Applied Mathematics E-Notes, 11(2011), 33-40 © ISSN 1607-2510
Available free at mirror sites of http://www.math.nthu.edu.tw/∼amen/

Oscillation Criteria For Second-order Impulsive Dynamic Equations On Time Scales*

Qiaoluan Li and Lina Zhou†

Received 4 March 2010

Abstract

Some oscillation criteria are established for second-order neutral impulsive dynamic equation with or without forcing term.

1 Introduction

In this paper, we are interested in obtaining oscillation criteria for second-order impulsive dynamic equations on time scales. We consider the following systems

$$(x(t)+px(t-\tau))^{\triangle\triangle}+q(t)x(\sigma(t))=0,\quad t\in\mathbb{J}_{\mathbb{T}}:=[t_0,\ \infty)\cap\mathbb{T},\ t\neq t_k,\ k=1,2,...,$$

$$x(t_k^+)=ax(t_k),\ \ x^{\triangle}(t_k^+)=bx^{\triangle}(t_k),\ k=1,2,..., \tag{1}$$

and

$$x^{\triangle\triangle}(t)+q(t)x(\sigma(t))=e(t),\quad t\in\mathbb{J}_{\mathbb{T}}:=[t_0,\ \infty)\cap\mathbb{T},\ t\neq t_k,\ k=1,2,...,$$

$$x(t_k^+)=a_kx(t_k),\ \ x^{\triangle}(t_k^+)=b_kx^{\triangle}(t_k),\ k=1,2,..., \tag{2}$$

where $\mathbb{T}$ is a unbounded-above time scale, with $t_k\in\mathbb{T}, 0\leq t_0<t_1<t_2<\cdots<t_k<\cdots$, $\lim\limits_{k\to\infty}t_k=\infty$ and $y(t_k^+)=\lim\limits_{h\to0^+}y(t_k+h)$, $y^{\triangle}(t_k^+)=\lim\limits_{h\to0^+}y^{\triangle}(t_k+h)$, which represent right limits of $y(t)$, $y^{\triangle}(t)$ at $t=t_k$ in the sense of time scales, and in addition, if t_k is right scattered, then $y(t_k^+)=y(t_k)$, $y^{\triangle}(t_k^+)=y^{\triangle}(t_k)$. We can define $y(t_k^-)$, $y^{\triangle}(t_k^-)$ similar to the above definitions.

We assume that $0\leq p<1$, $\tau>0$, $q(t)\in C_{rd}(\mathbb{T},\mathbb{R}^+)$, $a>0$, $b>0$, $a_k>0$, $b_k>0$, $e(t)\in C_{rd}(\mathbb{T},\mathbb{R})$, $\mathbb{R}^+=\{x|x>0\}$.

DEFINITION 1. A function x is said to be a solution of (1), if it satisfies

$$(x(t)+px(t-\tau))^{\triangle\triangle}+q(t)x(\sigma(t))=0$$

a.e. on $\mathbb{J}_{\mathbb{T}}\backslash\{t_k\}$, $k=1,2,...$, and for each $k=1,2,...$, x satisfies the impulsive conditions $x(t_k^+)=ax(t_k)$, $x^{\triangle}(t_k^+)=bx^{\triangle}(t_k)$.

*Mathematics Subject Classifications: 34A60, 39A12.

†College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang 050016, P. R. China

We can define the solution of (2) similar to Definition 1.

Recently, many results have been obtained on the oscillation and nonoscillation of dynamic equations on time scales, and we refer the reader to papers [3, 6, 7, 8] and references cited therein. Impulsive dynamic equations on time scales have been investigated by Agarwal et al. [1], Benchohra et al. [4] and so forth. Benchohra et al. [4] considered the existence of extremal solutions for a class of second order impulsive dynamic equations on time scales.

The oscillation of impulsive differential equations and impulsive difference equations has been investigated by many authors and good results were obtained (see [5, 10] etc. and the references cited therein). But fewer papers are on the oscillation of impulsive dynamic equations on time scales.

For example, Huang [9] considered the equation

$$y^{\triangle\triangle}(t) + f(t, y^{\sigma}(t)) = 0, \quad t \in \mathbb{J}_{\mathbb{T}} := [0,\ \infty) \cap \mathbb{T},\ t \neq t_k,\ k = 1, 2, ...,$$

$$y(t_k^+) = g_k(y(t_k)),\ \ y^{\triangle}(t_k^+) = h_k(y^{\triangle}(t_k)),\ k = 1, 2, ...,$$

$$y(t_0^+) = y(t_0),\ \ y^{\triangle}(t_0^+) = y^{\triangle}(t_0).$$

Using Riccati transformation techniques, they obtain sufficient conditions for oscillation of all solutions.

In the following, we always assume the solutions of (1)(or (2)) exist in $\mathbb{J}_{\mathbb{T}}$. To the best of our knowledge, the question of the oscillation for second order neutral impulsive dynamic equations on time scales has not been yet considered.

2 Main Results

We will briefly recall some basic definitions from the time scales calculus that we will use in the sequel. For more details see [2].

On any time scale $\mathbb{T}$, we define the forward and backward jump operators by

$$\sigma(t) = \inf\{s \in \mathbb{T}, s > t\},\ \ \rho(t) = \sup\{s \in \mathbb{T}, s < t\},$$

where $\inf \varnothing = \sup \mathbb{T}$, $\sup \varnothing = \inf \mathbb{T}$, and $\varnothing$ denotes the empty set. A nonmaximal element $t \in \mathbb{T}$ is called right-dense if $\sigma(t) = t$ and right-scattered if $\sigma(t) > t$. A nonminimal element $t \in \mathbb{T}$ is said to be left-dense if $\rho(t) = t$ and left-scattered if $\rho(t) < t$. The graininess μ of the time scale $\mathbb{T}$ is defined by $\mu(t) = \sigma(t) - t$.

A function $f : \mathbb{T} \to \mathbb{R}$ is called *rd-continuous* provided it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist(finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T}, \mathbb{R})$.

LEMMA 1 ([12]). Assume that $m \in PC^1[\mathbb{T}, \mathbb{R}]$ and

$$m^{\triangle}(t) \leq p(t)m(t) + q(t),\ t \in \mathbb{J}_{\mathbb{T}} := [0, \infty) \cap \mathbb{T},\ t \neq t_k,\ k = 1, 2, ...,$$

$$m(t_k^+) \leq d_k m(t_k^-) + b_k, \quad k = 1, 2, ...,$$

then for $t \geq t_0$,

$$m(t) \leq m(t_0) \prod_{t_0<t_k<t} d_k e_p(t,t_0) + \sum_{t_0<t_k<t} \Big(\prod_{t_k<t_j<t} d_j e_p(t,t_k) \Big) b_k + \int_{t_0}^{t} \prod_{s<t_k<t} d_k e_p(t,\sigma(s)) q(s) \Delta s,$$

where $PC = \{y : \mathbb{J}_{\mathbb{T}} \to \mathbb{R}$ which is rd-continuous except at t_k, $k = 1, 2, \ldots$, for which $y(t_k^-)$, $y(t_k^+)$, $y^{\triangle}(t_k^-)$, $y^{\triangle}(t_k^+)$ exist with $y(t_k^-) = y(t_k)$, $y^{\triangle}(t_k^-) = y^{\triangle}(t_k)\}$.

We first consider the Equation (1). Let $u(t) = x(t) + px(t-\tau)$.

LEMMA 2. Suppose that $x(t) > 0$, $t \geq T \geq t_0$, is a solution of (1), $t_{k+1} - t_k = \tau$. If

$$\int_{t_j}^{\infty} \prod_{t_j<t_k<s} \frac{b}{a} \triangle s = \infty, \tag{3}$$

holds for some $t_j \geq T$, then $u^{\triangle}(t_k^+) \geq 0$, $u^{\triangle}(t) \geq 0$ for $t \in (t_k,\ t_{k+1}]_{\mathbb{T}}$, where $t_k \geq T$, $k = 1, 2, \ldots$.

PROOF. From $u(t) = x(t) + px(t-\tau)$, we get

$$u(t_k^+) = x(t_k^+) + px(t_k^+ - \tau) = au(t_k),$$

$$u^{\triangle}(t_k^+) = x^{\triangle}(t_k^+) + px^{\triangle}(t_k^+ - \tau) = bu^{\triangle}(t_k).$$

We first prove that $u^{\triangle}(t_k) \geq 0$, $t_k \geq T$, $k = 1, 2, \ldots$. If not, then there exists a $j \in N$ such that $u^{\triangle}(t_j) < 0$, $t_j \geq T$, $u^{\triangle}(t_j^+) = bu^{\triangle}(t_j) < 0$. For $t \in (t_{j+i-1},\ t_{j+i}]_{\mathbb{T}}$, $i = 1, 2, \ldots$, we get

$$u^{\triangle\triangle}(t) = -q(t)x(\sigma(t)) \leq 0.$$

So $u^{\triangle}(t_{j+1}) \leq u^{\triangle}(t_j^+) = bu^{\triangle}(t_j)$, $u^{\triangle}(t_{j+2}) \leq u^{\triangle}(t_{j+1}^+) = bu^{\triangle}(t_{j+1}) \leq b^2 u^{\triangle}(t_j) < 0$. By induction, we obtain

$$u^{\triangle}(t) \leq u^{\triangle}(t_{j+n-1}^+) \leq b^n u^{\triangle}(t_j) \triangleq b^n(-\beta) < 0, \quad t \in (t_{j+n-1},\ t_{j+n}]_{\mathbb{T}}.$$

So

$$u^{\triangle}(t) \leq -\beta \prod_{t_j \leq t_k < t} b, \quad u(t_k^+) = au(t_k).$$

Applying Lemma 1, we obtain for $t > t_j$,

$$\begin{aligned} u(t) &\leq u(t_j^+) \prod_{t_j<t_k<t} a - \beta \int_{t_j}^{t} \prod_{s<t_k<t} a \prod_{t_j<t_l<s} b \triangle s \\ &= \prod_{t_j<t_k<t} a \left[u(t_j^+) - \beta \int_{t_j}^{t} \prod_{t_j<t_k<s} \frac{b}{a} \triangle s \right]. \end{aligned}$$

We get a contradiction as $t \to \infty$. Therefore, $u^{\triangle}(t_k) \geq 0, k = 1, 2, \cdots$. From $u^{\triangle\triangle}(t) \leq 0$, $u^{\triangle}(t_k^+) = bu^{\triangle}(t_k) \geq 0$, we have $u^{\triangle}(t) \geq 0$. The proof of Lemma 2 is complete.

THEOREM 1. Suppose that (3) holds, $t_{k+1} - t_k = \tau$, $a \geq 1$, and

$$\int_{t_0}^{\infty} \prod_{t_0<t_k<t} \frac{a}{b} q(t) \triangle t = \infty. \tag{4}$$

Then Eq.(1) is oscillatory.

PROOF. Suppose that Eq.(1) has a nonoscillatory solution x, without loss of generality, we assume that $x > 0,\ t \geq T$. From Lemma 2, we have

$$u^{\triangle\triangle}(t) + q(t)x(\sigma(t)) = 0, \quad t \in \mathbb{J}_{\mathbb{T}},\ t \neq t_k,\ k = 1, 2, ...,$$
$$u(t_k^+) = au(t_k),\ \ u^{\triangle}(t_k^+) = bu^{\triangle}(t_k),\ k = 1, 2, ..., \tag{5}$$

and $u^{\triangle}(t_k^+) \geq 0,\ u^{\triangle}(t) \geq 0,\ \ t \in (t_k, t_{k+1}]_{\mathbb{T}},\ t_k \geq T, k = 1, 2, ...$. Further we know

$$u^{\triangle\triangle}(t) + q(t)(1-p)u(\sigma(t)) \leq 0,\ t \in \mathbb{J}_{\mathbb{T}},\ \ t \neq t_k,\ k = 1, 2, ... \ .$$

Let $w(t) = \frac{u^{\triangle}(t)}{u(t)}$, then

$$w^{\triangle}(t) = \frac{u^{\triangle\triangle}(t)u(t) - (u^{\triangle}(t))^2}{u(t)u(\sigma(t))} \leq -q(t)(1-p), t \in \mathbb{J}_{\mathbb{T}},\ t \neq t_k,\ k = 1, 2, ...,$$

$$w(t_k^+) = \frac{u^{\triangle}(t_k^+)}{u(t_k^+)} = \frac{b}{a}w(t_k).$$

Applying Lemma 1, we get

$$\begin{aligned} w(t) &\leq w(t_0) \prod_{t_0 < t_k < t} \frac{b}{a} - (1-p) \int_{t_0}^{t} \prod_{s < t_k < t} \frac{b}{a} q(s) \triangle s \\ &= \prod_{t_0 < t_k < t} \frac{b}{a} \left[w(t_0) - (1-p) \int_{t_0}^{t} \prod_{t_0 < t_k < s} \frac{a}{b} q(s) \triangle s \right]. \end{aligned}$$

In view of (4), we get a contradiction as $t \to \infty$. The proof is complete.

Next, we discuss the Eq.(2). For Eq.(2), we assume that there exists a function $z(t)$, $z(t)$ is 2-times $\triangle$-differentiable, $z^{\triangle\triangle}(t) = e(t)$, *a.e.*, there exist two constants $p_1,\ p_2$ and two sequences $\{t_i^{'}\},\ \{t_i^{''}\},\ \lim_{i\to\infty} t_i^{'} = \lim_{i\to\infty} t_i^{''} = \infty$ such that $z(t_i^{'}) = p_1 \leq z(t) \leq p_2 = z(t_i^{''})$.

If Eq.(2) has an eventually positive solution $x(t)$, let $y(t) = x(t) - z(t) + p_1$, by Eq.(2), we have

$$y^{\triangle\triangle}(t) + q(t)y(\sigma(t)) \leq 0,\ t \in \mathbb{J}_{\mathbb{T}}\,,\ t \neq t_k,\ k = 1, 2, ...,$$
$$y(t_k^+) = a_k y(t_k) + c_k,\ y^{\triangle}(t_k^+) = b_k y^{\triangle}(t_k) + e_k,\ \ k = 1, 2, ..., \tag{6}$$

where $c_k = (a_k - 1)(z(t_k) - p_1),\ e_k = (b_k - 1)z^{\triangle}(t_k)$.

If Eq.(2) has an eventually negative solution $x(t)$, let $y(t) = x(t) - z(t) + p_2$, by Eq.(2), we have

$$y^{\triangle\triangle}(t) + q(t)y(\sigma(t)) \geq 0,\ t \in \mathbb{J}_{\mathbb{T}},\ t \neq t_k,\ k = 1, 2, ...,$$
$$y(t_k^+) = a_k y(t_k) + d_k,\ y^{\triangle}(t_k^+) = b_k y^{\triangle}(t_k) + e_k,\ \ k = 1, 2, ..., \tag{7}$$

where $d_k = (a_k - 1)(z(t_k) - p_2),\ e_k = (b_k - 1)z^{\triangle}(t_k)$.

LEMMA 3. Suppose $x(t)$ is an eventually positive solution of Eq.(2). If there exists a constant k_0, such that $z^{\triangle}(t_k) = 0,\ k \geq k_0$, and

$$(H1) \quad (t_1 - t_0) + \frac{b_1}{a_1}(t_2 - t_1) + \cdots + \frac{b_1 b_2 \cdots b_n}{a_1 a_2 \cdots a_n}(t_{n+1} - t_n) + \cdots = \infty,$$

$$(H2) \quad \frac{|c_1|}{a_1} + \frac{|c_2|}{a_1 a_2} + \cdots + \frac{|c_n|}{a_1 a_2 \cdots a_n} + \cdots < \infty,$$

then for Eq.(6), $y^{\triangle}(t_k^+) > 0,\ y^{\triangle}(t) > 0,\ t \in (t_k,\ t_{k+1}]_{\mathbb{T}},\ t_k \geq T_1 \geq t_{k_0}$.

The proof is similar to Lemma 2, so we omit it.

LEMMA 4. Suppose $x(t)$ is an eventually positive solution of Eq.(2). If the conditions (H1), (H2) hold, and there exists a k_0, such that $a_k \geq 1,\ z^{\triangle}(t_k) = 0,\ k \geq k_0$, then for Eq.(6), $y(t) > 0,\ t \geq T_1 \geq t_{k_0}$.

PROOF. Without loss of generality, we assume that $x(t) > 0,\ t \geq t_0$.

(I) If there exists a $t_j \geq t_{k_0}$ such that $y(t_j^+) = a_j y(t_j) + c_j \geq 0$, by Lemma 3, we have $y^{\triangle}(t) > 0,\ t \in (t_j,\ t_{j+1}]_{\mathbb{T}}$. So

$$y(t_{j+1}) > y(t_j^+) \geq 0,\ y(t_{j+1}^+) = a_{j+1} y(t_{j+1}) + c_{j+1} \geq a_{j+1} y(t_{j+1}).$$

By induction, there exists a $T_1 \geq t_{k_0}$, such that $y(t) > 0,\ t \geq T_1$.

(II) If all $t_j \geq t_{k_0}$ we have $y(t_j^+) = a_j y(t_j) + c_j < 0$, i.e., $y(t_j) = \frac{y(t_j^+) - c_j}{a_j} < 0$. Since $y(t)$ is monotonically increasing in $t \in (t_j,\ t_{j+1}]_{\mathbb{T}}$, $y(t) < y(t_{j+1}) < 0$. On the other hand, in $(t_k,\ t_{k+1}]_{\mathbb{T}}$, $t_k \geq t_{k_0}$, we take a point t_n', then $x(t_n') = y(t_n') + z(t_n') - p_1 = y(t_n') < 0$, this contradicts $x(t) > 0$.

Summing up the above consideration, we have $y(t) > 0,\ t \geq T_1$. The proof is complete.

For $x(t)$ which is an eventually negative solution of Eq.(2), we have similar results, we omit them.

THEOREM 2. Suppose (H1) and (H2) hold, and there exists a constant k_0 such that $a_k \geq 1, z^{\triangle}(t_k) = 0,\ k \geq k_0$, and

$$\int_{t_1}^{t_2} q(t)\triangle t + \frac{a_2}{b_2}\int_{t_2}^{t_3} q(t)\triangle t + \cdots + \frac{a_2 a_3 \cdots a_m}{b_2 b_3 \cdots b_m}\int_{t_m}^{t_{m+1}} q(t)\triangle t + \cdots = \infty,$$

then Eq.(2) is oscillatory.

PROOF. Let $x(t)$ be a nonoscillatory solution of Eq.(2). Without loss of generality, we assume $x(t) > 0$. By Lemma 3 and Lemma 4, we get

$$y(t) > 0,\ y^{\triangle}(t) > 0,\ y^{\triangle\triangle}(t) < 0, \quad t \geq T_1 \geq t_{k_0}.$$

Let $v(t) = \frac{y^{\triangle}(t)}{y(t)}$. Then

$$v^{\triangle}(t) \leq -q(t),\ t \in \mathbb{J}_{\mathbb{T}},\ t \neq t_k,\ k = 1, 2, ..., \tag{8}$$

$$v(t_k^+) = \frac{b_k y^{\triangle}(t_k)}{a_k y(t_k) + c_k} < \frac{b_k}{a_k} v(t_k).$$

Integrating (8), we have

$$\begin{aligned} v(t_2) &\leq v(t_1^+) - \int_{t_1}^{t_2} q(t)\triangle t, \\ v(t_3) &\leq v(t_2^+) - \int_{t_2}^{t_3} q(t)\triangle t \\ &\leq \frac{b_2}{a_2} v(t_2) - \int_{t_2}^{t_3} q(t)\triangle t \\ &\leq \frac{b_2}{a_2}[v(t_1^+) - \int_{t_1}^{t_2} q(t)\triangle t - \frac{a_2}{b_2}\int_{t_2}^{t_3} q(t)\triangle t]. \end{aligned}$$

Applying induction, for any natural number m,

$$v(t_{m+1}) \leq \frac{b_2 b_3 \cdots b_m}{a_2 a_3 \cdots a_m}[v(t_1^+) - \int_{t_1}^{t_2} q(t)\triangle t - \frac{a_2}{b_2}\int_{t_2}^{t_3} q(t)\triangle t - \cdots - \frac{a_2 a_3 \cdots a_m}{b_2 b_3 \cdots b_m}\int_{t_m}^{t_{m+1}} q(t)\triangle t].$$

Let $m \to \infty$, by the conditions of Theorem 2, we get a contradiction. The proof is complete.

THEOREM 3. If the conditions (H1),(H2) hold, and there exists a constant k_0 such that $a_k \geq 1,\ z^{\triangle}(t_k) = 0,\ k \geq k_0$, and

$$\lim_{t\to\infty} \int_{t_1}^{t} \prod_{t_1 < t_k < s} \frac{a_k}{b_k} q(s)\triangle s = \infty,$$

then the Eq.(2) is oscillatory.

PROOF. Let $x(t)$ be a positive solution of Eq.(2), similar to the proof of Theorem 2, we get

$$v^{\triangle}(t) \leq -q(t),$$

$$v(t_k^+) < \frac{b_k}{a_k} v(t_k).$$

By Lemma 1, we get

$$v(t) \leq \prod_{t_1 < t_k < t} \frac{b_k}{a_k}[v(t_1^+) - \int_{t_1}^{t} \prod_{t_1 < t_k < s} \frac{a_k}{b_k} q(s)\triangle s].$$

We get a contradiction as $t \to \infty$. The proof is complete.

EXAMPLE 1. Consider the system

$$(x(t) + \frac{1}{2}x(t-1))^{\triangle\triangle} + t^{[t]+1}x(\sigma(t)) = 0,\ t \in \mathbb{T} = \mathbb{P}_{\frac{1}{2},\frac{1}{2}},\ t \neq k + \frac{1}{3},\ k = 0, 1, 2, ...,$$

$$x((k + \frac{1}{3})^+) = ax(k + \frac{1}{3}),\ \ k = 0, 1, 2, ...,$$

$$x^{\triangle}((k+\frac{1}{3})^{+}) = 2ax^{\triangle}(k+\frac{1}{3}),\ \ k=0,1,2,...,$$

where $a \geq 1$, $\mathbb{P}_{\frac{1}{2},\frac{1}{2}} = \bigcup_{k=0}^{\infty}[k,\, k+\frac{1}{2}]$, $t_{k+1}-t_k = 1$. It is easy to show

$$\int_{t_1}^{\infty}\prod_{t_1<t_k<s}\frac{2a}{a}\triangle s > \int_{t_1}^{\infty}\triangle s = \infty,$$

$$\int_{0}^{\infty}\prod_{0<t_k<t}\frac{a}{2a}q(t)\triangle t = \int_{0}^{\frac{1}{2}}\frac{1}{2}t\triangle t + \int_{1}^{\frac{3}{2}}\frac{t^2}{2^2}\triangle t + \cdots + \int_{k}^{k+\frac{1}{2}}\frac{t^{k+1}}{2^{k+1}}\triangle t + \cdots = \infty,$$

by Theorem 1, we know our system is oscillatory.

EXAMPLE 2. Consider the system

$$x^{\triangle\triangle}(t) + t^2 x(\sigma(t)) = \sin t,\ t \in \mathbb{T} := \bigcup_{k=0}^{\infty}[k\pi,\, k\pi+\frac{3\pi}{4}],\ t \neq k\pi+\frac{\pi}{2}, k=0,1,2,...,$$

$$x((k\pi+\frac{\pi}{2})^{+}) = (1+\frac{1}{k})x(k\pi+\frac{\pi}{2}),\ \ k=0,1,2,...,$$
$$x^{\triangle}((k\pi+\frac{\pi}{2})^{+}) = (1+\frac{1}{k})x^{\triangle}(k\pi+\frac{\pi}{2}),\ \ k=0,1,2,\ldots .$$

Here $a_k = 1+\frac{1}{k} > 1$. Let $z(t) = -\sin t$ Then $z^{\triangle\triangle}(t) = \sin t$, *a.e.*, $z^{\triangle}(k\pi+\frac{\pi}{2}) = -\cos(k\pi+\frac{\pi}{2}) = 0$, $p_1 = -1$, $p_2 = 1$, and

$$c_k = \begin{cases} \frac{2}{k}, & k \text{ odd} \\ 0, & k \text{ even} \end{cases}.$$

Since

$$(t_1-t_0) + \frac{b_1}{a_1}(t_2-t_1) + \cdots + \frac{b_1 b_2\cdots b_n}{a_1 a_2\cdots a_n}(t_{n+1}-t_n) + \cdots = \pi+\pi+\cdots+\pi+\cdots = \infty,$$

$$\frac{|c_1|}{a_1} + \frac{|c_2|}{a_1 a_2} + \cdots + \frac{|c_n|}{a_1 a_2\cdots a_n} + \cdots \leq \frac{2}{1\cdot 2} + \frac{2}{2\cdot 3} + \cdots + \frac{2}{n\cdot(n+1)} + \cdots < \infty.$$

so the conditions (H1) and (H2) hold. Furthermore,

$$\begin{aligned} & \int_{t_1}^{t_2} q(t)\triangle t + \frac{a_2}{b_2}\int_{t_2}^{t_3} q(t)\triangle t + \cdots + \frac{a_2 a_3\cdots a_m}{b_2 b_3\cdots b_m}\int_{t_m}^{t_{m+1}} q(t)\triangle t + \cdots \\ = & \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} t^2\triangle t + \int_{\pi}^{\pi+\frac{3\pi}{4}} t^2\triangle t + \cdots + \int_{k\pi}^{k\pi+\frac{3\pi}{4}} t^2\triangle t + \cdots = \infty. \end{aligned}$$

The conditions of Theorem 2 are satisfied. Our system is oscillatory.

Acknowledgment. The authors are very grateful to the anonymous referee for his/her careful reading of the original manuscript, and for the helpful suggestions. This research is supported by the Main Foundation of Hebei Normal University (L2009Z02) and the Doctor Foundation of Hebei Normal University (L2006B01).

References

[1] R. P. Agarwal, M. Benchohra, D. O'Regan and A. Ouahab, Second order impulsive dynamic equations on time scales, Funct. Diff. Eq., 11(2004), 223–234.

[2] M. Bohner and A. Peterson, Dynamic Equations on Time Scales: An Introduction with Applications, Birkhäuser, Boston, 2001.

[3] L. Erbe, A. Peterson and S. H. Saker, Oscillation criteria for second order nonlinear dynamic equations on time scales, J. London Math., 3(2003), 701–714.

[4] M. Benchohra, S. K. Ntouyas and A. Ouahab, Extremal solutions of second order impulsive dynamic equations on time scales, J. Math. Anal. Appl., 324(1)(2006), 425–434.

[5] M. S. Peng, Oscillation criteria for second-order impulsive delay difference equations, Appl. Math. Comput., 146(1)(2003), 227–235.

[6] B. G. Zhang and Z. S. Liang, Oscillation of second-order nonlinear delay dynamic equations on time scales, Comput. Math. Appl., 49(2005), 599–609.

[7] S. H. Saker, Oscillation of second order forced nonlinear dynamic equations on time scales, Electronic J. Qualitative Theory Diff. Eq., 23(2005), 1–17.

[8] S. H. Saker, Oscillation criteria of second order half-linear dynamic equations on time scales, J. Comput. Appl. Math., 177(2005), 375–387.

[9] M. G. Huang and W. Z. Feng, Oscillation of second-order nonlinear impulsive dynamic equations on time scales, Electronic J. Diff. Eq., 2007(72)(2007), 1–13.

[10] L. P. Gimenes, M. Federson and P. Táboas, Impulsive stability for systems of second order retarded differential equations, Nonlinear Anal., 67(2007), 545–553.

[11] C. Zhang and H. X. Li, Dynamics of a rational difference equation of higher order, AMEN, 9(2009), 80–88.

[12] M. G. Huang and W. Z. Feng, Oscillation criteria for impulsive dynamic equations on time scales, Electronic J. Diff. Eq., 2007(169)(2007), 1–9.