $k = 1$, $(\alpha, \beta) = (1, 0)$, we have $\zeta_1^{(\alpha,\beta)} = [0.65, 0.67]$ and and $J_1(\zeta_1) = J_1(\zeta_2) = 1.32$. For $k = 2$, $J_2(\zeta_1) = 1.45$ and $J_2(\zeta_2) = 0.65$ i.e. $J_2(\zeta_1) > J_2(\zeta_2)$. Hence $\zeta_1 > \zeta_2$.

EXAMPLE 9. Let $\zeta_1 = \prec (0.2, 0.3, 0.4, 0.5), (0.7, 0.8, 0.85, 0.9) \succ$ and $\zeta_2 = \prec (0.3, 0.35, 0.4, 0.5), (0.2, 0.4, 0.45, 0.5) \succ$ be two bipolar fuzzy numbers. For $k = 1$ $(\alpha, \beta) = (1, 0)$, we have $J_1(\zeta_1) = 0.7$ and $J_1(\zeta_2) = 0.75$. i.e $J_1(\zeta_1) < J_1(\zeta_2)$. Hence $\zeta_1 < \zeta_2$.

4 Trapezoidal Bipolar Fuzzy Information System

In this section, the application of the proposed method in bipolar fuzzy information system (BFIS) is presented. An Information system is a decision model that helps to organize and analyze data and to take rapid decisions in the selection of a feasible alternative from given alternatives under certain evaluation criteria. In an information system, preeminence or dominance relation totally depends on the ranking of evaluated data or information. In this section, trapezoidal bipolar fuzzy information system is defined and decision making in trapezoidal bipolar fuzzy information system using preeminence value (based on preeminence relation on the set of objects) is described.

DEFINITION 19. An information system $I = (X, CR, D, \lambda)$ with $D = \bigcup_{c \in CR} D_c$, where D_c is domain for criterion c is called trapezoidal bipolar fuzzy information system (TrBFIS) if D is a set of trapezoidal bipolar fuzzy numbers.

We denote $\lambda(x, c) \in D_c$ by $\lambda(x, c) = \prec (o_1, o_2, o_3, o_4), (\tau_1, \tau_2, \tau_3, \tau_4) \succ$, where $o_i, \tau_i \in [0, 1]$.

DEFINITION 20. A trapezoidal bipolar fuzzy information system $I = (X, CR, D, \lambda)$ together with $W = \{w_c \mid c \in CR\}$ is called weighted trapezoidal bipolar fuzzy information system (WTrBFIS) and is denoted by $I = (X, CR, D, \lambda, W)$.

DEFINITION 21. Let $c \in CR$ be a criterion and $u_i, u_j \in X$. If $J_1(u_i) > J_1(u_j)$ or $J_1(u_i) = J_1(u_j)$, $J_2(u_i) > J_2(u_j)$ or $J_{k_\circ}(u_i) > J_{k_\circ}(u_j)$ and $J_k(u_i) = J_k(u_j)$, $\forall\ k < k_\circ$ for c, then $u_i >_c u_j$ which indicates that u_i is better than u_j with respect to criterion c. Also $u_i =_c u_j$ indicates that u_i is equal to u_j if $\lambda(u_i, c) = \lambda(u_j, c)$, with respect to criterion c.

DEFINITION 22. Let $I = (X, CR, D, \lambda, W)$ be a WTrBFIS and $C \subseteq CR$. Let $G_C(u_i, u_j) = \{c \in B_C \mid u_i >_c u_j\}$, $L_C(u_i, u_j) = \{c \in C_C \mid u_i <_c u_j\}$ and $E_C(u_i, u_j) = \{c \in C \mid u_i =_c u_j\}$. The weighted bipolar fuzzy preeminence relation $WR_C(u_i, u_j)$:

$X \times X \rightarrow [0,1]$ is defined by

$$WR_C(u_i,u_j) = \sum_{c \in G_C(u_i,u_j)} w_c + \frac{\sum\limits_{c \in E_C(u_i,u_j)} w_c}{2} + \sum_{c \in L_C(u_i,u_j)} w_c$$

where B_C represents benefit criteria and C_C represents cost criteria.

Let $I = (X, CR, D, \lambda, W)$ be a WTrBFIS and $C \subseteq CR$. The entire preeminence value of each object is defined as

$$WR_C(u_i) = \frac{1}{\mid X \mid} \sum_{j=1}^{|X|} WR_C(u_i,u_j).$$

We now give an algorithm for the ranking of objects in TrBFIS.

Algorithm 6.1

Let $I = (X, CR, D, \lambda, W)$ be a weighted TrBFIS. The objects in X are ranked using the following algorithm.

1. Using Definition 18 and 21, find $J_k^{\cdot}s$ accordingly, to decide whether $u_i >_c u_j$ or $u_j >_c u_i$ or $u_i =_c u_j$ for all $c \in C \subseteq CR$ and for all $u_i, u_j \in X$.

2. Numerate $G_C(u_i,u_j)$ using $G_C(u_i,u_j) = \{c \in B_C \mid u_i >_c u_j\}$, $L_C(u_i,u_j)$ using $L_C(u_i,u_j) = \{c \in C_C \mid u_i <_c u_j\}$ and $E_C(u_i,u_j)$ using $E_C(u_i,u_j) = \{c \in C \mid u_i =_c u_j\}$.

3. Calculatethe value of weighted bipolar fuzzy preeminence relation between each u_i and u_j using $WR_C(u_i,u_j) : X \times X \rightarrow [0,1]$, defined by

$$WR_C(u_i,u_j) = \sum_{c \in G_C(u_i,u_j)} w_c + \frac{\sum\limits_{c \in E_C(u_i,u_j)} w_c}{2} + \sum_{c \in L_C(u_i,u_j)} w_c.$$

4. Calculate the entire preeminence value of each object using

$$WR_C(u_i) = \frac{1}{\mid X \mid} \sum_{j=1}^{|X|} WR_C(u_i,u_j).$$

5. The objects are ranked using entire preeminence degree. The larger the value of $WR_C(u_i)$, the better the object is.

4.1 Numerical Illustration

In this subsection, the examples of selection of a textile mill for clothing brand and the best event organizer are presented to illustrate the Algorithm 6.1.

EXAMPLE 10. Consider a selection problem of a textile mill for clothing brand from the available alternatives (textile mills) $\{u_i \mid i = 1, \ldots, 6\}$ based on trapezoidal bipolar fuzzy information with attributes $\{c_j \mid j = 1, \ldots, 5\}$ as cost, fabric quality, sustainable textile supply chain, compliance with foreign textile regulations and risk. The attributes fabric quality, sustainable textile supply chain, compliance with foreign textile regulations are benefit criteria whereas cost and risk are cost criteria. A trapezoidal bipolar fuzzy information system with $X = \{u_1, u_2, ..., u_6\}$, $CR = \{c_1, c_2, ..., c_5\}$ is given in Table 1: and weights for each attribute w_c is given by

$$W = \{0.15, 0.25, 0.17, 0.25, 0.18\}.$$

Table 1: TrBFIS to evaluate alternatives with respect to criteria.

	c_1	c_2	c_3	c_4	c_5
u_1	$\prec$(0.2,0.3,0.4,0.5), (0.7,0.8,0.85,0.9)$\succ$	$\prec$(0.7,0.75,0.8,0.85), (0.5,0.55,0.6,0.62)$\succ$	$\prec$(0.1,0.2,0.3,0.35), (0.6,0.65,0.7,0.75)$\succ$	$\prec$(0.6,0.65,0.7,0.8), (0.2,0.3,0.35,0.4)$\succ$	$\prec$(0.2,0.3,0.4,0.7), (0.2,0.3,0.4,0.5)$\succ$
u_2	$\prec$(0.3,0.35,0.4,0.5), (0.2,0.4,0.45,0.5)$\succ$	$\prec$(0.6,0.7,0.75,0.8), (0.2,0.3,0.35,0.4)$\succ$	$\prec$(0.2,0.3,0.35,0.4), (0.2,0.3,0.3,0.4)$\succ$	$\prec$(0.7,0.8,0.85,0.9), (0.3,0.35,0.37,0.4)$\succ$	$\prec$(0.3,0.4,0.5,0.6), (0.5,0.6,0.7,0.8)$\succ$
u_3	$\prec$(0.6,0.65,0.67,0.75), (0.2,0.3,0.35,0.4)$\succ$	$\prec$(0.8,0.85,0.9,0.95), (0.1,0.2,0.3,0.4)$\succ$	$\prec$(0.2,0.22,0.25,0.3), (0.6,0.65,0.7,0.75)$\succ$	$\prec$(0.8,0.85,0.88,0.9), (0.2,0.25,0.3,0.35)$\succ$	$\prec$(0.2,0.3,0.4,0.8), (0.2,0.3,0.5,0.6)$\succ$
u_4	$\prec$(0.7,0.8,0.85,0.9), (0.5,0.6,0.65,0.7)$\succ$	$\prec$(0.5,0.55,0.6,0.7), (0.2,0.3,0.4,0.7)$\succ$	$\prec$(0.3,0.35,0.4,0.45), (0.6,0.7,0.75,0.8)$\succ$	$\prec$(0.67,0.7,0.75,0.8), (0.3,0.4,0.5,0.6)$\succ$	$\prec$(0.1,0.2,0.3,0.4), (0.2,0.25,0.5,0.6)$\succ$
u_5	$\prec$(0.3,0.4,0.45,0.5), (0.2,0.25,0.3,0.5)$\succ$	$\prec$(0.6,0.65,0.7,0.75), (0.3,0.35,0.4,0.45)$\succ$	$\prec$(0.2,0.25,0.27,0.3), (0.7,0.75,0.8,0.8)$\succ$	$\prec$(0.6,0.7,0.8,0.9), (0.5,0.55,0.6,0.65)$\succ$	$\prec$(0.3,0.4,0.5,0.8), (0.1,0.2,0.4,0.5)$\succ$
u_6	$\prec$(0.6,0.65,0.67,0.7), (0.6,0.7,0.75,0.8)$\succ$	$\prec$(0.8,0.85,0.88,0.9), (0.1,0.15,0.2,0.25)$\succ$	$\prec$(0.1,0.2,0.3,0.35), (0.6,0.65,0.67,0.7)$\succ$	$\prec$(0.7,0.75,0.8,0.85), (0.1,0.2,0.3,0.4)$\succ$	$\prec$(0.1,0.2,0.7,0.8), (0.4,0.5,0.6,0.7)$\succ$

For $k = 1$, $(\alpha, \beta) = (1, 0)$. By Step 1, $J_1(\lambda(u_i, c_j))$ using Definition 19, for all $c_i \in CR$ and for all $u_i \in X$ is found and tabulated in Table 2. The bold letters are used in Table 2 to represent the equality of scores. From calculations of J_1, we observe that in many places $J_k^{'}s$ are not distinguishable for different bipolar fuzzy numbers. Therefore J_2 for $k = 2$, $(\alpha, \beta) = (0, -1)$, is determined, whenever required.

Table 2: J_1 for $k = 1$, $(\alpha, \beta) = (1, 0)$ and J_2 for $k = 2, (\alpha, \beta) = (0, -1)$.

	c_1	c_2	c_3	c_4	c_5		c_1	c_2	c_3	c_4	c_5
J_1						J_2					
u_1	0.7	1.55	**0.5**	1.35	**0.7**				1.35		0.7
u_2	0.75	1.45	0.65	1.65	**0.9**						1.3
u_3	**1.32**	1.75	0.47	1.73	**0.7**		0.65				0.8
u_4	1.65	1.15	0.75	1.45	0.5						
u_5	0.85	1.35	0.52	1.5	**0.9**						0.6
u_6	**1.32**	1.73	**0.5**	1.55	**0.9**		1.45		1.32		1.1

The degree of weighted fuzzy preeminence relation between any two objects using

$$WR_C(u_i, u_j) = \sum_{c \in G_C(u_i,u_j)} w_c + \frac{\sum\limits_{c \in E_C(u_i,u_j)} w_c}{2} + \sum_{c \in L_C(u_i,u_j)} w_c$$

is calculated and is tabulated in Table 3. For example, $G_C(u_2, u_6) = \{c_3, c_4\}$, $L_C(u_2, u_6) = \{c_1\}$ and $E_C(u_2, u_6) = \varnothing$ and hence $WR_C(u_2, u_6) = 0.57$

Table 3: Weighted fuzzy preeminence relation.

$WR_C(u_i, u_j)$	u_1	u_2	u_3	u_4	u_5	u_6
u_1	0.5	0.53	0.5	0.4	0.53	0.5
u_2	0.42	0.5	0.32	0.65	0.82	0.57
u_3	0.5	0.68	0.5	0.65	0.68	0.83
u_4	0.6	0.35	0.35	0.5	0.35	0.35
u_5	0.42	0.18	0.32	0.65	0.5	0.5
u_6	0.5	0.43	0.17	0.65	0.5	0.5

Now the entire preeminence of each object using $WR_C(u_i) = \frac{1}{|X|} \sum\limits_{j=1}^{|X|} WR_C(u_i, u_j)$ is tabulated in Table 4. Hence u_3 is selected as the best project proposal for weighted trapezoidal bipolar fuzzy information system.

Table 4: Total preeminence degree.

X	u_1	u_2	u_3	u_4	u_5	u_6
$R_C(u_i)$	0.493	0.54	0.64	0.42	0.43	0.46

EXAMPLE 11. Consider a selection problem of best event organizer from the five pre-evaluated organizers $X = \{u_i : i = 1, ..., 5\}$ based on trapezoidal bipolar fuzzy information with attributes $\{c_j : j = 1, ..., 5\}$ as Personality, Experience, Recommendations, Reputation and Credibility to Subject. All the criteria are benefit criteria and are considered as linguistic variables with linguistic values as Good=G, Medium Good=MG, Fair=F, Medium Poor=MP, Poor=P. The evaluation of alternatives with respect to each criterion is given in Table 5. Now considering the attributes as bipolar fuzzy linguistic variables, a trapezoidal bipolar fuzzy information system with domain of linguistic values considered as closed interval $[0, 1]$ is shown in Table 6, and the weights for each attribute w_c is given as $\{0.1, 0.25, 0.2, 0.2, 0.25\}$. J_1 using Definition 19 is found and tabulated in Table 7. The bold letters are used for equal scores in Table 7. Equal scores in this table represent the objects having same bipolar fuzzy information. The preeminence value for weighted fuzzy preeminence relation between any two objects is given in Table 8. Now the entire preeminence value of each object is tabulated in Table 9. Hence u_4 is selected as the best event organizer for weighted

trapezoidal bipolar fuzzy information system.

Table 5: Rating of alternatives in terms of linguistic values.

	c_1	c_2	c_3	c_4	c_5
u_1	G	MG	F	MG	F
u_2	MG	G	G	MG	G
u_3	MP	G	G	P	P
u_4	MG	G	G	G	G
u_5	F	F	MG	G	MG

Table 6: TrBFIS to evaluate alternatives with respect to criteria.

	c_1	c_2	c_3	c_4	c_5
u_1	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.6,0.7,0.7,0.8), (0.1,0.2,0.3,0.3)$\succ$	$\prec$(0.4,0.5,0.6,0.7), (0.5,0.5,0.6,0.7)$\succ$	$\prec$(0.6,0.7,0.7,0.8), (0.1,0.2,0.3,0.3)$\succ$	$\prec$(0.4,0.5,0.6,0.7), (0.5,0.5,0.6,0.7)$\succ$
u_2	$\prec$(0.6,0.7,0.7,0.8), (0.1,0.2,0.3,0.3)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.6,0.7,0.7,0.8), (0.1,0.2,0.3,0.3)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$
u_3	$\prec$(0.1,0.2,0.3,0.4), (0.6,0.7,0.8,0.8)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.0,0.0,0.1,0.2), (0.7,0.8,0.8,0.9)$\succ$	$\prec$(0.0,0.0,0.1,0.2), (0.7,0.8,0.8,0.9)$\succ$
u_4	$\prec$(0.6,0.7,0.7,0.8), (0.1,0.2,0.3,0.3)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$
u_5	$\prec$(0.4,0.5,0.6,0.7), (0.5,0.5,0.6,0.7)$\succ$	$\prec$(0.4,0.5,0.6,0.7), (0.5,0.5,0.6,0.7)$\succ$	$\prec$(0.6,0.7,0.7,0.8), (0.1,0.2,0.3,0.3)$\succ$	$\prec$(0.8,0.9,1,1), (0.0,0.1,0.2,0.2)$\succ$	$\prec$(0.6,0.7,0.7,0.8), (0.1,0.2,0.3,0.3)$\succ$

Table 7:J_1 for $(\alpha, \beta) = (1, 0)$.

	c_1	c_2	c_3	c_4	c_5
u_1	1.9	1.4	1.1	1.4	1.1
u_2	1.4	1.9	1.9	1.4	1.9
u_3	0.5	1.9	1.9	1	1
u_4	1.4	1.9	1.9	1.9	1.9
u_5	1.1	1.1	1.4	1.9	1.4

Table 8: Weighted bipolar fuzzy preeminence relation.

$WR_C(u_i, u_j)$	u_1	u_2	u_3	u_4	u_5
u_1	0.5	0.2	0.55	0.1	0.35
u_2	0.8	0.5	0.775	0.4	0.8
u_3	0.45	0.225	0.5	0.225	0.45
u_4	0.9	0.6	0.775	0.5	0.9
u_5	0.65	0.2	0.55	0.1	0.5

Table 9: Total preeminence degree.

X	u_1	u_2	u_3	u_4	u_5
$R_A(u_i)$	0.34	0.655	0.585	0.735	0.4

5 Conclusion

A wide variety of human decision making is based on double-sided or bipolar judgmental thinking on a positive side and a negative side. The notion of bipolar fuzzy sets was introduced by Zhang [25]. In this paper, a simple and flexible method for defining total orderings on the set of all bipolar fuzzy numbers is introduced. In this work, a pattern of choosing a double upper lower dense sequence in $[-1,0]\times[0,1]$, to order closed intervals, determining (α,β)-cuts of the given bipolar fuzzy numbers with respect to each term of the sequence and then comparing these bipolar fuzzy number on the basis of their (α,β)-cuts is suggested. The total ordering defined in the context can order bipolar fuzzy numbers, either alone or as a supplementary means with other ranking methods, and may be adopted in decision making with trapezoidal bipolar fuzzy information.

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