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Some Results In Partial Metric Space Using Auxiliary Functions*

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Abstract

In this paper, we establish some results for the existence and uniqueness of a fixed point for a certain type operators on partial metric spaces. Our results generalize well-known results in metric spaces. Also, we provide an example to illustrate our result.

1 Introduction and Preliminaries

In the past few years, the extension of the theory of fixed point to generalized structures as cone metric spaces, partial metric spaces and ordered metric spaces has received much attention (see, for instance, [1]–[19] and references cited therein).

In 1992, Matthews [11] introduced the notion of a partial metric space, which is a generalization of usual metric spaces in which the self-distance for any point need not be equal to zero. The partial metric space has wide applications in many branches of mathematics as well as in the field of computer domain and semantics. After this remarkable contribution, many authors focused on partial metric spaces and its topological properties.

DEFINITION 1.1. Let X be a non-empty set and $p : X \times X \to [0, \infty)$ satisfies

(i) $x = y \Leftrightarrow p(x, x) = p(y, y) = p(x, y)$,

(ii) $p(x, x) \le p(x, y)$,

(iii) $p(x, y) = p(y, x)$,

(iv) $p(x, y) \le p(x, z) + p(z, y) - p(z, z)$,

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for all x, y and $z \in X$. Then the pair (X, p) is called a partial metric space and p is called a partial metric on X.

It is clear that, if $p(x, y) = 0$, then $x = y$. But if $x = y$, $p(x, y)$ may not be 0. Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(x, \epsilon) : x \in X, \epsilon > 0\}$ where

$$B_p(x, \epsilon) = \{y \in X : p(x, y) < p(x, x) + \epsilon\}$$

for all $x \in X$ and $\epsilon > 0$. Similarly, closed p-ball is defined as

$$B_p[x, \epsilon] = \{y \in X : p(x, y) \leq p(x, x) + \epsilon\}.$$

If p is a partial metric on X, then the function $d_p : X \times X \to \mathbb{R}^+$ given by

$$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y) \tag{1}$$

is a (usual) metric on X.

EXAMPLE 1.1. Let I denote the set of all intervals $[a, b]$ for any real numbers $a \leq b$. Let $p : I \times I \to [0, \infty)$ be a function such that

$$p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}.$$

Then (I, p) is a partial metric space.

EXAMPLE 1.2. Let $X = \mathbb{R}$ and $p(x, y) = e^{\max\{x,y\}}$ for all $x, y \in X$. Then (X, p) is a partial metric space.

Some basic concepts on partial metric spaces are defined as follows:

DEFINITION 1.2 (See [11, 12]).

(i) A sequence $\{x_n\}$ in a partial metric space (X, p) converges to $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

(ii) A sequence $\{x_n\}$ in a partial metric space (X, p) is called a *Cauchy sequence* if and only if $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite.

(iii) A partial metric space (X, p) is said to be *complete* if every cauchy sequence $\{x_n\}$ $\in X$ converges, with respect to τ_p, to a point $x \in X$ such that

$$p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m).$$

(iv) A mapping $f : X \to X$ is said to be *continuous* at $x_0 \in X$, if for every $\epsilon > 0$, there exists $\delta > 0$ such that $f(B(x_0, \delta)) \subset B(f(x_0), \epsilon)$.

In this paper, we obtained some results for the existence and uniqueness of a fixed point for a certain type operators on partial metric spaces. Our results generalize well-known results in (usual) metric spaces. Also, we introduce an example to illustrate the usability of our result.

2 Main Results

To begin with we have the following lemmas of [12] and [13] which will be used in the sequel.

LEMMA 2.1 (see [12]). (i) A sequence $\{x_n\}$ is Cauchy in a partial metric space (X,p) if and only if $\{x_n\}$ is Cauchy in a metric space (X,d_p). (ii) A partial metric space (X,p) is complete if a metric space (X,d_p) is complete. i.e

$$\lim_{n\to\infty} d_p(x,x_n) = 0 \Leftrightarrow p(x,x) = \lim_{n\to\infty} p(x,x_n) = \lim_{n,m\to\infty} p(x_n,x_m).$$

LEMMA 2.2 (see [13]). Let (X,p) be a partial metric space.

(i) If $p(x,y)=0$, then $x=y$.

(ii) If $x\neq y$, then $p(x,y)>0$.

LEMMA 2.3 (see [13]). Let $x_n \to z$ as $n\to\infty$ in a partial metric space (X,p) where $p(z,z)=0$. Then $\lim_{n\to\infty} p(x_n,y) = p(z,y)$ for every $y\in X$.

Denote by Ψ the family of continuous and monotone nondecreasing functions $\psi : [0,\infty)\to[0,\infty)$ such that $\psi(t)=0$ if and only if $t=0$ and by Φ the family of lower semi-continuous functions $\varphi:[0,\infty)\to[0,\infty)$ such that $\varphi(t)=0$ if and only if $t=0$.

THEOREM 2.4. Let (X,d) be a complete partial metric space and $T:X\to X$ satisfy

$$\psi(p(Tx,Ty)) \le \psi(M(x,y)) - \varphi(N(x,y)), \quad \forall x,y\in X, \tag{2}$$

where $\varphi\in\Phi$, $\psi\in\Psi$,

$$M(x,y) = \max\left\{\frac{p(y,Ty)[1+p(x,Tx)]}{1+p(x,y)}, \frac{p(x,Tx)[1+p(x,Tx)]}{1+p(x,y)}, p(x,y)\right\}$$

and

$$N(x,y) = \max\left\{\frac{p(y,Ty)[1+p(x,Tx)]}{1+p(x,y)}, p(x,y)\right\}.$$

Then T has a unique fixed point.

PROOF. Let x_0 be an arbitrary point in X. We construct the sequence $\{x_n\}$ in X as follows: $x_{n+1}=Tx_n$, for $n\ge 0$. If there exit n such that $x_n = x_{n+1}$ then x_n is a fixed point of T. Now suppose that $x_n\neq x_{n+1}$, for all $n\ge0$. Letting $x=x_{n-1}$, $y=x_n$ in the equation (2) respectively we have

$$\psi(p(Tx_{n-1},Tx_n)) \le \psi(M(x_{n-1},x_n)) - \varphi(N(x_{n-1},x_n)),$$

where

$$\begin{aligned} M(x_{n-1},x_n) &= \max\left\{p(x_{n-1},x_n), \frac{p(x_n,Tx_n)[1+p(x_{n-1},Tx_{n-1})]}{1+p(x_{n-1},x_n)},\right. \\ &\qquad \left.\frac{p(x_{n-1},Tx_{n-1})[1+p(x_{n-1},Tx_{n-1})]}{1+p(x_{n-1},x_n)}\right\} \\ &= \max\left\{p(x_{n-1},x_n), \frac{p(x_n,x_{n+1})[1+p(x_{n-1},x_n)]}{1+p(x_{n-1},x_n)},\right. \\ &\qquad \left.\frac{p(x_{n-1},x_n)[1+p(x_{n-1},x_n)]}{1+p(x_{n-1},x_n)}\right\} \\ &= \max\{p(x_n,x_{n+1}),p(x_{n-1},x_n),p(x_{n-1},x_n)\} \\ &= \max\{p(x_{n+1},x_n),p(x_{n-1},x_n)\} \end{aligned}$$

and

$$\begin{aligned} N(x_{n-1},x_n) &= \max\left\{\frac{p(x_n,Tx_n)[1+p(x_{n-1},Tx_{n-1})]}{1+p(x_{n-1},x_n)}, p(x_{n-1},x_n)\right\} \\ &= \max\left\{\frac{p(x_n,x_{n+1})[1+p(x_{n-1},x_n)]}{1+p(x_{n-1},x_n)}, p(x_{n-1},x_n)\right\} \\ &= \max\{p(x_n,x_{n+1}),p(x_{n-1},x_n)\}. \end{aligned}$$

Hence we obtain

$$\begin{aligned} \psi(p(x_n,x_{n+1})) &\leq \psi(\max\{p(x_n,x_{n+1}),p(x_{n-1},x_n)\}) \\ &\quad -\varphi(\max\{p(x_n,x_{n+1}),p(x_{n-1},x_n)\}). \end{aligned} \tag{3}$$

If $p(x_n,x_{n+1}) > p(x_{n-1},x_n)$, then from equation (3), we have

$$\psi(p(x_n,x_{n+1})) \leq \psi(p(x_n,x_{n+1})) - \varphi(p(x_n,x_{n+1})) < \psi(p(x_n,x_{n+1})),$$

which is contradiction since $p(x_n,x_{n+1}) > 0$ by Lemma 2.2. So, we have $p(x_n,x_{n+1}) \leq p(x_{n-1},x_n)$, that is, $\{p(x_n,x_{n+1})\}$ is a non-increasing sequence of positive real numbers. Thus, there exists $L \geq 0$ such that

$$\lim_{n\to\infty}(p(x_n,x_{n+1})) = L. \tag{4}$$

Suppose that $L > 0$. Taking the lower limit in equation (3) as $n \to \infty$ and using (4) and the properties of ψ,φ, we have

$$\psi(L) \leq \psi(L) - \lim_{n\to\infty}\inf\varphi(p(x_{n-1},x_n)) \leq \psi(L) - \varphi(L) < \psi(L),$$

which is contradiction. Therefore,

$$\lim_{n\to\infty} p(x_n,x_{n+1}) = 0. \tag{5}$$

Using inequality (1), we have $d_p(x_n,x_{n+1}) \leq 2p(x_n,x_{n+1})$ and hence

$$d_p(x_n,x_{n+1}) = 0. \tag{6}$$

Now, we will prove that $\lim_{n,m\to\infty} p(x_n, x_m) = 0$. Suppose to the contrary that $\lim_{n,m\to\infty} p(x_n, x_m) \neq 0$. Then there exists $\epsilon > 0$ for which we can find two subsequences $\{x_{m(k)}\}$, $\{x_{n(k)}\}$ of $\{x_n\}$ such that $n(k)$ is the smallest index for which

$$n(k) > m(k) > k, p(x_n(k), x_m(k)) \geq \epsilon. \tag{7}$$

This implies

$$p(x_{n(k)-1}, x_{m(k)}) < \epsilon. \tag{8}$$

From (7) and (8), we have

$$\begin{aligned} \epsilon &\leq p(x_{n(k)}, x_{m(k)}) \leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) - p(x_{n(k)-1}, x_{n(k)-1}) \\ &\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) < \epsilon + p(x_{n(k)}, x_{n(k)-1}). \end{aligned}$$

Taking $k \to \infty$ and using (5), we get

$$\lim_{k\to\infty} p(x_{n(k)}, x_{m(k)}) = \epsilon. \tag{9}$$

By triangle inequality, we have

$$\begin{aligned} p(x_{n(k)}, x_{m(k)}) &\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) - p(x_{n(k)-1}, x_{n(k)-1}) \\ &\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)}) \\ &\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)-1}) + p(x_{m(k)-1}, x_{m(k)}) \\ &\quad -p(x_{m(k)-1}, x_{m(k)-1}) \\ &\leq p(x_{n(k)}, x_{n(k)-1}) + p(x_{n(k)-1}, x_{m(k)-1}) + p(x_{m(k)-1}, x_{m(k)}) \end{aligned}$$

and

$$\begin{aligned} p(x_{n(k)-1}, x_{m(k)-1}) &\leq p(x_{n(k)-1}, x_{n(k)}) + p(x_{n(k)}, x_{m(k)-1}) - p(x_{n(k)}, x_{n(k)}) \\ &\leq p(x_{n(k)-1}, x_{n(k)}) + p(x_{n(k)}, x_{m(k)-1}) \\ &\leq p(x_{n(k)-1}, x_{n(k)}) + p(x_{n(k)}, x_{m(k)}) + p(x_{m(k)}, x_{m(k)-1}) \\ &\quad -p(x_{m(k)}, x_{m(k)}) \\ &\leq p(x_{n(k)-1}, x_{n(k)}) + p(x_{n(k)}, x_{m(k)}) + p(x_{m(k)}, x_{m(k)-1}). \end{aligned}$$

Taking $k \to \infty$ in the above two inequalities and using (5) and (9), we get

$$\lim_{k\to\infty} p(x_{n(k)-1}, x_{m(k)-1}) = \epsilon. \tag{10}$$

Now from (2), we have

$$\begin{aligned} \psi(p(x_{m(k)}, x_{n(k)})) &= \psi(p(Tx_{m(k)-1}, Tx_{n(k)-1})) \\ &\leq \psi(M(x_{m(k)-1}, x_{n(k)-1})) - \varphi(N(x_{m(k)-1}, x_{n(k)-1})), \end{aligned} \tag{11}$$

where

$$M(x_{m(k)-1}, x_{n(k)-1}) = \max\left\{ \frac{p(x_{n(k)-1}, Tx_{n(k)-1})[1 + p(x_{m(k)-1}, Tx_{m(k)-1})]}{1 + p(x_{m(k)-1}, x_{n(k)-1})}, \right.$$

$$\frac{p(x_{m(k)-1},Tx_{m(k)-1})[1+p(x_{m(k)-1},Tx_{m(k)-1})]}{1+p(x_{m(k)-1},x_{n(k)-1})}$$
$$,p(x_{m(k)-1},x_{n(k)-1})\Big\}.$$

Taking limit as $k\to\infty$ and using (5),(9) and (10), we have

$$\lim_{k\to\infty} M(x_{m(k)-1},x_{n(k)-1})=\epsilon \tag{12}$$

and

$$\begin{aligned} N(x_{m(k)-1},x_{n(k)-1}) &= \max\left\{\frac{p(x_{n(k)-1},Tx_{n(k)-1})[1+p(x_{m(k)-1},Tx_{m(k)-1})]}{1+p(x_{m(k)-1},x_{n(k)-1})},\right.\\ &\qquad \left. p(x_{m(k)-1},x_{n(k)-1})\right\}. \end{aligned}$$

Taking limit as $k\to\infty$ and using (5),(9) and (10), we have

$$\lim_{k\to\infty} N(x_{m(k)-1},x_{n(k)-1})=\epsilon. \tag{13}$$

Now taking the lower limit when $k\to\infty$ in (11) and using (9) and (12), we have

$$\psi(\epsilon)\le\psi(\epsilon)-\liminf_{k\to\infty}\varphi(N(x_{m(k)-1},x_{n(k)-1}))\le\psi(\epsilon)-\varphi(\epsilon)<\psi(\epsilon),$$

which is contradiction. So, we have

$$\lim_{n,m\to\infty} p(x_n,x_m)=0.$$

Since $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite, we conclude that $\{x_n\}$ is a cauchy sequence in (X,p). Using (1), we have $d_p(x_n,x_m)\le 2p(x_n,x_m)$, therefore,

$$\lim_{n,m\to\infty} d_p(x_n,x_m)=0. \tag{14}$$

Thus by Lemma 2.1, $\{x_n\}$ is a cauchy sequence in both (X,d_p) and (X,p). Since (X,p) is a complete partial metric space then there exist $x\in X$ such that $\lim_{n\to\infty} p(x_n,x)=p(x,x)$. Since $\lim_{n,m\to\infty} p(x_n,x_m)=0$, then again by using Lemma 2.1, we have $p(x,x)=0$. Now, we will prove that x is a fixed point of T. Suppose that $Tx\neq x$.

From the inequality (2) and using Lemma 2.3, we have

$$\begin{aligned} \psi(p(x_n,Tx)) &= \psi(p(Tx_{n-1},Tx)\\ &\le \psi\left(\max\left\{p(x_{n-1},x),\frac{p(x,Tx)[1+p(x_{n-1},Tx_{n-1})]}{1+p(x_{n-1},x)},\right.\right.\\ &\qquad \left.\left.\frac{p(x_{n-1},Tx_{n-1})[1+p(x_{n-1},Tx_{n-1})]}{1+p(x_{n-1},x)}\right\}\right)\\ &\qquad -\varphi\left(\max\left\{\frac{p(x,Tx)[1+p(x_{n-1},Tx_{n-1})]}{1+p(x_{n-1},x)},p(x_{n-1},x)\right\}\right). \end{aligned} \tag{15}$$

Letting $n \to \infty$ in the above inequality and regarding the property of φ, ψ, we obtain

$$\psi(p(x,Tx)) \leq \psi(p(x,Tx)) - \varphi(p(x,Tx)) < \psi(p(x,Tx)). \tag{16}$$

Then

$$\psi(p(x,Tx)) < \psi(p(x,Tx)),$$

which is contradiction. Thus $Tx = x$.

Finally, we shall prove the uniqueness of fixed point. Suppose that y is another fixed point of T such that $x \neq y$. From (2), we have

$$\begin{aligned} \psi(p(x,y)) &= \psi(p(Tx,Ty)) \leq \psi(M(x,y)) - \varphi(N(x,y)) \\ &\leq \psi(p(x,y)) - \varphi(p(x,y)) \\ &< \psi(p(x,y)), \end{aligned}$$

which is contradiction since $p(x,y) > 0$. Hence $x = y$.

COROLLARY 2.5 (see [13]). Let (X,d) be a complete partial metric space and $T : X \to X$ satisfies

$$\psi(p(Tx,Ty)) \leq \psi(N(x,y)) - \varphi(N(x,y)), \quad \forall x,y \in X, \tag{17}$$

where $\varphi \in \Phi$, $\psi \in \Psi$, and

$$N(x,y) = \max\left\{\frac{p(y,Ty)[1+p(x,Tx)]}{1+p(x,y)}, p(x,y)\right\}.$$

Then T has a unique fixed point.

Taking ψ to be the identity mapping and $\varphi(t) = (1-k)t$ for all $t \geq 0$, where $k \in (0,1)$, we have the following result.

COROLLARY 2.6. Let (X,d) be a complete partial metric space and $T : X \to X$ satisfy

$$p(Tx,Ty) \leq k \max\left\{\frac{p(y,Ty)[1+p(x,Tx)]}{1+p(x,y)}, p(x,y)\right\} \tag{18}$$

for each $x,y \in X$ and $k \in (0,1)$. Then T has a unique fixed point.

EXAMPLE 2.7 Consider $X = [0,1]$ and $p(x,y) = max\{x,y\}$, then (X,p) is a partial metric space. Suppose $T : X \to X$ such that $Tx = \frac{x^2}{k+x}$ for all $x \in X$ and $\varphi(t), \psi(t) : [0,\infty) \to [0,\infty)$, $\varphi(t) = \frac{t}{k+t}$ and $\psi(t) = rt$, where $k, r \in \mathbb{N}$ without loss of generality assume that $x \geq y$. Then we have

$$p(Tx,Ty) = \max\left\{\frac{x^2}{k+x}, \frac{y^2}{k+y}\right\} = \frac{x^2}{k+x},$$

$$M(x,y) = \max\left\{\frac{p(y,Ty)[1+p(x,Tx)]}{1+p(x,y)}, \frac{p(x,Tx)[1+p(x,Tx)]}{1+p(x,y)}, p(x,y)\right\}$$

$$= \max\left\{\frac{y(1+x)}{1+x}, \frac{x(1+x)}{1+x}, x\right\} = \max\{y, x, x\} = x,$$

and

$$\begin{aligned} N(x,y) &= \max\left\{\frac{p(y,Ty)[1+p(x,Tx)]}{1+p(x,y)}, p(x,y)\right\} = \max\left\{\frac{y(1+x)}{1+x}, x\right\} \\ &= \max\{y,x\} = x. \end{aligned}$$

Therefore

$$\psi(p(Tx,Ty) = \frac{rx^2}{k+x}$$

and

$$\psi(M(x,y)) - \varphi(N(x,y)) = \frac{rx^2}{k+x} + \frac{(rk-1)x}{k+x}.$$

Following cases arise:

Case 1. If $r = k = 1$ then $\psi(p(Tx,Ty)) = \psi(M(x,y)) - \varphi(N(x,y))$.
Case 2. If $r, k > 1$ then $\psi(p(Tx,Ty)) < \psi(M(x,y)) - \varphi(N(x,y))$.

Thus it satisfies all the conditions of Theorem 2.4. Hence, T has a unique fixed point, indeed $x = 0$ is the required point.

3 Remarks

Das et al. [10] proved a fixed point theorem for rational type mappings in complete metric spaces. Cabrera et al. [3] extend the result of Das et al. [10] in the context of metric spaces endowed with a partial order. Using the auxiliary functions, Chandok et al. [5] generalize some of the results of [3] in the framework of metric spaces endowed with a partial order. Theorem 2.4. generalize and extend the result of Chandok et al. [5] in a space having non-zero self distance, that is, partial metric space.

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