Extreme Values of Functions and Graphing

Def: local maximum, minimum
absolute maximum, ~~absolute~~ minimum

Prop: If f is continuous on $[a, b]$ (closed interval)
then f takes an absolute max and absolute min.

Possible extreme points

(a) interior points where $f' = 0$

(b) interior points where f' does not exist

(c) end points

Def: (a) and (b) are called critical points

Remark: If not on a closed interval, the extreme values need not exist

Example: $f(x) = x$ on $(0, 1)$

Methods of finding ~~extr~~ extreme values of continuous function.

<u>Prop</u>: <u>If</u> f has a local max or local min at an interior point c ~~and~~

<u>And</u> If $f'(c)$ exists,

<u>Then</u> $f'(c) = 0$

<u>pf</u>: Use linear approximation

<u>Prop</u>: f is continous on $[a, b]$ and differentiable on (a, b)

Then (1) $f'(x) > 0$ for all x in (a, b)

$\Rightarrow f$ is increasing on $[a, b]$

(2) $f'(x) < 0$ for all x in (a, b)

$\Rightarrow f$ is decreasing on $[a, b]$

<u>Remark</u>: "$\Leftarrow$" ~~may not~~ does not hold (can't exclude "$f' = 0$")

Application

At critical point c

f': "+" c "−" $\Rightarrow$ local max

f': "−" c "+" $\Rightarrow$ local min

f': "+" c "+" $\Rightarrow$ no local extreme

or "−" "−"

End point

f': [a "−" or f': "+" b] $\Rightarrow$ local max

f': [a "+" or f': "−" b] $\Rightarrow$ local min

Example $f(x) = x^{\frac{1}{3}}(x-4)$

find local extremes of f

(1) $f'(x) = \frac{4}{3} x^{-\frac{2}{3}} (x-1)$

critical points: $x = 0, 1$

(2)

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
f'	$-$	$-$	$+$
	$\searrow$	$\searrow$	$\nearrow$

(3) f decreasing on $(-\infty, 1)$

and increasing on $(1, \infty)$

Thus $x=1$ is a local min and absolute min as well

(4) $f''(x) = \frac{4}{9} x^{-\frac{5}{3}} (x+2)$

	$(-\infty, -2)$	$(-2, 0)$	$(0, \infty)$
$x^{-\frac{5}{3}}$	"$-$"	"$-$"	"$+$"
$x+2$	"$-$"	"$+$"	"$+$"
f''	"$+$"	"$-$"	"$+$"

Concave up on $(-\infty, -2), (0, \infty)$

Concave down on $(-2, 0)$

The Graph of f can be concluded from

(1), (2), (3)(4)