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Deficiency And Relative Deficiency Of E-Valued Meromorphic Functions*

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Abstract

The purpose of this paper is to discuss the deficiency and the relative deficiency of an E-valued meromorphic function f. Some inequalities on the deficiency and the relative deficiency of E-valued meromorphic functions are obtained. Results obtained extend some recent results by Wu and Xuan.

1 E-Valued Meromorphic Function

In this section, we shall introduce some fundamental definitions and notations of E-valued meromorphic functions (see e.g. [1, 2, 6, 7, 8]) which will be used in this paper.

Let $(E, \|\cdot\|)$ be a complex Banach space with Schauder basis $\{e_j\}$ and the norm $\|\cdot\|$. Then an E-valued function $f(z)$ defined in $C_R = \{|z| < R\}, 0 < R \leq +\infty, C_{+\infty} = \mathbb{C}$ can be written as $f(z) = (f_1(z), f_2(z), ..., f_k(z), ...) \in E$, where

$$f_1(z), f_2(z), ..., f_k(z), ...$$

are the component functions of $f(z)$. An E-valued function is called holomorphic (meromorphic) if all $f_j(z)$ are holomorphic (some of $f_j(z)$ are meromorphic). The j-th derivative of $f(z)$, where $j = 1, 2, ...$, is defined by

$$f^{(j)}(z) = (f_1^{(j)}(z), f_2^{(j)}(z), ..., f_k^{(j)}(z), ...).$$

We assume that $f^{(0)}(z) = f(z)$.

In this paper, we use the letters from the alphabet $a, b, c, ...$ to denote the elements of E which are called vectors. The symbol 0 denotes the zero vector of E. We denote vector infinity, complex number infinity, and the norm infinity by $\widehat{\infty}, \infty$, and $+\infty$, respectively.

Let $f(z)$ be an E-valued meromorphic function in C_R $(0 < R \leq +\infty)$. A point $z_0 \in C_R = \{|z| < r\}$ is called a pole of $f(z) = (f_1(z), f_2(z), ..., f_k(z), ...)$ if z_0 is a pole

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of at least one of the component functions $f_k(z)$ $(k = 1, 2, ...)$. We denote $\|f(z)\| = +\infty$ when z_0 is a pole. A point $z_0 \in C_R$ is called a zero of $f(z) = (f_1(z), f_2(z), ..., f_k(z), ...)$ if z_0 is a zero of all the component functions $f_k(z)$ $(k = 1, 2, ...)$. For any vector $a \in E$, A point $z_0 \in C_R$ is called an a-point of $f(z)$ if and only if z_0 a zero of $f(z) - a$.

Let $n(r, f)$ denote the number of poles of $f(z)$ in $|z| \leq r$, and $n(r, a)$ denote the number of a-points of $f(z)$ in $|z| \leq r$, counting with multiplicities. We define the volume function of $f(z)$ by

$$V(r, \widehat{\infty}, f) = V(r, f) = \frac{1}{2\pi} \int_{C_r} \log \left| \frac{r}{\xi} \right| \Delta \log \|f(\xi)\| dx \wedge dy,\ \xi = x + iy,$$

$$V(r, a) = V(r, a, f) = \frac{1}{2\pi} \int_{C_r} \log \left| \frac{r}{\xi} \right| \Delta \log \|f(\xi) - a\| dx \wedge dy,\ \xi = x + iy,$$

and the counting function of finite or infinite a-points by

$$N(r, f) = n(0, f) \log r + \int_0^r \frac{n(t, f) - n(0, f)}{t} dt$$

and

$$N(r, a) = n(0, a) \log r + \int_0^r \frac{n(t, a) - n(0, a)}{t} dt,$$

respectively. Next, we define

$$m(r, f) = m(r, \widehat{\infty}, f) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ \left\| f(re^{i\theta}) \right\| d\theta,$$

$$m(r, a) = m(r, a, f) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ \frac{1}{\|f(re^{i\theta}) - a\|} d\theta$$

and

$$T(r, f) = m(r, f) + N(r, f),$$

where $T(r, f)$ is called the Nevanlinna characteristic function. Let $\overline{n}(r, f)$ denote the number of poles of $f(z)$ in $|z| \leq r$, and $\overline{n}(r, a)$ denote the number of a-points of $f(z)$ in $|z| \leq r$, ignoring multiplicities. Similarly, we can define the counting function $\overline{N}(r, f)$ and $\overline{N}(r, a)$ of $\overline{n}(r, f)$ and $\overline{n}(r, a)$.

Let $f(z)$ be an E-valued meromorphic function in the whole complex plane. For any vector $a \in E$, we define the Nevanlinna deficiency $\delta(a) = \delta(a, f)$ and $\Theta(a) = \Theta(a, f)$ as following

$$\delta(a) = \delta(a, f) = \liminf_{r \to +\infty} \frac{m(r, a)}{T(r, f)} = 1 - \limsup_{r \to +\infty} \frac{V(r, a) + N(r, a)}{T(r, f)},$$

$$\delta(\widehat{\infty}) = \delta(\widehat{\infty}, f) = \liminf_{r \to +\infty} \frac{m(r, f)}{T(r, f)} = 1 - \limsup_{r \to +\infty} \frac{N(r, f)}{T(r, f)},$$

$$\Theta(a) = \Theta(a, f) = 1 - \limsup_{r \to +\infty} \frac{V(r, a) + \overline{N}(r, a)}{T(r, f)}$$

and

$$\Theta(\widehat{\infty}) = \Theta(\widehat{\infty}, f) = 1 - \limsup_{r\to+\infty} \frac{\overline{N}(r,f)}{T(r,f)}.$$

If $\delta(a) > 0$, we call a is a deficient vector of f.

If $f(z)$ is an E-valued meromorphic function in the whole complex plane, then the order and the lower order of $f(z)$ are defined by

$$\lambda(f) = \limsup_{r\to\infty} \frac{\log T(r,f)}{\log r} \text{ and } \mu(f) = \liminf_{r\to\infty} \frac{\log T(r,f)}{\log r}.$$

We call the E-valued meromorphic function f admissible if

$$\limsup_{r\to+\infty} \frac{T(r,f)}{\log r} = +\infty.$$

2 Lemmas

In this section, we shall introduce some fundamental results which will be used in the proof of the main results of this paper. For convenience, we give the following definitions.

DEFINITION 2.1 ([2]). For an E-valued meromorphic function $f(z)$, we denote by $S(r,f)$ any quantity such that

$$S(r,f) = O(\log T(r,f) + \log r),\ r \to +\infty$$

or

$$S(r,f) = o(T(r,f)),\ r \to +\infty$$

without restriction if $f(z)$ is of finite order and otherwise except possibly for a set of values of r of finite linear measure.

DEFINITION 2.2 ([2]) Let E_n be an n-dimensional projective space of E with a basis $\{e_j\}_1^n$. The projective operator $P_n : E \to E_n$ is a realization of E_n associated to the basis. An E-valued meromorphic function $f(z)$ in $C_R(0 < R \le +\infty)$ is of compact projection, if for any given $\varepsilon > 0$, $\|P_n(f(z)) - f(z)\| < \varepsilon$ has sufficiently large n in any fixed compact subset $D \subset C_R$.

LEMMA 2.3 ([2]). Let $f(z)$ be a nonconstant E-valued meromorphic function in C_R. Then for $0 < r < R$, $a \in E$, $f(z) \not\equiv a$,

$$T(r,f) = V(r,a) + N(r,a) + m(r,a) + \log^+ \|c_q(a)\| + \varepsilon(r,a).$$

Here $\varepsilon(r,a)$ is a function such that

$$|\varepsilon(r,a)| \le \log^+ \|a\| + \log 2, \varepsilon(r,0) \equiv 0$$

and $c_q(a) \in E$ is the coefficient of the first term in the Laurent series at the point a.

LEMMA 2.4 ([2]). Let $f(z)$ be a nonconstant E-valued meromorphic function of compact projection in $\mathbb{C}$ and $a^{[k]} \in E$ $(k = 1, 2, ..., q)$ be $q \geq 3$ distinct points. Then for $0 < r < R$,

$$(q-2)T(r,f) + G(r,f) \leq \sum_{k=1}^{q} [V(r,a^{[k]}) + \overline{N}(r,a^{[k]})] + S(r,f)$$

and

$$(q-1)T(r,f) + G(r,f) \leq \sum_{k=1}^{q} [V(r,a^{[k]}) + \overline{N}(r,a^{[k]})] + \overline{N}(r,f) + S(r,f)$$

where

$$G(r,f) = \int_0^r \frac{1}{2\pi} dt \int_{C_r} \Delta \log \|f(\xi)\| dx \wedge dy, \ \xi = x + iy.$$

LEMMA 2.5 ([6]). If an E-valued meromorphic function $f(z)$ in $\mathbb{C}$ is of compact projection, then for a positive integer k, we have

$$S(r, f^{(k)}) = S(r,f)$$

and

$$\frac{1}{2\pi} \int_0^{2\pi} \log^+ \frac{\|f^{(k)}(re^{i\theta})\|}{\|f(re^{i\theta})\|} d\theta = S(r,f).$$

3 Deficiency of E-Valued Meromorphic Function

In 1992, Lahiri [3] proved the following theorem on vector valued transcendental integral function connecting the deficiencies of the function with its derivative.

THEOREM A. Let $f(z) = (f_1(z), f_2(z), ..., f_n(z))$ be a vector-valued transcendental integral function of finite order. Then

$$\sum_{a \in \mathbb{C}^n} \delta(a) \leq \delta(0, f').$$

Extending Theorem A recently, Wu and Xuan [6] prove the following result.

THEOREM B. Let $f(z)$ be a finite order admissible E-valued meromorphic function of compact projection in $\mathbb{C}$ and assume $\delta(\widehat{\infty}) = 1$. Then

$$\sum_{a \in E} \delta(a) \leq \delta(0, f').$$

We extend the above result to the higher derivatives as follows.

THEOREM 3.1. Let $f(z)$ be a finite order admissible E-valued meromorphic function of compact projection in $\mathbb{C}$ and assume $\delta(\widehat{\infty}) = 1$. Then for any positive integer k, we have

$$\sum_{a \in E} \delta(a) \leq \delta(0, f^{(k)}).$$

PROOF. For any $q \geq 2$ vectors $\{a^{[\mu]}\}$ in E, put

$$F(z) = \sum_{j=1}^{q} \frac{1}{\|f(z) - a^{[j]}\|}.$$

We can get

$$\frac{1}{2\pi} \int_0^{2\pi} \log^+ F(re^{i\theta}) d\theta \leq m\left(r, 0, f^{(k)}\right) + \frac{1}{2\pi} \int_0^{2\pi} \log^+ \{F(re^{i\theta}) \|f^{(k)}(re^{i\theta})\|\} d\theta. \quad (1)$$

By Wu and Xuan [6], we have

$$\frac{1}{2\pi} \int_0^{2\pi} \log^+ F(re^{i\theta}) d\theta \geq \sum_{\mu=1}^{q} m(r, a^{[\mu]}) - \log^+ \frac{2q}{\delta}. \quad (2)$$

So that by (1) and (2), we can get

$$\sum_{\mu=1}^{q} m(r, a^{[\mu]}) \leq m\left(r, 0, f^{(k)}\right) + \frac{1}{2\pi} \int_0^{2\pi} \log^+ \{F(re^{i\theta}) \|f^{(k)}(re^{i\theta})\|\} d\theta + \log^+ \frac{2q}{\delta}.$$

Thus by Lemma 2.5, we have

$$\sum_{\mu=1}^{q} m(r, a^{[\mu]}) \leq m\left(r, 0, f^{(k)}\right) + S(r, f). \quad (3)$$

It follows from Lemma 2.3 that

$$m\left(r, 0, f^{(k)}\right) + N\left(r, 0, f^{(k)}\right) + V\left(r, 0, f^{(k)}\right) = T(r, f^{(k)}) + O(1). \quad (4)$$

Thus from (3) and (4), we deduce

$$\sum_{\mu=1}^{q} m(r, a^{[\mu]}) + N\left(r, 0, f^{(k)}\right) + V\left(r, 0, f^{(k)}\right) \leq T\left(r, f^{(k)}\right) + S(r, f).$$

Therefore, we have

$$\frac{N\left(r, 0, f^{(k)}\right) + V\left(r, 0, f^{(k)}\right)}{T(r, f^{(k)})} + \frac{T(r, f)}{T(r, f^{(k)})} \left(\frac{\sum\limits_{\mu=1}^{q} m(r, a^{[\mu]})}{T(r, f)} - o(1) \right) \leq 1. \quad (5)$$

Now, basic estimates in E-valued Nevanlinna theory yield

$$\begin{aligned} T(r,f^{(k)}) &= m(r,f^{(k)})+N(r,f^{(k)}) \\ &= m(r,f)+N(r,f^{(k)})+\frac{1}{2\pi}\int_0^{2\pi}\log^+\frac{\|f^{(k)}(re^{i\theta})\|}{\|f(re^{i\theta})\|}d\theta \\ &\le m(r,f)+N(r,f)+k\overline{N}(r,f)+\frac{1}{2\pi}\int_0^{2\pi}\log^+\frac{\|f^{(k)}(re^{i\theta})\|}{\|f(re^{i\theta})\|}d\theta \\ &\le T(r,f)+kN(r,f)+\frac{1}{2\pi}\int_0^{2\pi}\log^+\frac{\|f^{(k)}(re^{i\theta})\|}{\|f(re^{i\theta})\|}d\theta. \end{aligned}$$

Thus we can get from the above inequality and Lemma 2.5 that

$$\limsup_{r\to+\infty}\frac{T(r,f^{(k)})}{T(r,f)}\le (k+1)-k\delta(\widehat{\infty}). \tag{6}$$

Hence from (5) and (6), we can get

$$\begin{aligned} 1 &\ge \limsup_{r\to+\infty}\left[\frac{N\left(r,0,f^{(k)}\right)+V\left(r,0,f^{(k)}\right)}{T(r,f^{(k)})}+\frac{T(r,f)}{T(r,f^{(k)})}\left(\frac{\sum\limits_{\mu=1}^{q} m(r,a^{[\mu]})}{T(r,f)}-o(1)\right)\right] \\ &\ge \limsup_{r\to+\infty}\frac{N\left(r,0,f^{(k)}\right)+V\left(r,0,f^{(k)}\right)}{T(r,f^{(k)})}+\liminf_{r\to+\infty}\frac{T(r,f)}{T(r,f^{(k)})}\left(\frac{\sum\limits_{\mu=1}^{q} m(r,a^{[\mu]})}{T(r,f)}-o(1)\right) \\ &\ge \limsup_{r\to+\infty}\frac{N\left(r,0,f^{(k)}\right)+V\left(r,0,f^{(k)}\right)}{T(r,f^{(k)})}+\liminf_{r\to+\infty}\frac{T(r,f)}{T(r,f^{(k)})}\liminf_{r\to+\infty}\frac{\sum\limits_{\mu=1}^{q} m(r,a^{[\mu]})}{T(r,f)} \\ &\ge \limsup_{r\to+\infty}\frac{N\left(r,0,f^{(k)}\right)+V\left(r,0,f^{(k)}\right)}{T(r,f^{(k)})}+\frac{\sum\limits_{j=1}^{q}\delta\left(a^{[j]}\right)}{(k+1)-k\delta(\widehat{\infty})}. \end{aligned}$$

Since $q>0$ was arbitrary and $\delta(\widehat{\infty})=1$, we have

$$\sum_{a\in E}\delta(a)\le\delta(0,f^{(k)}).$$

From Theorem 3.1, we have

COROLLARY 3.2. Let $f(z)$ be a finite order admissible E-valued meromorphic function of compact projection in $\mathbb{C}$ and assume $\delta(\widehat{\infty})=1$. If $f(z)$ has at least one deficient vector $a\in E$, then for any positive integer k, the vector 0 is the deficient vector of $f^{(k)}$.

COROLLARY 3.3. Let $f(z)$ be a finite order admissible E-valued meromorphic function of compact projection in $\mathbb{C}$ and assume $\sum_{a\in E}\delta(a)=1$ and $\delta(\widehat{\infty})=1$. Then for any positive integer k, $\delta(0,f^{(k)})=1$.

EXAMPLE 3.4. Put $f(z)=(e^z,e^z,...,e^z,...)$. Then

$$f^{(j)}(z)=(e^z,e^z,...,e^z,...),\quad j=1,2,\ldots\ .$$

For any non-zero vector $a\in E$, we have $\delta(a)=0$, $\delta(\widehat{\infty})=1,\delta(0)=1$ and $\delta(0,f^{(k)})=1$ holds for any positive integer k. Thus

$$\sum_{a\in E}\delta(a)\le\delta(0,f^{(k)}).$$

4 Relative Deficiency

The concept of the relative Nevanlinna defect of meromorphic function was due to Milloux [4] and Xiong Qinglai (also Hiong Qinglai [5]). Lahiri [3] extended this concept to meromorphic vector valued function in 1990. Most recently, Wu and Xuan [6] considered the relative deficiency of E-valued meromorphic function and gave the following definition and theorem.

DEFINITION 4.1. If k is a positive integer then the number

$$\Theta^{(k)}(a,f)=1-\limsup_{r\to+\infty}\frac{V(r,a,f^{(k)})+\overline{N}(r,a,f^{(k)})}{T(r,f)}$$

is called the relative deficiency of the value $a\in E$ with respect to distinct zeros.

THEOREM C. Let $f(z)$ be an admissible E-valued meromorphic function of compact projection in $\mathbb{C}$ and let $a^{[\mu]}$ $(\mu=1,2,\cdots,p)$ and $b^{[\lambda]}$ $(\lambda=1,2,\cdots,q)$, $q\ge2$, be elements of E, distinct within each set. Then for all positive integers k,

$$\sum_{\lambda=1}^{q}\Theta^{(k)}(b^{[\lambda]},f)+(q-2)\sum_{\mu=1}^{p}\delta(a^{[\mu]},f)\le q.$$

In this section, we continue to study the relative deficiency of E-valued meromorphic function and prove the following theorems.

THEOREM 4.2. Let $f(z)$ be a finite order admissible E-valued meromorphic function of compact projection in $\mathbb{C}$ and let a,b,c be three distinct elements in $E\cup\{\widehat{\infty}\}$ with $a\ne\widehat{\infty}$. Then

$$\delta(a,f)+\Theta^{(k)}(b,f)+\Theta^{(k)}(c,f)\le2.$$

PROOF. By Lemma 2.5, we get that

$$\begin{aligned}
m(r,a,f) &= \frac{1}{2\pi}\int_0^{2\pi} \log^+ \frac{\|f^{(k)}(re^{i\theta})\|}{\|f(re^{i\theta})-a\|}\frac{1}{\|f^{(k)}(re^{i\theta})\|}d\theta \\
&\le m(r,0,f^{(k)}) + \int_0^{2\pi} \log^+ \frac{\|f^{(k)}(re^{i\theta})\|}{\|f(re^{i\theta})-a\|}d\theta \\
&= m(r,0,f^{(k)}) + S(r,f).
\end{aligned}$$

Now by Lemma 2.3-2.5, we can get

$$\begin{aligned}
T(r,f) &\le T(r,f^{(k)}) - \Big(N(r,0,f^{(k)}) + V(r,0,f^{(k)})\Big) \\
&\quad + (N(r,a,f) + V(r,a,f)) + S(r,f) \\
&\le (N(r,a,f) + V(r,a,f)) - \Big(N(r,0,f^{(k)}) + V(r,0,f^{(k)})\Big) \\
&\quad + \Big(\overline{N}(r,0,f^{(k)}) + V(r,0,f^{(k)})\Big) + \Big(\overline{N}(r,b,f^{(k)}) + V(r,b,f^{(k)})\Big) \\
&\quad + \Big(\overline{N}(r,c,f^{(k)}) + V(r,c,f^{(k)})\Big) + S(r,f) \\
&\le (N(r,a,f) + V(r,a,f)) + \Big(\overline{N}(r,b,f^{(k)}) + V(r,b,f^{(k)})\Big) \\
&\quad + \Big(\overline{N}(r,c,f^{(k)}) + V(r,c,f^{(k)})\Big) + S(r,f).
\end{aligned}$$

Dividing both sides of the above inequality by $T(r,f)$ and taking limit superior we have

$$\begin{aligned}
1 &\le \limsup_{r\to+\infty}\frac{N(r,a,f)+V(r,a,f)}{T(r,f)} + \limsup_{r\to+\infty}\frac{\overline{N}(r,b,f^{(k)})+V(r,b,f^{(k)})}{T(r,f)} \\
&\quad + \limsup_{r\to+\infty}\frac{\overline{N}(r,c,f^{(k)})+V(r,c,f^{(k)})}{T(r,f)} + \limsup_{r\to+\infty}\frac{S(r,f)}{T(r,f)},
\end{aligned}$$

i. e.

$$1 \le (1-\delta(a,f)) + (1-\Theta^{(k)}(b,f)) + (1-\Theta^{(k)}(b,f)) + 0,$$

or

$$\delta(a,f) + \Theta^{(k)}(b,f) + \Theta^{(k)}(c,f) \le 2.$$

THEOREM 4.3. Let $a \ne 0$ be any vector in E and f be a finite order admissible E-valued meromorphic function of compact projection in $\mathbb{C}$. Then

$$\Theta(0,f) + \Theta(a,f) + \Theta^{(k)}(\widehat{\infty},f) \le 2.$$

PROOF. By Lemma 2.4, we have

$$\begin{aligned}
T(r,f) &\le \big(\overline{N}(r,0,f) + V(r,0,f)\big) + \big(\overline{N}(r,a,f) + V(r,a,f)\big) + \overline{N}(r,f) + S(r,f) \\
&\le \big(\overline{N}(r,0,f) + V(r,0,f)\big) + \big(\overline{N}(r,a,f) + V(r,a,f)\big) + \overline{N}(r,f^{(k)}) + S(r,f).
\end{aligned}$$

Dividing both sides of the above inequality by $T(r, f)$ and taking limit superior we have

$$\begin{aligned} 1 \quad \leq \quad & \limsup_{r\to+\infty} \frac{\overline{N}(r,0,f)+V(r,0,f)}{T(r,f)} + \limsup_{r\to+\infty} \frac{\overline{N}(r,a,f)+V(r,a,f)}{T(r,f)} \\ & + \limsup_{r\to+\infty} \frac{\overline{N}(r,f^{(k)})}{T(r,f)} + \limsup_{r\to+\infty} \frac{S(r,f)}{T(r,f)}, \end{aligned}$$

i. e.

$$1 \leq (1-\Theta(0,f)) + (1-\Theta(a,f)) + (1-\Theta^{(k)}(\widehat{\infty},f)) + 0,$$

or

$$\Theta(0,f) + \Theta(a,f) + \Theta^{(k)}(\widehat{\infty},f) \leq 2.$$

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