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A Further Improved Tanh Function Method Exactly Solving The (2+1)-Dimensional Dispersive Long Wave Equations*

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Abstract

In this paper, a further improved tanh function method is used to construct exact solutions of the (2+1)-dimensional dispersive long wave equations. As a result, many new and more general solutions are obtained including soliton-like solutions, periodic formal solutions and rational function solutions. Compared with most existing tanh function methods, the proposed method gives new and more general exact solutions. More importantly, with the aid of symbolic computation, this method provides a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

1 Introduction

It is well known that nonlinear complex physics phenomena are related to nonlinear partial differential equations (NLPDEs) which are involved in many fields from physics to biology, chemistry, mechanics, etc. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs will help us to understand these phenomena better. Many effective methods for obtaining exact solutions of NLPDEs have been presented, such as Bäcklund transformation [1], hyperbolic function method [2], sine-cosine method [3], Jacobi elliptic function expansion method [4], homotopy perturbation method [5], F-expansion method [6] and so on.

One of the most effective direct methods to construct exact solutions of NLPDEs is tanh function method [7–9], the method was later extended in different manners [10–16]. Recently, Xie et al. [17] generalized the work made in [10–16]. Very recently, by using a more general transformation we improved the method [17] and proposed a further improved tanh function method [18] to seek more general exact solutions of NLPDEs. The present paper is motivated by the desire to extend the further improved tanh function method to the (2+1)-dimensional dispersive long wave (DLW) equations:

$$u_{yt} + H_{xx} + \frac{1}{2}(u^2)_{xy} = 0, \tag{1}$$

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$$H_t + (uH + u + u_{xy})_x = 0. \quad (2)$$

Soliton-like solutions, Jacobi elliptic function solutions and other exact solutions can be found in [19–21].

2 A Further Improved Tanh Function Method

Given a system of NLPDEs with independent variables $x = (t, x_1, x_2, \ldots, x_m)$ and dependent variables u, v:

$$F(u, v, u_t, v_t, u_{x_1}, v_{x_1}, \ldots, u_{x_1t}, v_{x_1t} \ldots, u_{tt}, v_{tt}, \ldots, u_{x_mx_m}, v_{x_mx_m}, \ldots) = 0, \quad (3)$$

$$G(u, v, u_t, v_t, u_{x_1}, v_{x_1}, \ldots, u_{x_1t}, v_{x_1t} \ldots, u_{tt}, v_{tt}, \ldots, u_{x_mx_m}, v_{x_mx_m}, \ldots) = 0, \quad (4)$$

we seek its solutions in the more general forms:

$$u = a_0(x) + \sum_{i=1}^{N_1} \left\{ a_{-i}(x)\phi^{-i}(\xi) + a_i(x)\phi^i(\xi) \right\}, \quad (5)$$

$$v = b_0(x) + \sum_{j=1}^{N_2} \left\{ b_{-j}(x)\phi^{-j}(\xi) + b_j(x)\phi^j(\xi) \right\}, \quad (6)$$

with

$$\phi'(\xi) = r + p\phi(\xi) + q\phi^2(\xi), \quad (7)$$

where the prime denotes $\mathrm{d}/\mathrm{d}\xi$, r, p and q are all real constants, while $a_0(x)$, $a_{-i}(x)$, $a_i(x)$, $b_0(x)$, $b_{-j}(x)$, $b_j(x)$ $(i = 1, 2, \ldots, N_1; j = 1, 2, \ldots, N_2)$ and $\xi = \xi(x)$ are all differentiable functions to be determined later. Given different values of r, p and q, equation (7) has twenty seven special solutions which are listed in [18]. To determine u explicitly, we take the following four steps:

Step 1. Determine the integers N_1 and N_2 by balancing the highest order nonlinear term(s) and the highest order partial derivative term(s) in equations (3) and (4).

Step 2. Substitute (5) and (6) along with (7) into equations (3) and (4), then collect coefficients of the same order of $\phi^l(\xi)$ $(l = \pm 1, \pm 2, \ldots)$ and set each coefficient to zero to derive a set of over-determined partial differential equations for $a_0(x)$, $a_{-i}(x)$, $a_i(x)$, $b_0(x)$, $b_{-j}(x)$, $b_j(x)$ and ξ.

Step 3. Solve the over-determined partial differential equations obtained in Step 2 by use of Mathematica and using Wu elimination method.

Step 4. Substitute $a_0(x)$, $a_{-i}(x)$, $a_i(x)$, $b_0(x)$, $b_{-j}(x)$, $b_j(x)$ and ξ along with one solution $\phi(\xi)$ of equation (7) into (5) and (6), we then obtain soliton-like solutions, periodic formal solutions and rational function solutions of equations (3) and (4).

3 Exact Solutions of the DLW Equations

According to Step 1, we get $N_1 = 2$ for H and $N_2 = 1$ for u. In order to search for explicit solutions of equations (1) and (2), we set $a_0 = a_0(y, t)$, $a_{-2} = a_{-2}(y, t)$,

$a_{-1}=a_{-1}(y,t)$, $a_1=a_1(y,t)$, $a_2=a_2(y,t)$, $b_0=b_0(y,t)$, $b_{-1}=b_{-1}(y,t)$, $b_1=b_1(y,t)$, $\xi=kx+\eta$, $\eta=\eta(y,t)$, k is a nonzero constant. Thus we have

$$H=a_0+a_{-2}\phi^{-2}(\xi)+a_{-1}\phi^{-1}(\xi)+a_1\phi(\xi)+a_2\phi^2(\xi), \tag{8}$$

$$u=b_0+b_{-1}\phi^{-1}(\xi)+b_1\phi(\xi). \tag{9}$$

Substituting (8) and (9) along with (7) into equations (1) and (2), then collecting coefficients of the same order of $\phi(\xi)^l$ $(l=\pm1,\pm2,\ldots)$ and setting each coefficient to zero, we derive a set of over-determined partial differential equations for a_0, a_{-2}, a_{-1}, a_1, a_2, b_0, b_{-1}, b_1 and η as follows:

$$\begin{gathered}
2k^2r^2a_{-1}+10k^2pra_{-2}-2krb_{-1}b_{-1,y}+2kr^2b_0b_{-1}\eta_y+5kprb_{-1}^2\eta_y\\
+2r^2b_{-1}\eta_t\eta_y=0,\quad 6k^2r^2a_{-2}+3kr^2b_{-1}^2\eta_y=0,\\
3k^2pra_{-1}+4k^2p^2a_{-2}+8k^2qra_{-2}-krb_{-1}b_{0,y}-krb_0b_{-1,y}-2kpb_{-1}b_{-1,y}\\
-r\eta_tb_{-1,y}+3kprb_0b_{-1}\eta_y+2kp^2b_{-1}^2\eta_y+4kqrb_{-1}^2\eta_y-rb_{-1,t}\eta_y+3prb_{-1}\eta_t\eta_y\\
-rb_{-1}\eta_{t,y}=0,\quad 3kqa_2b_1+6k^2q^3b_1\eta_y=0,\\
k^2p^2a_{-1}+2k^2qra_{-1}+6k^2pqa_{-2}-kpb_{-1}b_{0,y}-kpb_0b_{-1,y}-2kqb_{-1}b_{-1,y}\\
-p\eta_tb_{-1,y}+kp^2b_0b_{-1}\eta_y+2kqrb_0b_{-1}\eta_y+3kpqb_{-1}^2\eta_y-pb_{-1,t}\eta_y+p^2b_{-1}\eta_t\eta_y\\
+2qrb_{-1}\eta_t\eta_y+b_{-1,t,y}-pb_{-1}\eta_{t,y}=0,\quad -3kra_{-2}b_{-1}-6k^2r^3b_{-1}\eta_y=0,\\
k^2pra_1+k^2pqa_{-1}+2k^2q^2a_{-2}+2k^2r^2a_2+krb_1b_{0,y}-kqb_{-1}b_{0,y}+krb_0b_{1,y}\\
+r\eta_tb_{1,y}-kqb_0b_{-1,y}-q\eta_tb_{-1,y}+kprb_0b_1\eta_y+kr^2b_1^2\eta_y+kpqb_0b_{-1}\eta_y+rb_1\eta_{t,y}\\
+rb_{1,t}\eta_y-qb_{-1,t}\eta_y+prb_1\eta_t\eta_y-qb_{-1}\eta_{t,y}+pqb_{-1}\eta_t\eta_y+b_{0,t,y}+kq^2b_{-1}^2\eta_y=0,\\
k^2p^2a_1+2k^2qra_1+6k^2pra_2+kpb_1b_{0,y}+kpb_0b_{1,y}+2krb_1b_{1,y}+p\eta_tb_{1,y}\\
+kp^2b_0b_1\eta_y+2kqrb_0b_1\eta_y+3kprb_1^2\eta_y+pb_{1,t}\eta_y+p^2b_1\eta_t\eta_y+2qrb_1\eta_t\eta_y\\
+b_{1,t,y}+pb_1\eta_{t,y}=0,\quad 6k^2q^2a_2+3kq^2b_1^2\eta_y=0,\\
3k^2pqa_1+4k^2p^2a_2+8k^2qra_2+kqb_1b_{0,y}+kqb_0b_{1,y}+2kpb_1b_{1,y}+q\eta_tb_{1,y}\\
+3kpqb_0b_1\eta_y+2kp^2b_1^2\eta_y+4kqrb_1^2\eta_y+qb_{1,t}\eta_y+3pqb_1\eta_t\eta_y+qb_1\eta_{t,y}=0,\\
2k^2q^2a_1+10k^2pqa_2+2kqb_1b_{1,y}+2kq^2b_0b_1\eta_y+5kpqb_1^2\eta_y+2q^2b_1\eta_t\eta_y=0,\\
2k^2r^2b_{-1,y}-2kra_{-2}b_0-2kra_{-1}b_{-1}-3kpa_{-2}b_{-1}-2ra_{-2}\eta_t-12k^2pr^2b_{-1}\eta_y=0,\\
-kra_{-1}b_0-2kpa_{-2}b_0-kra_{-2}b_1-krb_{-1}-kra_0b_{-1}-2kpa_{-1}b_{-1}-3kqa_{-2}b_{-1}\\
+a_{-2,t}-ra_{-1}\eta_t-2pa_{-2}\eta_t+3k^2prb_{-1,y}-7k^2p^2rb_{-1}\eta_y-8k^2qr^2b_{-1}\eta_y=0,\\
-kpa_{-1}b_0-2kqa_{-2}b_0-kpa_{-2}b_1-kpb_{-1}-kpa_0b_{-1}-2kqa_{-1}b_{-1}+a_{-1,t}\\
-pa_{-1}\eta_t-2qa_{-2}\eta_t+k^2p^2b_{-1,y}+2k^2qrb_{-1,y}-k^2p^3b_{-1}\eta_y-8k^2pqrb_{-1}\eta_y=0,\\
kra_1b_0-kqa_{-1}b_0+krb_1+kra_0b_1-kqa_{-2}b_1-kqb_{-1}-kqa_0b_{-1}+kra_2b_{-1}+a_{0,t}\\
+ra_1\eta_t-qa_{-1}\eta_t+k^2prb_{1,y}+k^2pqb_{-1,y}+k^2p^2rb_1\eta_y+2k^2qr^2b_1\eta_y
\end{gathered}$$

$$-k^2p^2qb_{-1}\eta_y - 2k^2q^2rb_{-1}\eta_y = 0,$$

$$kpa_1b_0 + 2kra_2b_0 + kpa_2b_1 + kpb_1 + kpa_0b_1 + 2kra_1b_1 + kpa_2b_{-1} + a_{1,t} + pa_1\eta_t$$
$$+2ra_2\eta_t + k^2p^2b_{1,y} + 2k^2qrb_{1,y} + k^2p^3b_1\eta_y + 8k^2pqrb_1\eta_y = 0,$$
$$kqa_1b_0 + 2kpa_2b_0 + kqb_1 + kqa_0b_1 + 2kpa_1b_1 + 3kra_2b_1 + kqa_2b_{-1} + a_{2,t}$$
$$+qa_1\eta_t + 2pa_2\eta_t + 3k^2pqb_{1,y} + 7k^2p^2qb_1\eta_y + 8k^2q^2rb_1\eta_y = 0,$$
$$2kqa_2b_0 + 2kqa_1b_1 + 3kpa_2b_1 + 2qa_2\eta_t + 2k^2q^2b_{1,y} + 12k^2pq^2b_1\eta_y = 0.$$

Solving above over-determined partial differential equations by use of Mathematica, we get the following nontrivial results:

Case 1:

$$a_0 = -1 - 2kqr(f_1'(y)t + f_2'(y)) \pm \frac{f_1'(y)}{k}, \quad a_{-2} = 0, \quad a_{-1} = 0,$$
$$a_1 = -2kpq(f_1'(y)t + f_2'(y)), \quad a_2 = -2kq^2(f_1'(y)t + f_2'(y)), \quad b_{-1} = 0,$$
$$b_0 = \pm kp - \frac{f_1(y) + f_3'(t)}{k}, \quad b_1 = \pm 2kq, \quad \eta = f_1(y)t + f_2(y) + f_3(t),$$

where $f_1(y)$, $f_2(y)$ and $f_3(t)$ are arbitrary functions, $f_1'(y) = \mathrm{d}f_1(y)/\mathrm{d}y$, $f_3'(t) = \mathrm{d}f_3(t)/\mathrm{d}t$. The sign "$\pm$" means that the same sign must be used in a_0, b_0 and b_1.

Case 2:

$$a_0 = -1 - 2kqr(f_1'(y)t + f_2'(y)) \pm \frac{f_1'(y)}{k}, \quad a_1 = 0, \quad a_2 = 0,$$
$$a_{-1} = -2kpr(f_1'(y)t + f_2'(y)), \quad a_{-2} = -2kr^2(f_1'(y)t + f_2'(y)), \quad b_1 = 0,$$
$$b_0 = \mp kp - \frac{f_1(y) + f_3'(t)}{k}, \quad b_{-1} = \mp 2kr, \quad \eta = f_1(y)t + f_2(y) + f_3(t),$$

where the signs "$\pm$" and "$\mp$" mean that different signs must be used in a_0 and b_{-1}, furthermore the same sign must be used in b_0 and b_{-1}.

Case 3:

$$a_0 = -1, \quad a_{-2} = -2kr^2f_1'(y), \quad a_{-1} = -2kprf_1'(y),$$
$$a_1 = -2kpqf_1'(y), \quad a_2 = -2kq^2f_1'(y), \quad b_0 = \pm kp - \frac{f_3'(t)}{k},$$
$$b_{-1} = \pm 2kr, \quad b_1 = \pm 2kq, \quad \eta = f_1(y) + f_3(t),$$

where the sign "$\pm$" means that the same sign must be used in b_0, b_1 and b_{-1}.

For simplification, in the rest of this paper, we introduce the notations:

$$M = \frac{\sqrt{p^2 - 4qr}}{2}, \quad N = \frac{\sqrt{4qr - p^2}}{2}. \tag{10}$$

From Cases 1–3, Appendix 1 in [18] and (8)–(10), we can obtain many more general exact solutions of equations (1) and (2):

Family 1. When $p^2 - 4qr > 0$ and $pq \neq 0$ (or $qr \neq 0$)

For example, if we select $\phi_8(\xi) = \frac{2r\cosh(\sqrt{p^2-4qr}\xi/2)}{\sqrt{p^2-4qr}\sinh(\sqrt{p^2-4qr}\xi/2)-p\cosh(\sqrt{p^2-4qr}\xi/2)}$ from Appendix 1 in [18], and use Case 1 and (8)–(10), we then obtain soliton-like solutions

$$H_{1.1} = -1 - 2kqr\Xi \pm \frac{f_1'(y)}{k} - 2kpq\Xi\phi_8(\xi) - 2kq^2\Xi\phi_8^2(\xi)$$

$$= -1 - 2kqr\Xi \pm \frac{f_1'(y)}{k} - \frac{2kqr\Xi\cosh(M\xi)[4pM\sinh(M\xi) - (p^2+4M^2)\cosh(M\xi)]}{[2M\sinh(M\xi) - p\cosh(M\xi)]^2},$$

$$u_{1.1} = \pm kp - \frac{f_1(y) + f_3'(t)}{k} \pm 2kq\phi_8(\xi)$$

$$= \pm kp - \frac{f_1(y) + f_3'(t)}{k} \pm \frac{4kqr\cosh(M\xi)}{2M\sinh(M\xi) - p\cosh(M\xi)},$$

where $\xi = kx + f_1(y)t + f_2(y) + f_3(t)$, $\Xi = f_1'(y)t + f_2'(y)$.

Family 2. When $p^2 - 4qr < 0$ and $pq \neq 0$ (or $qr \neq 0$)

For example, if we select $\phi_{22}(\xi)$, from Case 1 we obtain periodic formal solutions

$$H_{2.1} = -1 - 2kqr\Xi \pm \frac{f_1'(y)}{k} + \frac{2kqr\Xi\cos(2N\xi)[4pN\sin(2N\xi) + (p^2-4N^2)\cos(2N\xi) \pm 4pN]}{[2N\sin(2N\xi) + p\cos(2N\xi) \pm 2N]^2},$$

$$u_{2.1} = \pm kp - \frac{f_1(y) + f_3'(t)}{k} \mp \frac{4kqr\cos(2N\xi)}{2N\sin(2N\xi) + p\cos(2N\xi) \pm 2N},$$

where $\xi = kx + f_1(y)t + f_2(y) + f_3(t)$, $\Xi = f_1'(y)t + f_2'(y)$.

Family 3. When $r = 0$ and $pq \neq 0$

For example, if we select $\phi_{25}(\xi)$, from Case 1 we obtain soliton-like solutions

$$H_{3.1} = -1 \pm \frac{f_1'(y)}{k} + \frac{2kp^2d\Xi[\cosh(p\xi) - \sinh(p\xi)]}{[d + \cosh(p\xi) - \sinh(p\xi)]^2},$$

$$u_{3.1} = \pm kp - \frac{f_1(y) + f_3'(t)}{k} \mp \frac{2kpd}{d + \cosh(p\xi) - \sinh(p\xi)},$$

where $\xi = kx + f_1(y)t + f_2(y) + f_3(t)$, $\Xi = f_1'(y)t + f_2'(y)$, d is an arbitrary constant.

Family 4. When $q \neq 0$ and $r = p = 0$

If we select $\phi_{27}(\xi)$, from Case 1 we obtain rational function solutions

$$H_{4.1} = -1 \pm \frac{f_1'(y)}{k} - \frac{2kq^2\Xi}{(q\xi + c)^2}, \quad u_{4.1} = -\frac{f_1(y) + f_3'(t)}{k} \mp \frac{2kq}{(q\xi + c)},$$

where $\xi = kx + f_1(y)t + f_2(y) + f_3(t)$, $\Xi = f_1'(y)t + f_2'(y)$, c is an arbitrary constant.

Family 5. When $p^2 - 4qr > 0$ and $pq \neq 0$ (or $qr \neq 0$)

For example, if we select $\phi_1(\xi)$, from Case 2 we obtain soliton-like solutions

$$H_{5.1} = -1 - 2kqr\Xi \pm \frac{f_1'(y)}{k} - \frac{2kqr\Xi[4pM\tanh(M\xi) + p^2 + 4M^2]}{[p + 2M\tanh(M\xi)]^2},$$

$$u_{5.1} = \mp kp - \frac{f_1(y) + f_3'(t)}{k} \pm \frac{4kqr}{p + 2M\tanh(M\xi)},$$

where $\xi = kx + f_1(y)t + f_2(y) + f_3(t)$, $\Xi = f_1'(y)t + f_2'(y)$.

Family 6. When $p^2 - 4qr < 0$ and $pq \neq 0$ (or $qr \neq 0$)

For example, if we select $\phi_{15}(\xi)$, from Case 2 we obtain periodic formal solutions

$$H_{6.1} = -1 - 2kqr\Xi \pm \frac{f_1'(y)}{k} - \frac{2kqr\Xi[4pN(\tan(2N\xi) \pm \sec(2N\xi)) - p^2 + 4N^2]}{[-p + 2N(\tan(2N\xi) \pm \sec(2N\xi))]^2},$$

$$u_{6.1} = \mp kp - \frac{f_1(y) + f_3'(t)}{k} \mp \frac{4kqr}{-p + 2N[\tan(2N\xi) \pm \sec(2N\xi)]},$$

where $\xi = kx + f_1(y)t + f_2(y) + f_3(t)$, $\Xi = f_1'(y)t + f_2'(y)$.

Family 7. When $r = 0$ and $pq \neq 0$

From Case 2 we obtain rational function solutions

$$H_{7.1} = -1 \pm \frac{f_1'(y)}{k}, \quad u_{7.1} = \mp kp - \frac{f_1(y) + f_3'(t)}{k}.$$

Family 8. When $q \neq 0$ and $r = p = 0$

From Case 2 we obtain rational function solutions

$$H_{8.1} = -1 \pm \frac{f_1'(y)}{k}, \quad u_{8.1} = -\frac{f_1(y) + f_3'(t)}{k}.$$

Family 9. When $p^2 - 4qr > 0$ and $pq \neq 0$ (or $qr \neq 0$)

For example, if we select $\phi_2(\xi)$, from Case 3 we get soliton-like solutions

$$H_{9.1} = -1 - \frac{8kq^2r^2 f_1'(y)}{[p + 2M\coth(M\xi)]^2} + \frac{4kpqr f_1'(y)}{p + 2M\coth(M\xi)} + \frac{1}{2}kf_1'(y)[p^2 - 4M^2\coth^2(M\xi)],$$

$$u_{9.1} = -\frac{f_3'(t)}{k} \mp \frac{4kqr}{p + 2M\coth(M\xi)} \mp 2kM\coth(M\xi),$$

where $\xi = kx + f_1(y) + f_3(t)$

Family 10. When $p^2 - 4qr < 0$ and $pq \neq 0$ (or $qr \neq 0$)

For example, if we select $\phi_{13}(\xi)$, from Case 3 we get periodic formal solutions

$$H_{10.1} = -1 - \frac{8kq^2r^2 f_1'(y)}{[-p + 2N\tan(N\xi)]^2} - \frac{4kpqr f_1'(y)}{-p + 2N\tan(N\xi)} - \frac{1}{2}kf_1'(y)[-p^2 + 4N^2\tan^2(N\xi)],$$

$$u_{10.1} = -\frac{f_3'(t)}{k} \pm \frac{4kqr}{-p + 2N\tan(N\xi)} \pm 2kN\tan(N\xi),$$

where $\xi = kx + f_1(y) + f_3(t)$.

Family 11. When $r = 0$ and $pq \neq 0$

For example, if we select $\phi_{25}(\xi)$, from Case 3 we get soliton-like solutions

$$H_{11.1} = -1 + \frac{2kp^2 d f_1'(y)[\cosh(p\xi) - \sinh(p\xi)]}{[d + \cosh(p\xi) - \sinh(p\xi)]^2},$$

$$u_{11.1} = \pm kp - \frac{f_3'(t)}{k} \mp \frac{2kpd}{d + \cosh(p\xi) - \sinh(p\xi)},$$

where $\xi = kx + f_1(y) + f_3(t)$, d is an arbitrary constant.

Family 12. When $q \neq 0$ and $r = p = 0$

If we select $\phi_{27}(\xi)$, from Case 3 we get rational function solutions

$$H_{12.1} = -1 - \frac{2kq^2 f_1'(y)}{(q\xi + c)^2}, \quad u_{12.1} = -\frac{f_3'(t)}{k} \mp \frac{2kq}{(q\xi + c)},$$

where $\xi = kx + f_1(y) + f_3(t)$, c is an arbitrary constant.

From Cases 1–3, Appendix 1 in [18] and (8)–(10), we can also obtain other more general exact solutions of equations (1) and (2), we omit them here for simplicity. These solutions obtained contain some arbitrary functions, which can make us discuss the behaviors of solutions and also provide us enough freedom to construct solutions that may be related to real physical problem. As an illustrative example, one plot of $H_{1.1}$ is shown in Figure 1, from which we can see that $H_{1.1}$ possesses solitonic features.

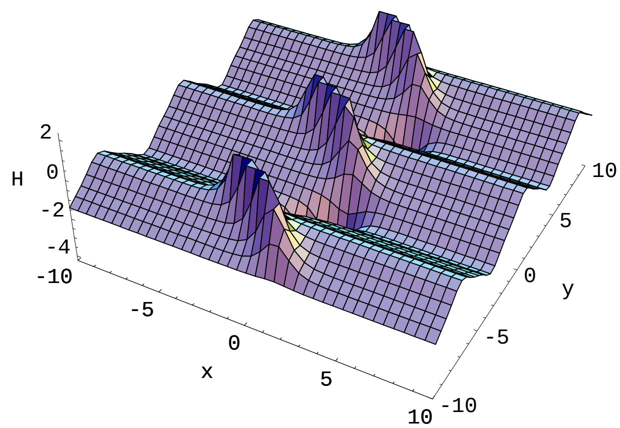

Figure 1. Plot of $H_{1.1}$ is shown at $f_1(y) = \mathrm{sn}(y|0.5)$, $f_2(y) = \mathrm{cn}(y|0.5)$, $f_3(t) = \sin(t)$, $k = q = r = 1$, $p = 3$, $t = \pi/2$, and the sign "$\pm$" selected by "$+$".

REMARK 1. These solutions presented above have not been obtained in [19–21]. Case 1 can be obtained by using the method [17]. However, Cases 2 and 3 can not be obtained by the methods [10–17]. It shows that our method is more powerful in constructing exact solutions of NLPDEs. All solutions reported in this paper have been checked with Mathematica by putting them back into equations (1) and (2).

4 Conclusion

By using a further improved tanh function method, we have constructed many more general exact solutions of the (2+1)-dimensional DLW equations. The arbitrary functions in the solutions imply that these solutions have rich local structures. It may be important to explain some physical phenomena.

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