Homework Assignment 3.

Given Mar 05, due Mar 16.

1. Section 9.7: problems 1, 3, 7, 25, 29, 33, 50, 54, 57, 58.

2. Section 9.8: problems 10, 14, 28, 29, 31, 32, 33, 34(a).

3. Chap 9: 27, 28,29, 30, 32, 34, 49-54.

4. Read p 619, which says the Taylor series generated by $(1+x)^m$, $m \in R$, converges on $|x| < 1$. It is not clear the series converges to $(1+x)^m$ itself since analyzing $R_n(x)$ directly is not quite easy (try it and you'll see why). An alternative approach is through the following steps:

(a) Verify that

$$(k+1)\binom{m}{k+1} + k\binom{m}{k} = m\binom{m}{k}$$

(b) Define, for $|x| < 1$,

$$f(x) = \sum_{k=0}^{\infty} \binom{m}{k} x^k = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots.$$

Show that $f(0) = 1$ and

$$(1+x)f'(x) = mf(x)$$

on $|x| < 1$.

(c) Show that $f(x) = (1+x)^m$.