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Positive Periodic Solutions Of Periodic Retarded Functional Differential Equations *

Liang-long Wang† , Zhi-cheng Wang‡

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Abstract

This paper is concerned with the periodic functional differential equations of retarded type (RFDEs). Sufficient conditions for the existence of positive periodic solutions are established by combining the theory of monotone semiflows generated by RFDEs and fixed point theorems. Nontrivial applications of our results to some periodic ecologic systems are also presented.

1 Introduction

Beginning with the path-breaking work of Hirsch [3, 4] for monotone semiflows, there is now an extensive literature on monotone dynamical systems. Smith, Wu et al., have successfully established the theory of monotone semiflows generated by functional differential equations or FDEs (see [6-9, 12-14] and references cited therein). Recently, there has been a remarkable advance in our understanding of the qualitative as well as the asymptotic of semiflows generated by FDEs on partially ordered spaces which preserve the partial order. One of the most striking results should be that almost every precompact orbit of solutions of FDEs converges to the set of equilibria [6-7]. It is now well-known that the theory of monotone dynamical systems provides a powerful tool for the study of the global dynamics of multi-species cooperative systems, and that a two-species competition system can be transformed into a cooperative system by a simple change of variable. In addition, the theory of monotone semiflows generated by FDEs has also been applied to investigate the asymptotic periodicity of solutions of periodic FDEs [12].

It should be pointed out that Tang and Kuang [10] studied the existence of periodic solutions of general Lotka-Volterra type n-dimensional periodic RFDEs

$$\dot{x}_i(t) = x_i(t)F_i(t, x_1(t), \cdots, x_n(t), x_1(t-\tau_1(t)), \cdots, x_n(t-\tau_n(t))),\ 1 \le i \le n, \quad (1)$$

by combining the theory of monotone semiflow and Horn's fixed point theorem. To some extent, the results of [10] indicated that the uniform persistence and uniform

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†Department of Mathematics, University of Science and Technology of China, Hefei 230026, and Department of Mathematics, Anhui Uiversity, Heifei 230039, P. R. China

‡Department of Applied Mathematics, Hunan University, Changsha 410082, P. R. China

boundedness implied the existence of periodic solutions of the system (1). The paper [11] applied the Horn's asymptotic fixed theorem in Banach spaces to periodic Kolmogorov type system with finite delays

$$\dot{x}_i(t) = x_i(t) f_i(t, x_t), \qquad i = 1, 2, ..., n \tag{2}$$

to obtain the existence of positive periodic solutions under the fundamental assumption that system (2) is uniformly persistent.

Motivated by [10,11], this paper is concerned with general periodic RFDEs

$$\dot{x}(t) = F(t, x_t), \qquad t \geq t_0 \tag{3}$$

where $x \in R^n$ and $F(t+\omega, \varphi) = F(t, \varphi)$ for all $t \in R$ and $\varphi \in C([-r, 0]; R^n)$. We make full use of instinctive merits of monotone semiflow generated by the system (3), define an abstract operator U which is similar to Poincare mapping of dynamical system, look for the fixed point of U and obtain the existence of positive periodic solutions of the system (3) under suitable conditions which are easy to be verified in practice.

This paper is organized as follows. In the next section, we present some notations and preliminaries. The general existence of periodic solutions of the system (3) is given in Section 3. The final section contains applications of our results to some well-known ecologic systems.

2 Preliminaries

Let R^n_+ be the cone of nonnegative vectors in R^n. Let $x, y \in R^n$. We write $x \leq y$ if $x_i \leq y_i$ for each $i \in N = \{1, 2, ..., n\}$. $C = C([-r, 0]; R^n)$ is the Banach space of continuous function mappings $\varphi : [-r, 0] \to R^n$ with supremum norm. If $\varphi, \psi \in C$, we write $\varphi \leq \psi$ (or $\psi \geq \varphi$) in case the indicated inequality holds at each point of $[-r, 0]$. Let $C^+ = \{\varphi \in C : \varphi \geq 0\}$, then C^+ is a positive cone of C which induces the above order relation. Obviously, the cone C^+ is normal, that is, for any $\varphi, \psi \in C^+$ with $||\varphi|| = 1$, $||\psi|| = 1$, there exists a positive constant $\delta > 0$ such that $||\varphi + \psi|| \geq \delta$ [1]. If $\varphi, \psi \in C$ with $\varphi \leq \psi$, we denote $[\varphi, \psi] = \{\xi \in C : \varphi \leq \xi \leq \psi\}$. Let $\hat{}$ denote the inclusion $R^n \to C$ by $x \to \hat{x}$, $\hat{x}(\theta) \equiv x$, $\theta \in [-r, 0]$. For any continuous function $x(\cdot) : [t_0 - r, \sigma) \to R^n$, x_t denotes the element of C, given by $x_t(\theta) = x(t + \theta)$, $-r \leq \theta \leq 0, t \in [t_0, \sigma)$.

Suppose that Ω is an open subset of $R \times C$ and $F : \Omega \to R^n$ is continuous. Consider the retarded functional differential equation (RFDE)

$$\dot{x}(t) = F(t, x_t), \qquad t \geq t_0. \tag{4}$$

We assume throughout this paper that solutions of the initial value problem (4) together with $x_{t_0} = \varphi$, for any $(t_0, \varphi) \in \Omega$, exist and are unique in $[t_0 - r, \infty)$. We refer to [2] for the fundamental theory of solutions of the system (4). We write $x(t; t_0, \varphi, F)$ (or $x_t(t_0, \varphi, F)$) for the solution of the initial value problem (4) together with $x_{t_0} = \varphi$, and we may drop the last argument F from $x(t; t_0, \varphi, F)$ (resp. $x_t(t_0, \varphi, F)$) when no confusion is caused.

We make the following assumptions:

(H_1) $F(t,\varphi)$ is ω-periodic in $t \in R$, that is, $F(t+\omega,\varphi) = F(t,\varphi)$ for any $t \in R$ and $\varphi \in C$.

(H_2) $F(t,\varphi)$ is continuous in $R \times C$.

(H_3) F maps bounded sets of $R \times C$ into bounded sets of R^n.

(H_4) If $(t,\varphi),(t,\psi) \in \Omega$, $\varphi \le \psi$ and $\varphi_i(0) = \psi_i(0)$ for some $i \in N$, then

$$F_i(t,\varphi) \le F_i(t,\psi) \text{ for all } \mathrm{t} \ge \mathrm{t}_0.$$

(H_5) There exist $a, b \in R$, $0 \le a < b$, such that

$$F(t,\hat{A}) \ge 0 \text{ and } F(t,\hat{B}) \le 0 \text{ for any } t \ge t_0,$$

where $A = (a,...,a) \in R^n$, $B = (b,...,b) \in R^n$.

We would like to point out that the assumptions (H_1) and (H_2) are almost always satisfied by systems described in [5] since most of them are Lotka-Volterra type systems. For Lotka-Volterra type systems, (H_3) also holds as long as the coefficient functions are bounded. The assumption (H_4) guarantees that system (4) generates monotone semiflows while (H_5) is satisfied since there are effects of intraspecific crowdings.

In the study of realistic models, only nonnegative solutions of the system are of interest. If (H_4) is replaced by the following assumption

$(H_4)'$ For any $\varphi \in C^+$, if $\varphi_i(0) = 0$ for some $i \in N$, one has $F_i(t,\varphi) \ge 0$ for all $t \ge t_0$, then our results is also true in C^+.

In order to establish the existence of periodic solutions of the system (4), we need the following lemma which is due to Smith [6,7].

LEMMA 2.1. Let Ω be an open subset of $R \times C$, and $F, G : \Omega \to R^n$ be continuous. Suppose that either F or G satisfies (H_4) and $F(t,\varphi) \le G(t,\varphi)$ for all $(t,\varphi) \in \Omega$. Then we have

$$x_t(t_0,\varphi,F) \le x_t(t_0,\varphi,G)$$

for all $t \ge t_0$, whenever both are defined.

Lemma 2.1 not only establishes the desired monotonicity of the semiflow Φ_t but also allows comparisons of solutions between related RFDEs, where a (local) semiflow Φ_t on C can be defined by $\Phi_t(\varphi) = x_t(t_0,\varphi,F)$.

LEMMA 2.2. Let (H_4) and (H_5) hold. Then the order interval $[\hat{A},\hat{B}] = \{\varphi \in C : \hat{A} \le \varphi \le \hat{B}\}$ is positively invariant under the solution semiflow generated by the system (4), that is, $x_t(t_0,\varphi) \in [\hat{A},\hat{B}]$ for any $\varphi \in [\hat{A},\hat{B}]$ and all $t \ge t_0$.

PROOF. For any $\varphi \in [\hat{A},\hat{B}], t \in R$, if $\varphi_i(0) = a$ (resp. $\varphi_i(0) = b$) for some $i \in N$, then we have $F_i(t,\varphi) \ge F_i(t,\hat{A}) \ge 0$ (resp. $F_i(t,\varphi) \le F_i(t,\hat{B}) \le 0$), where the inequality follows from (H_4) and (H_5). From Remark 5.2.1 of [7], we know that the conclusion of the lemma is true and the proof is completed.

3 General Results

In this section, we establish the existence of periodic solutions of system (4). Let K be a positive cone of a real Banach space E and the order relation in E be induced by K.

Let D be a subset of E. An operator $P : D \to E$ is said to be monotone if $\varphi \le \xi$ (where $\varphi, \xi \in D$) implies $P\varphi \le P\xi$. The sequence $\{\varphi^m\}_{m=0}^{\infty} \subset E$ is said to be nondecreasing (resp. nonincreasing) with respect to m, if the order inequality $\varphi^m \le \varphi^{m+1}$ (resp. $\varphi^m \ge \varphi^{m+1}$) holds for any integer $m \ge 0$.

LEMMA 3.1. Let (H_1)-(H_5) hold. Then

$$\{x_{t_0+m\omega}(t_0, \hat{A})\}_{m=0}^{\infty} \ \text{(resp. } \{x_{t_0+m\omega}(t_0, \hat{B})\}_{m=0}^{\infty})$$

in C is nondecreasing (resp. nonincreasing) with respect to m.

PROOF. For any $\varphi \in C$, if $\varphi \ge \hat{A}$ and $\varphi_i(0) = a$ for some $i \in N$, then by (H_4) and (H_5), we know that $F_i(t, \varphi) \ge F_i(t.\hat{A}) \ge 0$. Therefore, by a similar proof of Lemma 2.2, it follows that $[\hat{A}, \infty) = \{\varphi \in C : \varphi \ge \hat{A}\}$ is positively invariant under the system (4), that is, $x_t(t_0, \hat{A}) \ge \hat{A}$ holds for all $t \ge t_0$. In particular, $x_{t_0+\omega}(t_0, \hat{A}) \ge \hat{A}$. Then Lemma 2.1 ($F = G$) implies that $x_{t_0+\omega}(t_0, x_{t_0+\omega}(t_0, \hat{A})) \ge x_{t_0+\omega}(t_0, \hat{A}) \ge \hat{A}$. By (H_1) and the uniqueness of solutions, it follows that

$$x_{t_0+2\omega}(t_0, \hat{A}) \ge x_{t_0+\omega}(t_0, \hat{A}) \ge \hat{A}.$$

Continuing in this manner, we obtain that $\{x_{t_0+m\omega}(t_0, \hat{A})\}_{m=0}^{\infty}$ is nondecreasing with respect to m. Similar arguments apply to $\{x_{t_0+m\omega}(t_0, \hat{B})\}_{m=0}^{\infty}$ and the proof is completed.

LEMMA 3.2. If (H_1)-(H_5) hold, then both $\{x_t(t_0, \hat{A}) : t \ge t_0\}$ and $\{x_t(t_0, \hat{B}) : t \ge t_0\}$ are precompact in C.

PROOF. From Lemma 2.2, we know that $\{x_t(t_0, \hat{A})\}$ is uniformly bounded because of $x_t(t_0, \hat{A}) \in [\hat{A}, \hat{B}]$ for all $t \ge t_0$ and the fact that $[\hat{A}, \hat{B}]$ is bounded since C^+ is a normal cone of C [1]. For any $s \in R$, there exists an integer m such that $s = m\omega + s_0$, where $s_0 \in [0, \omega)$. Hence, from (H_1) and (H_3), there exists a constant number $M > 0$ such that $|F(s, x_s(t_0, \hat{A}))| = |F(s_0, x_s(t_0, \hat{A}))| \le M$ for any $s \ge t_0$. From (4), we know that

$$x(t; t_0, \hat{A}) = x(t_0; t_0, \hat{A}) + \int_{t_0}^{t} F(s, x_s(t_0, \hat{A}))ds$$

from which it follows that

$$|x_t(t_0, \hat{A})(\theta_2) - x_t(t_0, \hat{A})(\theta_1)| = \left| \int_{t+\theta_1}^{t+\theta_2} F(s, x_s(t_0, \hat{A}))ds \right| \le M|\theta_2 - \theta_1|$$

for any $\theta_1, \theta_2 \in [-r, 0]$, that is, $\{x_t(t_0, \hat{A}) : t \ge t_0\}$ is equicontinuous. Hence, $\{x_t(t_0, \hat{A}) : t \ge t_0\}$ is precompact in C. Similar arguments apply to $\{x_t(t_0, \hat{B}) : t \ge 0\}$ and the proof is completed.

In order to show the existence of periodic solutions for the system (4), we define a special solution operator U which is similar to the Poincare mapping of (4). Let $U : C \to C$ be defined by

$$U\varphi = x_\omega(\varphi) \ \text{ for any } \ \varphi \in \mathrm{C}. \tag{5}$$

Here and in what follows, we write $x(t; \varphi)$ for $x(t; 0, \varphi)$, which denotes the unique solution of system (4) together with $x_0 = \varphi$. Then we have the following results.

PROPOSITION 3.3. If (H_1)-(H_5) hold, then the operator U defined by (5) is monotone in C and U maps $[\hat{A}, \hat{B}]$ into itself.

The proof is an immediate consequence of Lemmas 2.1 and 2.2.

PROPOSITION 3.4. If (H_1)-(H_5) hold, then the operator U defined by (5) has a maximal fixed point φ^* and a minimal fixed point φ_* in $[\hat{A}, \hat{B}]$. Moreover, let

$$A^{(0)} = \hat{A}, B^{(0)} = \hat{B}, A^{(m)} = UA^{(m-1)} \quad \text{and} \quad B^{(m)} = UB^{(m-1)}, \ m = 1, 2, \ldots .$$

Then we have

$$A^{(0)} \leq A^{(1)} \leq \cdots \leq A^{(m)} \leq \cdots \leq B^{(m)} \leq \cdots \leq B^{(1)} \leq B^{(0)}, \tag{6}$$

$$\varphi_* = \lim_{m \to \infty} A^{(m)}, \qquad \varphi^* = \lim_{m \to \infty} B^{(m)}. \tag{7}$$

PROOF. From Proposition 3.3 and the fact that the solutions of (4) depend continuously on the initial data [2], we know that $U : [\hat{A}, \hat{B}] \to [\hat{A}, \hat{B}]$ is monotone and continuous. Again by Proposition 3.3 and induction, we see that (6) holds. Thus, we have proved that $\{A^{(m)}\}_{m=0}^{\infty}$ is nondecreasing with respect to m in C. By Lemma 1.1.2 of [7] and Lemma 3.2, $\{A^{(m)}\}_{m=0}^{\infty}$ is convergent in $[\hat{A}, \hat{B}]$ and there exists a point $\varphi_* \in [\hat{A}, \hat{B}]$ such that $A^{(m)} \to \varphi_*$ as $m \to \infty$ in C. Since U is continuous, we get $U\varphi_* = \varphi_*$. Similar arguments apply to $\{B^{(m)}\}_{m=0}^{\infty}$ and hence there is a point $\varphi^* \in [\hat{A}, \hat{B}]$ such that $B^{(m)} \to \varphi^*$ in C and $U\varphi^* = \varphi^*$.

Next, we prove that φ^* and φ_* are the maximal and minimal fixed points of U in $[\hat{A}, \hat{B}]$ respectively. Let $\varphi \in [\hat{A}, \hat{B}]$ and $U\varphi = \varphi$. Since U is monotone, it is easy to see that $U\hat{A} \leq U\varphi \leq U\hat{B}$, i.e., $A^{(1)} \leq \varphi \leq B^{(1)}$. By induction, we obtain $A^{(m)} \leq \varphi \leq B^{(m)}$ for $m = 0, 1, 2, \ldots$. Now, taking limit $m \to \infty$, it follows from the normality of cone C^+ that $\varphi_* \leq \varphi \leq \varphi^*$, and the proof is completed.

COROLLARY 3.5. Let (H_1)-(H_5) hold. If U has only one fixed point φ in $[\hat{A}, \hat{B}]$, then for any $\psi \in [\hat{A}, \hat{B}]$, the successive iterates

$$\psi^{(m)} = U\psi^{(m-1)} \quad (m = 1, 2, \ldots) \quad \text{with} \quad \psi^{(0)} = \psi \tag{8}$$

converge to φ in C, that is, $||\psi^{(m)} - \varphi|| \to 0$ as $m \to \infty$.

PROOF. Since $\hat{A} \leq \psi = \psi^{(0)} \leq \hat{B}$ and U is monotone, we have

$$A^{(m)} \leq \psi^{(m)} \leq B^{(m)}, \qquad m = 0, 1, 2, \ldots . \tag{9}$$

By hypotheses, we must have $\varphi^* = \varphi_* = \varphi$. It follows therefore from (7), (9), the normality of cone C^+ of C and Proposition 3.4 that $\psi^{(m)} \to \varphi$ as $m \to \infty$. The proof is completed.

THEOREM 3.6. Let (H_1)-(H_5) hold. Then $x(t; \hat{A})$ and $x(t; \hat{B})$ converge to positive ω-periodic solutions as $t \to \infty$.

PROOF. Let $y(t) = A$ for $t \in R$, then

$$y'(t) = 0 \leq F(t, \hat{A}) = F(t, y_t).$$

It follows from Lemma 2.1 that

$$A = y(t) \le x(t; \hat{A}) \quad \text{for all} \quad t \ge 0.$$

In a similar way we know $0 \le x(t; \hat{B}) \le B$ for all $t \ge 0$. Again by Lemma 2.1, it follows that

$$A \le x(t; \hat{A}) \le x(t; \hat{B}) \le B \quad \text{for all } t \ge 0. \tag{10}$$

From Proposition 3.4, we know that there exist $\varphi_*, \varphi^* \in [\hat{A}, \hat{B}]$ such that

$$\lim_{m\to\infty} U^m(\hat{A}) = \varphi_*, \quad \lim_{m\to\infty} U^m(\hat{B}) = \varphi^*, \quad U\varphi_* = \varphi_* \quad \text{and} \quad U\varphi^* = \varphi^*$$

where $U^m(\varphi)$ denotes the m-th iterate of φ under U. It is easy to see from (10) that $x(t; \hat{A})$ converges to the positive ω-periodic solution $x(t; \varphi_*)$ and $x(t; \hat{B})$ tends to the positive ω-periodic solution $x(t; \varphi^*)$ as $t \to \infty$. The proof is completed.

To conclude this section, we give the following remark.

REMARK 3.7. First, Theorem 3.6 implies that the system (4) has at least one positive ω-periodic solution. Next, suppose the assumption (H_5) is replaced by the following:

$(H_5)'$ For any $s \in (0, 1]$, $\xi \in [1 + \infty)$, $F(t, \hat{A}s) \ge 0$ and $F(t, \hat{B}\xi) \le 0$ for any $t \ge t_0$, where $As = (sa, \cdots, sa) \in R^n$, $B\xi = (\xi b, \cdots, \xi b) \in R^+$.

Then from Theorem 3.6 and Corollary3.5, we know that if the system (4) admits a unique positive ω-periodic solution, then this periodic solution attracts each solution $x(t; \varphi)$ of system (4) with $\varphi \in C^+$ and $\varphi \ne 0$, that is, this periodic solution is globally attractive in $C^+ \setminus \{0\}$.

4 Applications

The object of this section is to apply the results in the previous section to some well-known population models.

We first consider the following n-dimensional delay Lotka-Volterra system

$$\dot{x}_i(t) = x_i(t) \left[c_i(t) - a_{ii}(t)x_i(t) + \sum_{j=1, j\ne i}^{n} a_{ij}(t)x_j(t) + \sum_{j=1}^{n} b_{ij}(t)x_j(t - \tau(t)) \right], \tag{11}$$

where $1 \le i \le n$. For the system (11) we assume that

(i) $a_{ij}(t), c_i(t), b_{ij}(t)$, $i, j = 1, ..., n$, and $\tau(t)$ are continuous, ω-periodic functions with $c_i(t) > 0$, $a_{ii}(t) > 0$, $a_{ij}(t) \ge 0$, $b_{ij}(t) \ge 0$ and $\tau(t) \ge 0$ for $t \in R$.

(ii) For $t \in R$ and $i = 1, ..., n$,

$$a_{ii}(t) > \sum_{j=1, j\ne i}^{n} a_{ij}(t) + \sum_{j=1}^{n} b_{ij}(t). \tag{12}$$

THEOREM 4.1. Under the above assumptions, the system (11) has positive ω-periodic solutions.

PROOF. Let $r = \max\{\tau(t) : t \in [0,\omega]\}$, $C = C([-r,0]; R^n)$, $C^+ = C([-r,0]; R^n_+)$, and for $i = 1, 2, ..., n$,

$$F_i(t,\varphi) = \varphi_i(0)\left[c_i(t) - a_{ii}(t)\varphi_i(0) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)\varphi_j(0) + \sum_{j=1}^{n} b_{ij}(t)\varphi_j(-\tau(t))\right].$$

Obviously, the assumption (i) yields (H_1)-(H_3). For any $\varphi, \psi \in C^+, \varphi \le \psi$ and $\varphi_i(0) = \psi_i(0)$ for some $i \in N$, we have

$$F_i(t,\psi) - F_i(t,\varphi) \ge \varphi_i(0)\sum_{j=1}^{n} b_{ij}(t)[\psi_i(-\tau(t)) - \varphi_j(-\tau(t))] \ge 0,$$

that is, (H_4) holds. Furthermore, if $A = (a, ..., a)$ and $a > 0$ is sufficiently small, by the assumption (ii), we have

$$F_i(t,\hat{A}) = a\left\{c_i(t) - a\left[a_{ii}(t) - \sum_{j=1, j\neq i}^{n} a_{ij}(t) - \sum_{j=1}^{n} b_{ij}(t)\right]\right\} \ge 0$$

for $t \in R$ and $i = 1, 2, ..., n$. If $B = (b, ..., b)$ and $b > 0$ is sufficiently large, we have

$$F_i(t,\hat{B}) = b\left\{c_i(t) - b\left[a_{ii}(t) - \sum_{j=1, j\neq i}^{n} a_{ij}(t) - \sum_{j=1}^{n} b_{ij}(t)\right]\right\} \le 0$$

for $t \in R$ and $i = 1, 2, ..., n$, that is, $(H_5)'$ is valid. Hence the conclusion of theorem is true and the proof is completed.

For single-species models with periodic delay and coefficients, we have the following result.

COROLLARY 4.2. Consider the equation

$$\dot{x}(t) = x(t)[c(t) - a(t)x(t) + b(t)x(t - \tau(t))] \tag{13}$$

where $c(t), a(t), b(t), \tau(t)$ are continuous, ω-periodic. Suppose that (i) $c(t) > 0, a(t) > 0, b(t) \ge 0$ and $\tau(t) \ge 0$ for $t \in R$, and (ii) $a(t) > b(t)$ for $t \in R$. Then (13) has positive ω-periodic solutions.

Finally, we consider a model to describe the growth of a single-species population dispersing in an n patch environment which is periodically fluctuating as follows

$$\dot{M}_i(t) = -\beta_i(t)M_i^2(t) + \sum_{j=1, j\neq i}^{n} d_{ij}(t)[M_j(t) - M_i(t)] + \sum_{j=1}^{n} b_{ij}(t)M_j(t - \tau(t)) \tag{14}$$

when $t \ge 0$ and $i = 1, \cdots, n$, $M_i(t)$ denotes the concentration of mature populations in the i-th patch. When the coefficients of system (14) are constant, (14) has been widely investigated (see paper [5] and references cited therein). There are sufficient reasons to consider periodic cases of (14) (e.g., seasonal effects of weather, food supplies, mating habits, etc.). For details, we refer to [5].

For system (14), we make the following assumptions:

(i) $\beta_i(t), d_{ij}(t)$ and $\tau(t)$ are continuous, ω-periodic in $t \in R$, $d_{ii}(t) = 0$, $i, j = 1, 2, ..., n$.

(ii) $\beta_i(t) > 0, d_{ij}(t) \geq 0, b_{ij}(t) \geq 0$ and $\sum_{j=1}^n b_{ij}(t) > 0$ for $t \in R$.

THEOREM 4.3. Under the above assumptions,the system (14) has positive ω-periodic solutions.

PROOF. Let $r = \max\{\tau(t); t \in [0, \omega]\} > 0$, $C = C([-r, 0]; R^n)$, $C^+ = C([-r, 0]; R^n_+)$, and

$$F_i(t, \varphi) = -\beta_i(t)\varphi_i^2(0) + \sum_{j=1, j\neq i}^n d_{ij}(t)[\varphi_j(0) - \varphi_i(0)] + \sum_{j=1}^n b_{ij}(t)\varphi_j(-\tau(t)).$$

Obviously, (H_1)-(H_3) is satisfied. For any $\varphi \in C^+$, if $\varphi_i(0) = 0$ for some $i \in N$, we have

$$F_i(t, \varphi) = \sum_{j=1, j\neq i}^n d_{ij}(t)\varphi_j(0) + \sum_{j=1}^n b_{ij}\varphi_j(-\tau(t)) \geq 0,$$

that is, $(H_4)'$ is valid. Clearly, $(H_5)'$ holds. Hence the conclusion of the theorem is true from Theorem 3.6 and Remark 3.7.

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