Scalar Wave Scattering By Two-Layer Radial Inhomogeneities*

Umaporn Nuntaplook†, John Adam‡

Received 6 June 2014

Abstract

It is shown that the iteration technique gives a better approximation for the problem with long wavelengths.

1 Introduction

This paper is based on the study of the scattering of a scalar plane wave by an inhomogeneous medium which can be applied in many interests such as scattering by inhomogeneous spheres and scattering of acoustic waves in the ocean. Since the analogous classical problems with scattering by spherically symmetric inhomogeneities have not been thoroughly studied, the purpose of this paper is to show that the classical problems and some simple solvable problems can be simply treated by a quantum-mechanical method. Moreover, in practice (in optical and industrial applications at least) the inhomogeneous scattering media will be piecewise constant continuous, so a useful approach to the problem may be to mimic the continuous cases for piecewise increasing or decreasing refractive index profiles. Therefore, the application of the Jost function formulation of potential scattering theory [1] to the scattering of a scalar plane wave by a medium with a piecewise constant two-layer spherical inhomogeneity is of interest.

2 Scattering From a Piecewise Constant by Multi-Layer Spherically Symmetric Inhomogeneities

We are now in a position to apply the method outlined in [1] to the problem of scattering from a piecewise constant in a multi-layer spherical inhomogeneities. For a three-layer inhomogeneity we define the following potential

\begin{align*}
\text{Region 1} : V(r) &= -V_1, k(r) = k_1, r < R_1; \\
\text{Region 2} : V(r) &= -V_2, k(r) = k_2, R_1 < r < R_2; \\
\text{Region 3} : V(r) &= 0, k(r) = k, r > R_2.
\end{align*}

*(Mathematics Subject Classifications: 35C20, 35D10.
†Department of Mathematics, Faculty of Science, Mahidol University, Bangkok, Thailand
‡Department of Mathematics, Old Dominion University, Norfolk, Virginia, USA)
The solutions in the three regions are:

Region 1: \( u^{(1)}_{\lambda-\frac{1}{2}}(k_1, r) = r[A j_{\lambda-\frac{1}{2}}(k_1 r) + B y_{\lambda-\frac{1}{2}}(k_1 r)] \);

Region 2: \( u^{(2)}_{\lambda-\frac{1}{2}}(k_2, r) = r[C j_{\lambda-\frac{1}{2}}(k_2 r) + D y_{\lambda-\frac{1}{2}}(k_2 r)] \);

Region 3: \( u^{(3)}_{\lambda-\frac{1}{2}}(k, r) = r[E h^{(1)}_{\lambda-\frac{1}{2}}(k r) + F h^{(2)}_{\lambda-\frac{1}{2}}(k r)] \).

where again \( j_{\lambda-\frac{1}{2}}(k_1 r) \), \( y_{\lambda-\frac{1}{2}}(k_2 r) \), \( h^{(1)}_{\lambda-\frac{1}{2}}(k r) \), and \( h^{(2)}_{\lambda-\frac{1}{2}}(k r) \) are spherical Bessel, Neumann, and Hankel functions of the first kind and second kind, respectively.

Choosing \( u^{(1)}_{\lambda-\frac{1}{2}}(k_1 r) \) to be \( \phi_1(\lambda, k_1, r) \) and imposing the boundary conditions at \( r = 0 \) (see [1, (13)]), we find that \( B = 0 \) and

\[
\phi_1(\lambda, k_1, r) = 2^{\lambda+\frac{1}{2}} \pi^{-\frac{1}{2}} k_1^{-\lambda+\frac{1}{2}} \Gamma(\lambda + 1) r j_{\lambda-\frac{1}{2}}(k_1 r)
\]

and

\[
\phi'_1(\lambda, k_1, r) = 2^{\lambda+\frac{1}{2}} \pi^{-\frac{1}{2}} k_1^{-\lambda+\frac{1}{2}} \Gamma(\lambda + 1) \times [j_{\lambda-\frac{1}{2}}(k_1 r) + k_1 r j'_{\lambda-\frac{1}{2}}(k_1 r)],
\]

where the prime denotes differentiation with respect to the argument of the function, \( \Gamma \) is the gamma function, and we have used the following series representation for \( j_{\lambda-\frac{1}{2}}(k_1 r) \) [Handbook of Mathematical Functions (McGraw Hill Book, p. 263)]:

\[
j_{\lambda-\frac{1}{2}}(k_1 r) = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{\frac{1}{2}} (k_1 r/2)^{\lambda+2n-\frac{1}{2}}}{2^n \Gamma(\lambda + n + 1)}, \quad \lambda - \frac{1}{2} \neq -1, -2, -3, \ldots.
\]

Choosing \( u^{(2)}_{\lambda-\frac{1}{2}}(k_2 r) \) to be \( \phi_2(\lambda, k_2, r) \) and imposing the continuity at the boundary \( r = R_1 \) by matching the continuity of \( \phi_1 \) with \( \phi_2 \) and \( \phi'_1 \) with \( \phi'_2 \), we have

\[
\phi_2(\lambda, k_2, r) = r \left[ C j_{\lambda-\frac{1}{2}}(k_2 r) + D y_{\lambda-\frac{1}{2}}(k_2 r) \right]
\]

and

\[
\phi'_2(\lambda, k_2, r) = C \left[ j'_{\lambda-\frac{1}{2}}(k_2 r) + k_2 r j''_{\lambda-\frac{1}{2}}(k_2 r) \right] + D \left[ y'_{\lambda-\frac{1}{2}}(k_2 r) + k_2 r y''_{\lambda-\frac{1}{2}}(k_2 r) \right],
\]

where

\[
C = -\frac{m \left( j_{\lambda-\frac{1}{2}}(k_1 R_1)y'_{\lambda-\frac{1}{2}}(k_2 R_1) - k_2 j'_{\lambda-\frac{1}{2}}(k_1 R_1)y_{\lambda-\frac{1}{2}}(k_2 R_1) \right)}{j'_{\lambda-\frac{1}{2}}(k_2 R_1)y_{\lambda-\frac{1}{2}}(k_2 R_1) - j_{\lambda-\frac{1}{2}}(k_2 R_1)y'_{\lambda-\frac{1}{2}}(k_2 R_1)},
\]

\[
D = \frac{m \left( j'_{\lambda-\frac{1}{2}}(k_1 R_1)j''_{\lambda-\frac{1}{2}}(k_2 R_1) - k_2 j''_{\lambda-\frac{1}{2}}(k_1 R_1)j'_{\lambda-\frac{1}{2}}(k_2 R_1) \right)}{j'_{\lambda-\frac{1}{2}}(k_2 R_1)y_{\lambda-\frac{1}{2}}(k_2 R_1) - j_{\lambda-\frac{1}{2}}(k_2 R_1)y'_{\lambda-\frac{1}{2}}(k_2 R_1)},
\]

and

\[
m = 2^{\lambda+\frac{1}{2}} \pi^{-\frac{1}{2}} k_1^{-\lambda+\frac{1}{2}} \Gamma(\lambda + 1).
\]
Choosing $u^{(3)}_{\lambda - \frac{1}{2}}(k, r)$ to be $f(\lambda, k, r)$ and imposing the Jost solution condition at infinity (see \cite[(20)]{1}) we find that $E = 0$, $F = ke^{-i\frac{\pi}{2}(\lambda + \frac{1}{2})}$, and hence

$$f(\lambda, k, r) = ke^{-i\frac{\pi}{2}(\lambda + \frac{1}{2})}rh^{(2)}_{\lambda - \frac{1}{2}}(kr)$$

and

$$f'(\lambda, k, r) = ke^{-i\frac{\pi}{2}(\lambda + \frac{1}{2})}\left[h^{(2)}_{\lambda - \frac{1}{2}}(kr) + krh^{(2)'}_{\lambda - \frac{1}{2}}(kr)\right],$$

where we have used the following asymptotic form for $h^{(2)}_{\lambda - \frac{1}{2}}(kr)$:

$$\lim_{kr \to \infty} h^{(2)}_{\lambda - \frac{1}{2}}(kr) = \frac{1}{kr}e^{-i\left[kr - \frac{\pi}{2}(\lambda + \frac{1}{2})\right]}.$$ 

Since the point $r = R_2$ is the common domain of $\phi_2(\lambda, k_2, r)$ and $f(\lambda, k, r)$, we evaluate the Jost function at $r = R_2$ and thus obtain

$$f(\lambda, k) = W[f(\lambda, k, r), \phi_2(\lambda, k_2, r)]_{r=R_2} = f(\lambda, k, r)\phi_2(\lambda, k_2, r) - f'(\lambda, k, r)\phi_2(\lambda, k_2, r) =$$

$$= \frac{2^{\lambda + \frac{1}{2}}\pi^{-\frac{1}{2}}\Gamma(\lambda + 1)k_1^{-\lambda + \frac{1}{2}}ke^{-i\frac{\pi}{2}(\lambda + \frac{1}{2})}R_2^2}{j_{\lambda - \frac{1}{2}}(k_2R_1)j_{\lambda - \frac{1}{2}}(k_2R_1) - j_{\lambda - \frac{1}{2}}(k_2R_1)j_{\lambda - \frac{1}{2}}(k_2R_1)}$$

$$\times \left\{h^{(2)}_{\lambda - \frac{1}{2}}(kR_2)k_2 \left[a_1j_{\lambda - \frac{1}{2}}(k_1R_1) + \frac{k_1}{k_2}a_3j_{\lambda - \frac{1}{2}}(k_1R_1)\right] -$$

$$-h^{(2)'}_{\lambda - \frac{1}{2}}(kR_2)k \left[a_2j_{\lambda - \frac{1}{2}}(k_1R_1) + \frac{k_1}{k_2}a_4j_{\lambda - \frac{1}{2}}(k_1R_1)\right]\right\}. \quad (2)$$

We also have that

$$f(\lambda, -k) = \frac{2^{\lambda + \frac{1}{2}}\pi^{-\frac{1}{2}}\Gamma(\lambda + 1)k_1^{-\lambda + \frac{1}{2}}ke^{-i\frac{\pi}{2}(\lambda + \frac{1}{2})}R_2^2e^{i\pi(\lambda - \frac{1}{2})}}{j_{\lambda - \frac{1}{2}}(k_2R_1)j_{\lambda - \frac{1}{2}}(k_2R_1) - j_{\lambda - \frac{1}{2}}(k_2R_1)j_{\lambda - \frac{1}{2}}(k_2R_1)}$$

$$\times \left\{-h^{(1)}_{\lambda - \frac{1}{2}}(kR_2)k_2 \left[a_1j_{\lambda - \frac{1}{2}}(k_1R_1) + \frac{k_1}{k_2}a_3j_{\lambda - \frac{1}{2}}(k_1R_1)\right] +$$

$$+h^{(1)'}_{\lambda - \frac{1}{2}}(kR_2)k \left[a_2j_{\lambda - \frac{1}{2}}(k_1R_1) + \frac{k_1}{k_2}a_4j_{\lambda - \frac{1}{2}}(k_1R_1)\right]\right\},$$

where we have used the following identities:

$$h^{(2)}_{\lambda - \frac{1}{2}}(kre^{i\pi}) = h^{(2)}_{\lambda - \frac{1}{2}}(-kr) = (-1)^{\lambda - \frac{1}{2}}h^{(1)}_{\lambda - \frac{1}{2}}(kr) = e^{i\pi(\lambda - \frac{1}{2})}h^{(1)}_{\lambda - \frac{1}{2}}(kr)$$

and

$$h^{(2)'}_{\lambda - \frac{1}{2}}(-kr) = (-1)^{\lambda + \frac{1}{2}}h^{(1)'}_{\lambda - \frac{1}{2}}(kr) = e^{i\pi(\lambda + \frac{1}{2})}h^{(1)'}_{\lambda - \frac{1}{2}}(kr) = -e^{i\pi(\lambda - \frac{1}{2})}h^{(1)'}_{\lambda - \frac{1}{2}}(kr),$$

where $\lambda - \frac{1}{2} = 0, 1, 2, \ldots$. The $S$-matrix is then given by

$$S(\lambda, k) = -\left\{kh^{(2)'}_{\lambda - \frac{1}{2}}(kR_2) \left[a_2j_{\lambda - \frac{1}{2}}(k_1R_1) + \frac{k_1}{k_2}a_4j_{\lambda - \frac{1}{2}}(k_1R_1)\right]\right\}.$$
where we have used the following relations:

$$a_1 = j_{\frac{\lambda - 1}{2}}(k_2 R_1) y_{\frac{\lambda - 1}{2}}(k_2 R_2) - y_{\frac{\lambda - 1}{2}}(k_2 R_1) j_{\frac{\lambda - 1}{2}}(k_2 R_2),$$

$$a_2 = j_{\frac{\lambda - 1}{2}}(k_2 R_1) y_{\frac{\lambda - 1}{2}}(k_2 R_2) - j_{\frac{\lambda - 1}{2}}(k_2 R_2) y_{\frac{\lambda - 1}{2}}(k_2 R_1),$$

$$a_3 = y_{\frac{\lambda - 1}{2}}(k_2 R_1) j_{\frac{\lambda - 1}{2}}(k_2 R_2) - j_{\frac{\lambda - 1}{2}}(k_2 R_1) y_{\frac{\lambda - 1}{2}}(k_2 R_2),$$

$$a_4 = j_{\frac{\lambda - 1}{2}}(k_2 R_1) y_{\frac{\lambda - 1}{2}}(k_2 R_2) - y_{\frac{\lambda - 1}{2}}(k_2 R_1) j_{\frac{\lambda - 1}{2}}(k_2 R_2).$$

We can calculate the Jost function for $\lambda = \frac{1}{2}$ from (2):

$$f \left( \frac{1}{2}, k \right) = \frac{1}{4} e^{-ikR_2} \left\{ \left[ \frac{k - ik_2}{k_1} + \frac{ik + k_2}{k_2} \right] e^{ik_1(R_2 - R_1)} \right. \right.

$$+ \left( \frac{k + ik_2}{k_1} + \frac{k_2 - ik}{k_2} \right) e^{-ik_2(R_2 - R_1)} + \left[ \left( \frac{ik + k_2}{k_2} - \frac{k - ik}{k_1} \right) e^{ik_2(R_2 - R_1)} \right. \right.

$$+ \left( \frac{k_2 - ik}{k_2} - \frac{k + ik_2}{k_1} \right) e^{-ik_2(R_2 - R_1)} \right\} e^{-iR_1},$$

(3)

where we have used the following relations:

$$j_0(kR) = \frac{\sin kR}{kR}, \quad j'_0(kR) = \frac{\cos kR}{kR} - \frac{\sin kR}{(kR)^2},$$

$$h_0^{(2)}(kR) = \frac{-e^{-ikR}}{ikR}, \quad \text{and} \quad h_0^{(2)'}(kR) = \frac{e^{-ikR}}{kR} \left( 1 + \frac{1}{iR} \right).$$

### 3 The Jost Integral Equation for $\lambda = \frac{1}{2}$ and Some Approximate Solutions for The Three-Layer Model

We now apply the method in section I [1] to the case of scattering from a piecewise constant by multi-layer spherical inhomogeneity. We have already calculated $f \left( \frac{1}{2}, k \right)$ exactly in equation (3). Elsewhere we use the exact solution of the Jost function to check for the accuracy of the iteration procedure. If we assume there is an $R$ such that $V(r) = 0$ for $r > R$ (certainly true in optics!) and let

$$g \left( \frac{1}{2}, k, r \right) = e^{ikr} f \left( \frac{1}{2}, k, r \right),$$

(4)
then (24) in [1] becomes the Jost integral equation for $\lambda = \frac{1}{2}$:

$$g\left(\frac{1}{2}, k, r\right) = 1 + (2ik)^{-1} \int_r^R [1 - e^{2ik(r-r')} V(r') g\left(\frac{1}{2}, k, r'\right)] dr'$$

$$= 1 - V_1(2ik)^{-1} \int_r^{R_1} [1 - e^{2ik(r-r')} V(r') g\left(\frac{1}{2}, k, r'\right)] dr',$$

(5)

for the potential defined by region 1 in equation (1). Next we write the solution of (5) as a perturbation expansion

$$g\left(\frac{1}{2}, k, r\right) = \sum_{n=0}^{\infty} g_n \left(\frac{1}{2}, k, r\right),$$

where

$$g_0 \left(\frac{1}{2}, k, r\right) = 1$$

and

$$g_n \left(\frac{1}{2}, k, r\right) = 1 + (2ik)^{-1} \int_r^R [1 - e^{2ik(r-r')} V(r') g_{n-1} \left(\frac{1}{2}, k, r'\right)] dr'.$$

From (13) in [1], we have

$$\lim_{r \to 0} \phi \left(\frac{1}{2}, k, r\right) = 0 \quad \text{and} \quad \lim_{r \to 0} \frac{\phi' \left(\frac{1}{2}, k, r\right)}{dr} = 1.$$  \hspace{1cm} (6)

$f\left(\frac{1}{2}, k, r\right)$ and $f'\left(\frac{1}{2}, k, r\right)$ are finite and we can evaluate $f\left(\frac{1}{2}, k\right)$ at $r = 0$ using (6), thus obtaining the useful relation

$$f \left(\frac{1}{2}, k\right) = f \left(\frac{1}{2}, k, 0\right) = g \left(\frac{1}{2}, k, 0\right).$$

The first iteration $g_1\left(\frac{1}{2}, k, 0\right)$ of (5) is

$$g_1 \left(\frac{1}{2}, k, 0\right)$$
$$= g_0 \left(\frac{1}{2}, k, 0\right) + g_1 \left(\frac{1}{2}, k, 0\right)$$
$$= 1 - \frac{1}{4} \left\{ \left[ \frac{k_1}{k} \right]^2 - 1 \right\} (1 - \cos 2kR_1) + \left[ 1 - \left( \frac{k_2}{k} \right)^2 \right] (\cos 2kR_2 - \cos 2kR_1)$$
$$+ \frac{i}{2} \left\{ \left[ \frac{k_1}{k} \right]^2 - 1 \right\} \left[ kR_1 - \frac{1}{2} \sin 2kR_1 \right] + \left[ 1 - \left( \frac{k_2}{k} \right)^2 \right] \left[ k(R_2 - R_1) \right.$$
$$- \frac{1}{2} (\sin 2kR_2 - \sin 2kR_1) \right\}.$$
The second iteration $g_{II}(\frac{1}{2}, k, 0)$ is

$$g_{II}\left(\frac{1}{2}, k, 0\right)$$

$$= g_0\left(\frac{1}{2}, k, 0\right) + g_1\left(\frac{1}{2}, k, 0\right) + g_2\left(\frac{1}{2}, k, 0\right)$$

$$= 1 + \frac{1}{4} \left\{ \left[ \left(\frac{k_1}{k}\right)^2 - 1 \right] (\cos 2kR_1 - 1) + \left[ 1 - \left(\frac{k_2}{k}\right)^2 \right] (\cos 2kR_2 - \cos 2kR_1) \right\}$$

$$- \frac{1}{8} \left\{ \left[ \left(\frac{k_1}{k}\right)^2 - 1 \right]^2 kR_1 (kR_1 + \sin 2kR_1) + \frac{3}{2} (\cos 2kR_1 - 1) \right\}$$

$$+ \left[ \left(\frac{k_1}{k}\right)^2 - 1 \right] \left[ 1 - \left(\frac{k_2}{k}\right)^2 \right] \left[ k(R_2 - R_1) (2kR_1 - k(R_2 - R_1) - \sin 2kR_1) \right.$$

$$- (\cos 2k(R_2 - R_1) - 1) + \frac{3}{2} (\cos 2kR_2 - \cos 2kR_1)$$

$$+ k(R_2 - R_1) (\sin 2kR_2 + \sin 2kR_1) + kR_1 (\sin 2kR_2 - \sin 2kR_1) \right]$$

$$+ \left[ 1 - \left(\frac{k_2}{k}\right)^2 \right]^2 \left[ k(R_2 - R_1)[2k(R_2 - R_1) - \sin 2k(R_2 - R_1) \right.$$

$$+ \sin 2kR_2 - \sin 2kR_1 + (\cos 2k(R_2 - R_1) - 1) \right]\}$$

$$+ i \left\{ \frac{1}{2} \left\{ \left[ \left(\frac{k_1}{k}\right)^2 - 1 \right] (kR_1 - \frac{1}{2} \sin 2kR_1 \right.$$

$$+ \left[ 1 - \left(\frac{k_2}{k}\right)^2 \right] \left[ k(R_2 - R_1) - \frac{1}{2} (\sin 2kR_2 - \sin 2kR_1) \right] \right\}\}$$

$$- \frac{1}{8} \left\{ \left[ \left(\frac{k_1}{k}\right)^2 - 1 \right]^2 kR_1 (\cos 2kR_1 + 2) - \frac{3}{2} \sin 2kR_1 \right\}$$

$$+ \left[ \left(\frac{k_1}{k}\right)^2 - 1 \right] \left[ 1 - \left(\frac{k_2}{k}\right)^2 \right] \left[ kR_1 (\cos 2kR_2 - \cos 2kR_1) \right.$$

$$+ k(R_2 - R_1) (\cos 2kR_2 + \cos 2kR_1)$$

$$- k(R_2 - R_1) (\cos 2kR_1 - 1) - \frac{3}{2} (\sin 2kR_2 - \sin 2kR_1) \right]$$

$$- \left[ 1 - \left(\frac{k_2}{k}\right)^2 \right]^2 \left[ k(R_2 - R_1) (\cos 2k(R_2 - R_1) - \cos 2kR_2 + \cos 2kR_1) \right] \right\}. \right\}$$

For real $\lambda$ and $k$, we have $f(\lambda, -k) = f^*(\lambda, k),}$
and therefore

\[
\frac{\sigma_0}{\pi R_1^2} = \left| 1 - e^{2i\beta(k,R)} \right|^2 = \left| 1 - \frac{f\left(\frac{1}{2},k\right)}{f\left(\frac{1}{2},k\right)} \right|^2 \frac{1}{(kR_1)^2},
\]

where \( \sigma_0 \) is the \( l = 0 \) total cross section. The accuracy of these approximations as functions of \( k, k_1 \) and \( k_2 \) will be reported elsewhere.

4 Jost Integral Equation Formulation for Arbitrary \( \lambda \): 3-Layer Model

In case of scattering from a piecewise constant spherical inhomogeneity, the two integral equations (60) and (61) in [1] become:

\[
\phi(\lambda, k, r) = r^{\lambda + \frac{1}{2}} + \frac{1}{2} \lambda^{-1} \int_0^{R_1} \left[ (\xi/r)^\lambda - (r/\xi)^\lambda \right] \times (r/\xi)^\frac{1}{2}[k^2 + V_1] \phi(\lambda, k, \xi)d\xi 
+ \frac{1}{2} \lambda^{-1} \int_{R_1}^{R_2} \left[ (\xi/r)^\lambda - (r/\xi)^\lambda \right] \times (r/\xi)^\frac{1}{2}[k^2 + V_2] \phi(\lambda, k, \xi)d\xi 
+ \frac{1}{2} \lambda^{-1} \int_R^{R_2} \left[ (\xi/r)^\lambda - (r/\xi)^\lambda \right][k^2 d\xi] \phi(\lambda, k, \xi)d\xi 
+ \frac{1}{2} \lambda^{-1} \int_{R_2}^{\infty} \left[ (\xi/r)^\lambda - (r/\xi)^\lambda \right][k^2 d\xi] \phi(\lambda, k, \xi)d\xi 
\]

\[
f(\lambda, k, r) = e^{-ikr} + k^{-1} \int_r^{R_1} [\sin k(r' - r)] \left[ -V_1 + \frac{\lambda^2 - \frac{1}{4}}{r'} \right] f(\lambda, k, r')dr' 
+ k^{-1} \int_{R_1}^{R_2} [\sin k(r' - r)] \left[ -V_2 + \frac{\lambda^2 - \frac{1}{4}}{r'} \right] f(\lambda, k, r')dr' 
+ k^{-1} \int_{R_2}^{\infty} [\sin k(r' - r)] \left( \frac{\lambda^2 - \frac{1}{4}}{r'^2} \right) f(\lambda, k, r')dr'.
\]

5 Summary

This iterative technique may be most useful when the scattering system is more complicated than those discussed here. By comparing the present formulation with the numerical results obtained for a constant spherical inhomogeneity [1], it appears that the iteration technique is good for problems with long wavelengths \((kR_1 << 1)\) for any \( k_1/k \). For shorter wavelengths, small \( k_1/k \) (e.g., \( k_1/k = 1.1 \)) gives a good approximation to \( \sigma_0 \) for the entire range of \( kR_1 \) considered \((0 \leq R_1 \leq 2\pi)\); however, large \( k_1/k \) (e.g., \( k_1/k = 1.5, 2.0 \)) gives a good approximation to \( \sigma_0 \) in the range of \( 0 < kR_1 < 3\pi/4 \). In case of a piecewise constant spherical inhomogeneity, the iteration procedure gives a better approximation for the problem with long wavelengths \((kR_1 << 1)\) only for small
ratios of $k_1/k$ and $k_2/k$ (e.g., $k_1/k = 0.7$, $k_2/k = 0.9$; $k_1/k = 1.1$, $k_2/k = 1.3$). For a larger $k_1/k$ and $k_2/k$ (e.g., $k_1/k = 1.5$, $k_2/k = 1.2$), it gives a good approximation when $kR_1 < 2\pi/3$. The approximation for the Jost function becomes less accurate for larger ratios of wavenumber $k_1/k$ and $k_2/k$ (e.g., $k_1/k = 1.0$, $k_2/k = 1.5$). When the ratios of wavenumbers $k_1/k$ is greater than $k_2/k$, we have a better approximation. However, the approximation for the Jost function is still better than the total cross section for the large wavelengths. For shorter wavelengths, all ratios of the wavenumbers give a better approximation to $\sigma_0$ for approximately $kR_1 > 2\pi/3$ [4]. These results will be reported in more detail elsewhere.

Acknowledgements. This research is supported by full scholarship from the Commission on Higher Education Congress: University Staff Development Consortium of Thailand, 2006–2012.

References


