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Coincidence Points And Common Fixed Points For Expansive Type Mappings In Cone b-Metric Spaces*

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Abstract

In this paper we prove coincidence point and common fixed point results for mappings satisfying some expansive type contractions in the setting of a cone b-metric space. Our results improve and supplement some recent results in the literature. Some examples are also provided to illustrate our results.

1 Introduction and Preliminaries

Metric fixed point theory is playing an increasing role in mathematics because of its wide range of applications in applied mathematics and sciences. There has been a number of generalizations of the usual notion of a metric space. One such generalization is a b-metric space introduced and studied by Bakhtin [3] and Czerwik [4]. In [6], Huang and Zhang introduced the concept of cone metric spaces as a generalization of metric spaces and proved some fixed point theorems for contractive mappings that extend certain results of fixed points in metric spaces. Recently, Hussain and Shah [7] introduced the concept of cone b-metric spaces as a generalization of b-metric spaces and cone metric spaces. There are many related works about the fixed point of contractive mappings (see, for example [1, 5, 10]). The aim of this work is to obtain sufficient conditions for existence of points of coincidence and common fixed points for a pair of self mappings satisfying some expansive type conditions in cone b-metric spaces.

We need to recall some basic notations, definitions, and necessary results from existing literature. Let E be a real Banach space and θ denote the zero vector of E. A cone P is a subset of E such that

(i) P is closed, nonempty and $P \neq \{\theta\}$,

(ii) $ax + by \in P$ for $a, b \in \mathbb{R}$, $a, b \geq 0$, $x, y \in P$,

(iii) $P \cap (-P) = \{\theta\}$.

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For any cone $P \subseteq E$, we can define a partial ordering $\preceq$ on E with respect to P by $x \preceq y$ (equivalently, $y \succeq x$) if and only if $y - x \in P$. We shall write $x \prec y$ (equivalently, $y \succ x$) if $x \preceq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in int(P)$, where $int(P)$ denotes the interior of P. The cone P is called normal if there is a number $k > 0$ such that for all $x, y \in E$,

$$\theta \preceq x \preceq y \text{ implies } \|x\| \leq k \|y\|.$$

The least positive number satisfying the above inequality is called the normal constant of P. Throughout this paper, we suppose that E is a real Banach space, P is a cone in E with $int(P) \neq \emptyset$ and $\preceq$ is a partial ordering on E with respect to P.

DEFINITION 1.1 ([6]). Let E be a real Banach space with cone P and let X be a nonempty set. Suppose the mapping $d : X \times X \to E$ satisfies

(i) $\theta \preceq d(x,y)$ for all $x, y \in X$ and $d(x,y) = \theta$ if and only if $x = y$,

(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$,

(iii) $d(x,y) \preceq d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X, and (X, d) is called a cone metric space.

DEFINITION 1.2 ([7]). Let X be a nonempty set and E a real Banach space with cone P. A vector valued function $d : X \times X \to E$ is said to be a cone b-metric function on X with the constant $s \geq 1$ if the following conditions are satisfied:

(i) $\theta \preceq d(x,y)$ for all $x, y \in X$ and $d(x,y) = \theta$ if and only if $x = y$,

(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$,

(iii) $d(x,y) \preceq s\,(d(x,z) + d(z,y))$ for all $x, y, z \in X$.

The pair (X, d) is called a cone b-metric space.

Observe that if $s = 1$, then the ordinary triangle inequality in a cone metric space is satisfied, however it does not hold true when $s > 1$. Thus the class of cone b-metric spaces is effectively larger than that of the ordinary cone metric spaces. That is, every cone metric space is a cone b-metric space, but its converse need not be true. The following examples illustrate these facts.

EXAMPLE 1.3 ([7]). Let $X = \{-1, 0, 1\}$, $E = \mathbb{R}^2$, $P = \{(x,y) : x \geq 0, y \geq 0\}$. Define $d : X \times X \to P$ by $d(x,y) = d(y,x)$ for all $x, y \in X$, $d(x,x) = \theta$, $x \in X$ and $d(-1,0) = (3,3)$, $d(-1,1) = d(0,1) = (1,1)$. Then (X, d) is a cone b-metric space, but not a cone metric space since the triangle inequality is not satisfied. Indeed, we have

$$d(-1,1) + d(1,0) = (1,1) + (1,1) = (2,2) \prec (3,3) = d(-1,0).$$

It is easy to verify that $s = \frac{3}{2}$.

EXAMPLE 1.4 ([8]). Let $E = \mathbb{R}^2$, $P = \{(x, y) : x \geq 0, y \geq 0\} \subseteq E$, $X = \mathbb{R}$ and $d : X \times X \to E$ such that $d(x, y) = (| x - y |^p, \alpha | x - y |^p)$ where $\alpha \geq 0$ and $p > 1$ are two constants. Then (X, d) is a cone b-metric space with $s = 2^{p-1}$, but not a cone metric space.

DEFINITION 1.5 ([7]). Let (X, d) be a cone b-metric space, $x \in X$ and (x_n) be a sequence in X. Then

(i) (x_n) converges to x whenever, for every $c \in E$ with $\theta \ll c$, there is a natural number n_0 such that for all $n > n_0$, $d(x_n, x) \ll c$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ $(n \to \infty)$;

(ii) (x_n) is a Cauchy sequence whenever, for every $c \in E$ with $\theta \ll c$, there is a natural number n_0 such that $d(x_n, x_m) \ll c$ for all $n, m > n_0$;

(iii) (X, d) is a complete cone b-metric space if every Cauchy sequence is convergent.

REMARK 1.6 ([7]). Let (X, d) be a cone b-metric space over the ordered real Banach space E with a cone P. Then the following properties are often used:

(i) If $a \preceq b$ and $b \ll c$, then $a \ll c$.

(ii) If $a \ll b$ and $b \ll c$, then $a \ll c$.

(iii) If $\theta \preceq u \ll c$ for each $c \in int(P)$, then $u = \theta$.

(iv) If $c \in int(P)$, $\theta \preceq a_n$ and $a_n \to \theta$, then there exists n_0 such that for all $n > n_0$ we have $a_n \ll c$.

(v) Let $\theta \ll c$. If $\theta \preceq d(x_n, x) \preceq b_n$ and $b_n \to \theta$, then eventually $d(x_n, x) \ll c$, where (x_n), x are a sequence and a given point in X.

(vi) If $\theta \preceq a_n \preceq b_n$ and $a_n \to a$, $b_n \to b$, then $a \preceq b$, for each cone P.

(vii) If E is a real Banach space with cone P and if $a \preceq \lambda a$ where $a \in P$ and $0 \leq \lambda < 1$, then $a = \theta$.

(viii) $\alpha\, int(P) \subseteq int(P)$ for $\alpha > 0$.

(ix) For each $\delta > 0$ and $x \in int(P)$ there is $0 < \gamma < 1$ such that $\| \gamma x \| < \delta$.

(x) For each $\theta \ll c_1$ and $c_2 \in P$, there is an element $\theta \ll d$ such that $c_1 \ll d$ and $c_2 \ll d$.

(xi) For each $\theta \ll c_1$ and $\theta \ll c_2$, there is an element $\theta \ll e$ such that $e \ll c_1$ and $e \ll c_2$.

DEFINITION 1.7. Let (X, d) be a cone b-metric space and let $T : X \to X$ be a given mapping. We say that T is continuous at $x_0 \in X$ if $Tx_n \to Tx_0$ as $n \to \infty$ for every sequences (x_n) in X satisfying $x_n \to x_0$ as $n \to \infty$. If T is continuous at each point $x_0 \in X$, then we say that T is continuous on X.

DEFINITION 1.8. Let (X, d) be a cone b-metric space with the constant $s \geq 1$. A mapping $T : X \to X$ is called expansive if there exists a real constant $k > s$ such that

$$d(Tx, Ty) \succeq k\, d(x, y) \text{ for all } x, y \in X.$$

DEFINITION 1.9 ([2]). Let T and S be self mappings of a set X. If $y = Tx = Sx$ for some x in X, then x is called a coincidence point of T and S and y is called a point of coincidence of T and S.

DEFINITION 1.10 ([9]). The mappings $T, S : X \to X$ are weakly compatible, if for every $x \in X$, the following holds:

$$T(Sx) = S(Tx) \text{ whenever } Sx = Tx.$$

PROPOSITION 1.11 ([2]). Let S and T be weakly compatible selfmaps of a nonempty set X. If S and T have a unique point of coincidence $y = Sx = Tx$, then y is the unique common fixed point of S and T.

2 Main Results

In this section, we prove point of coincidence and common fixed point results in cone b-metric spaces.

THEOREM 2.1. Let (X, d) be a cone b-metric space with the constant $s \geq 1$. Suppose the mappings f, $g : X \to X$ satisfy $g(X) \subseteq f(X)$, either $f(X)$ or $g(X)$ is complete, and

$$d(fx, fy) \succeq \alpha\, d(gx, gy) + \beta\, d(fx, gx) + \gamma\, d(fy, gy) \text{ for all } x, y \in X, \quad (1)$$

where α, β, γ are nonnegative real numbers with $\alpha + \beta + \gamma > s$, $\beta < 1$ and $\alpha \neq 0$. Then f and g have a point of coincidence in X. Moreover, if $\alpha > 1$, then the point of coincidence is unique. If f and g are weakly compatible and $\alpha > 1$, then f and g have a unique common fixed point in X.

PROOF. Let $x_0 \in X$ and choose $x_1 \in X$ such that $gx_0 = fx_1$. This is possible since $g(X) \subseteq f(X)$. Continuing this process, we can construct a sequence (x_n) in X such that $fx_n = gx_{n-1}$, for all $n \geq 1$. By (1), we have

$$\begin{aligned} d(gx_{n-1}, gx_n) &= d(fx_n, fx_{n+1}) \\ &\succeq \alpha d(gx_n, gx_{n+1}) + \beta d(fx_n, gx_n) + \gamma d(fx_{n+1}, gx_{n+1}) \\ &= \alpha d(gx_n, gx_{n+1}) + \beta d(gx_{n-1}, gx_n) + \gamma d(gx_n, gx_{n+1}) \end{aligned}$$

which gives that

$$d(gx_n, gx_{n+1}) \preceq \lambda d(gx_{n-1}, gx_n)$$

where $\lambda = \frac{1-\beta}{\alpha+\gamma}$. It is easy to see that $\lambda \in (0, \frac{1}{s})$. By induction, we get that

$$d(gx_n, gx_{n+1}) \preceq \lambda^n d(gx_0, gx_1) \tag{2}$$

for all $n \geq 0$. Let $m, n \in \mathbb{N}$ with $m > n$. Then, by using condition (2) we have

$$\begin{aligned}
d(gx_n, gx_m) &\preceq s\left[d(gx_n, gx_{n+1}) + d(gx_{n+1}, gx_m)\right] \\
&\preceq sd(gx_n, gx_{n+1}) + s^2 d(gx_{n+1}, gx_{n+2}) + \cdots \\
&\quad + s^{m-n-1}\left[d(gx_{m-2}, gx_{m-1}) + d(gx_{m-1}, gx_m)\right] \\
&\preceq \left[s\lambda^n + s^2\lambda^{n+1} + \cdots + s^{m-n-1}\lambda^{m-2} + s^{m-n-1}\lambda^{m-1}\right] d(gx_0, gx_1) \\
&\preceq \left[s\lambda^n + s^2\lambda^{n+1} + \cdots + s^{m-n-1}\lambda^{m-2} + s^{m-n}\lambda^{m-1}\right] d(gx_0, gx_1) \\
&= s\lambda^n \left[1 + s\lambda + (s\lambda)^2 + \cdots + (s\lambda)^{m-n-2} + (s\lambda)^{m-n-1}\right] d(gx_0, gx_1) \\
&\preceq \frac{s\lambda^n}{1 - s\lambda} d(gx_0, gx_1). \qquad (3)
\end{aligned}$$

It is to be noted that $\frac{s\lambda^n}{1-s\lambda} d(gx_0, gx_1) \to \theta$ as $n \to \infty$. Let $\theta \ll c$ be given. Then we can find $m_0 \in \mathbb{N}$ such that

$$\frac{s\lambda^n}{1 - s\lambda} d(gx_0, gx_1) \ll c \text{ for each } n > m_0.$$

Therefore, it follows from (3) that

$$d(gx_n, gx_m) \preceq \frac{s\lambda^n}{1 - s\lambda} d(gx_0, gx_1) \ll c \text{ for all } m > n > m_0.$$

So (gx_n) is a Cauchy sequence in $g(X)$. Suppose that $g(X)$ is a complete subspace of X. Then there exists $y \in g(X) \subseteq f(X)$ such that $gx_n \to y$ and also $fx_n \to y$. In case, $f(X)$ is complete, this holds also with $y \in f(X)$. Let $u \in X$ be such that $fu = y$. For $\theta \ll c$, one can choose a natural number $n_0 \in \mathbb{N}$ such that $d(y, gx_n) \ll \frac{c}{2s}$ and $d(fx_n, fu) \ll \frac{\alpha c}{2s}$ for all $n > n_0$. By (1), we have

$$\begin{aligned}
d(gx_{n-1}, fu) &= d(fx_n, fu) \\
&\succeq \alpha d(gx_n, gu) + \beta d(fx_n, gx_n) + \gamma d(fu, gu) \\
&\succeq \alpha d(gx_n, gu).
\end{aligned}$$

If $\alpha \neq 0$, then

$$d(gx_n, gu) \preceq \frac{1}{\alpha} d(gx_{n-1}, fu).$$

Therefore,

$$\begin{aligned}
d(y, gu) &\preceq s[d(y, gx_n) + d(gx_n, gu)] \\
&\preceq s[d(y, gx_n) + \frac{1}{\alpha} d(gx_{n-1}, fu)] \\
&= s[d(y, gx_n) + \frac{1}{\alpha} d(fx_n, fu)] \\
&\ll c, \ \textit{for all } n > n_0.
\end{aligned}$$

This gives that $d(y, gu) = \theta$, i.e., $gu = y$ and hence $fu = gu = y$. Therefore, y is a point of coincidence of f and g.

Now we suppose that $\alpha > 1$. Let v be another point of coincidence of f and g. So $fx = gx = v$ for some $x \in X$. Then

$$d(y, v) = d(fu, fx) \succeq \alpha d(gu, gx) + \beta d(fu, gu) + \gamma d(fx, gx) = \alpha d(y, v),$$

which implies that

$$d(y, v) \preceq \frac{1}{\alpha} d(y, v).$$

By Remark 1.6(vii), we have $d(v, y) = \theta$ i.e., $v = y$. Therefore, f and g have a unique point of coincidence in X. If f and g are weakly compatible, then by Proposition 1.11, f and g have a unique common fixed point in X. The proof is complete.

COROLLARY 2.2. Let (X, d) be a cone b-metric space with the constant $s \geq 1$. Suppose the mappings $f, g : X \to X$ satisfy the condition

$$d(fx, fy) \succeq \alpha d(gx, gy) \text{ for all } x, y \in X,$$

where $\alpha > s$ is a constant. If $g(X) \subseteq f(X)$ and $f(X)$ or $g(X)$ is complete, then f and g have a unique point of coincidence in X. Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point in X.

PROOF. It follows by taking $\beta = \gamma = 0$ in Theorem 2.1.

The following corollary is the Theorem 2.1 [8].

COROLLARY 2.3. Let (X, d) be a complete cone b-metric space with the constant $s \geq 1$. Suppose the mapping $g : X \to X$ satisfies the contractive condition

$$d(gx, gy) \preceq \lambda\, d(x, y) \text{ for all } x, y \in X,$$

where $\lambda \in [0, \frac{1}{s})$ is a constant. Then g has a unique fixed point in X. Furthermore, the iterative sequence $(g^n x)$ converges to the fixed point.

PROOF. It follows by taking $\beta = \gamma = 0$ and $f = I$, the identity mapping on X, in Theorem 2.1.

COROLLARY 2.4. Let (X, d) be a complete cone b-metric space with the constant $s \geq 1$. Suppose the mapping $f : X \to X$ is onto and satisfies

$$d(fx, fy) \succeq \alpha d(x, y) \text{ for all } x, y \in X,$$

where $\alpha > s$ is a constant. Then f has a unique fixed point in X.

PROOF. Taking $g = I$ and $\beta = \gamma = 0$ in Theorem 2.1, we obtain the desired result.

REMARK 2.5. Corollary 2.4 gives a sufficient condition for the existence of unique fixed point of an expansive mapping in cone b-metric spaces.

COROLLARY 2.6. Let (X, d) be a complete cone b-metric space with the constant $s \geq 1$. Suppose the mapping $f : X \to X$ is onto and satisfies the condition

$$d(fx, fy) \succeq \alpha\, d(x, y) + \beta\, d(fx, x) + \gamma\, d(fy, y) \text{ for } x, y \in X,$$

where α, β, γ are nonnegative real numbers with $\alpha \neq 0,\ \beta < 1,\ \alpha + \beta + \gamma > s$. Then f has a fixed point in X. Moreover, if $\alpha > 1$, then the fixed point of f is unique.

PROOF. It follows by taking $g = I$ in Theorem 2.1.

THEOREM 2.7. Let (X, d) be a complete cone b-metric space with the constant $s \geq 1$. Suppose the mappings S, $T : X \to X$ satisfy the following conditions:

$$d(T(Sx), Sx) + \frac{k}{s} d(T(Sx), x) \succeq \alpha d(Sx, x) \tag{4}$$

and

$$d(S(Tx), Tx) + \frac{k}{s} d(S(Tx), x) \succeq \beta d(Tx, x) \tag{5}$$

for all $x \in X$, where α, β, k are nonnegative real numbers with $\alpha > s + (1 + s)k$ and $\beta > s + (1 + s)k$. If S and T are continuous and surjective, then S and T have a common fixed point in X.

PROOF. Let $x_0 \in X$ be arbitrary and choose $x_1 \in X$ such that $x_0 = Tx_1$. This is possible since T is surjective. Since S is also surjective, there exists $x_2 \in X$ such that $x_1 = Sx_2$. Continuing this process, we can construct a sequence (x_n) in X such that $x_{2n} = Tx_{2n+1}$ and $x_{2n-1} = Sx_{2n}$ for all $n \in \mathbb{N}$. Using (4), we have for $n \in \mathbb{N} \cup \{0\}$

$$d(T(Sx_{2n+2}), Sx_{2n+2}) + \frac{k}{s} d(T(Sx_{2n+2}), x_{2n+2}) \succeq \alpha d(Sx_{2n+2}, x_{2n+2})$$

which implies that

$$d(x_{2n}, x_{2n+1}) + \frac{k}{s} d(x_{2n}, x_{2n+2}) \succeq \alpha d(x_{2n+1}, x_{2n+2}).$$

Hence, we have

$$\alpha d(x_{2n+1}, x_{2n+2}) \preceq d(x_{2n}, x_{2n+1}) + kd(x_{2n}, x_{2n+1}) + kd(x_{2n+1}, x_{2n+2}).$$

Therefore,

$$d(x_{2n+1}, x_{2n+2}) \preceq \frac{1 + k}{\alpha - k} d(x_{2n}, x_{2n+1}). \tag{6}$$

Using (5) and by an argument similar to that used above, we obtain that

$$d(x_{2n}, x_{2n+1}) \preceq \frac{1 + k}{\beta - k} d(x_{2n-1}, x_{2n}). \tag{7}$$

Let $\lambda = max\left(\frac{1+k}{\alpha-k}, \frac{1+k}{\beta-k}\right)$. It is easy to see that $\lambda \in (0, \frac{1}{s})$. Then, by combining (6) and (7), we get

$$d(x_n, x_{n+1}) \preceq \lambda d(x_{n-1}, x_n) \tag{8}$$

for all $n \geq 1$. By repeated application of (8), we obtain

$$d(x_n, x_{n+1}) \preceq \lambda^n d(x_0, x_1).$$

By an argument similar to that used in Theorem 2.1, it follows that (x_n) is a Cauchy sequence in X. Since X is complete, there exists $u \in X$ such that $x_n \to u$ as $n \to \infty$. Now, $x_{2n+1} \to u$ and $x_{2n} \to u$ as $n \to \infty$. The continuity of S and T imply that $Tx_{2n+1} \to Tu$ and $Sx_{2n} \to Su$ as $n \to \infty$ i.e., $x_{2n} \to Tu$ and $x_{2n-1} \to Su$ as $n \to \infty$. The uniqueness of limit yields that $u = Su = Tu$. Hence, u is a common fixed point of S and T. The proof is complete.

COROLLARY 2.8. Let (X, d) be a complete cone b-metric space with the constant $s \geq 1$. Let $T : X \to X$ be a continuous surjective mapping such that

$$d(T^2x, Tx) + \frac{k}{s} d(T^2x, x) \succeq \alpha d(Tx, x) \text{ for all } x \in X,$$

where α, k are nonnegative real numbers with $\alpha > s + (1 + s)k$. Then T has a fixed point in X.

PROOF. It follows from Theorem 2.7 by taking $S = T$ and $\beta = \alpha$.

We conclude this paper with the following two examples.

EXAMPLE 2.9. Let $E = \mathbb{R}^2$, the Euclidean plane and $P = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0\}$ a cone in E. Let $X = [0, 1]$ and $p > 1$ be a constant. We define $d : X \times X \to E$ as

$$d(x, y) = (| x - y |^p, | x - y |^p) \text{ for all } x, y \in X.$$

Then (X, d) is a cone b-metric space with the constant $s = 2^{p-1}$. Let us define $f, g : X \to X$ as $fx = \frac{x}{3}$ and $gx = \frac{x}{9} - \frac{x^2}{27}$ for all $x \in X$. Then, for every $x, y \in X$ one has $d(fx, fy) \succeq 3^p d(gx, gy)$ i.e., the condition (1) holds for $\alpha = 3^p, \beta = \gamma = 0$. Thus, we have all the conditions of Theorem 2.1 and $0 \in X$ is the unique common fixed point of f and g.

EXAMPLE 2.10. Let $E = \mathbb{R}^2$ and $P = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0\}$ a cone in E. Let $X = [0, \infty)$. We define $d : X \times X \to E$ as

$$d(x, y) = (| x - y |^2, | x - y |^2) \text{ for all } x, y \in X.$$

Then (X, d) is a complete cone b-metric space with the constant $s = 2$. Let us define $S, T : X \to X$ as $Sx = 3x$ and $Tx = 4x$ for all $x \in X$. Then, the conditions (4) and (5) hold for $\alpha = \beta = 3 + 3k > s + (1 + s)k$, where k is a nonnegative real number. We see that all hypotheses of Theorem 2.7 are satisfied and $0 \in X$ is a common fixed point of S and T.

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