

# Extreme Values of Functions and Graphing

Def: local maximum, minimum  
absolute maximum, ~~absolute~~ minimum

Prop: If  $f$  is continuous on  $[a, b]$  (closed interval)  
then  $f$  takes an absolute max and absolute min.

Possible extreme points

(a) interior points where  $f' = 0$

(b) interior points where  $f'$  does not exist

(c) end points

Def: (a) and (b) are called critical points

Remark: ~~A~~ If not on a closed interval,  
the extreme values need not exist

Example:  $f(x) = x$  on  $(0, 1)$

Methods of finding ~~ext~~ extreme values  
of continuous function.

Prop: If  $f$  has a local max or local min  
at an interior point  $c$  ~~and~~

And If  $f'(c)$  exists,

Then  $f'(c) = 0$

pf: Use linear approximation

Prop:  $f$  is continuous on  $[a, b]$  and differentiable  
on  $(a, b)$

Then (1)  $f'(x) > 0$  for all  $x$  in  $(a, b)$

$\Rightarrow f$  is increasing on  $[a, b]$

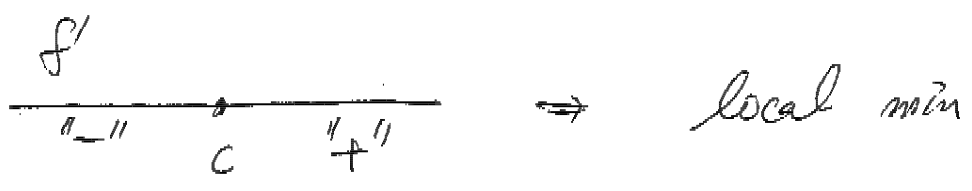
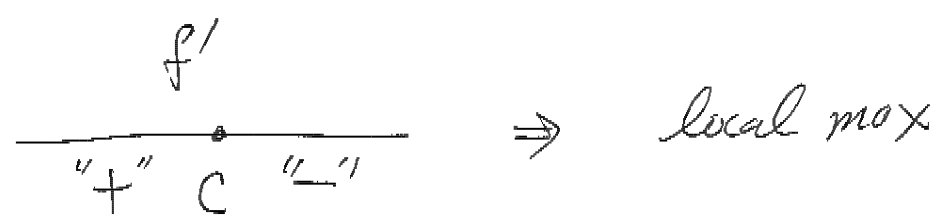
(2)  $f'(x) < 0$  for all  $x$  in  $(a, b)$

$\Rightarrow f$  is decreasing on  $[a, b]$

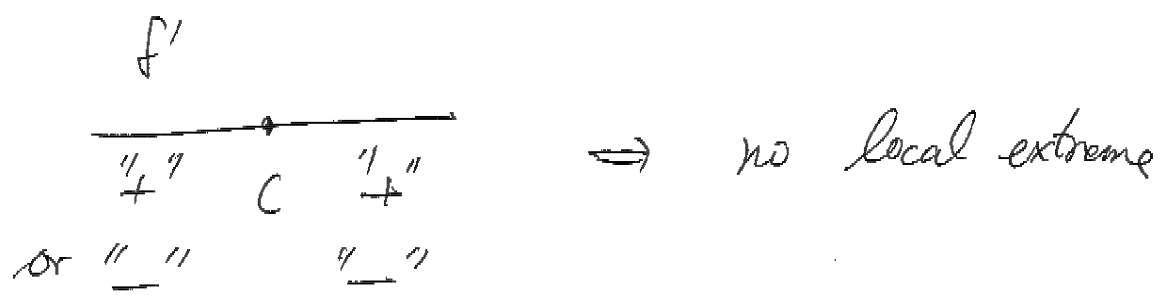
Remark: " $\Leftarrow$ " ~~may not~~  
does not hold (can't exclude " $f' = 0$ ")

Application

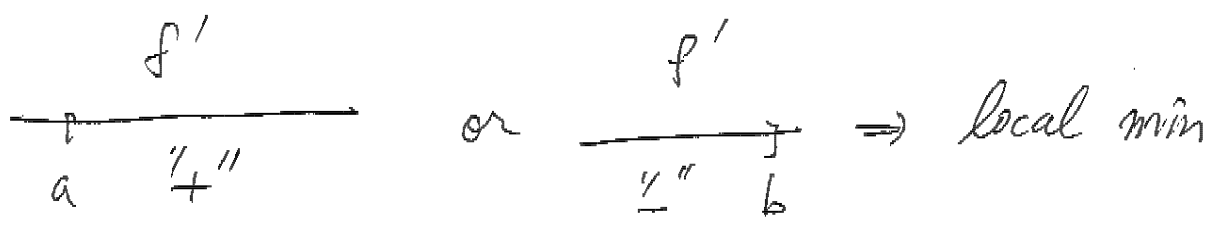
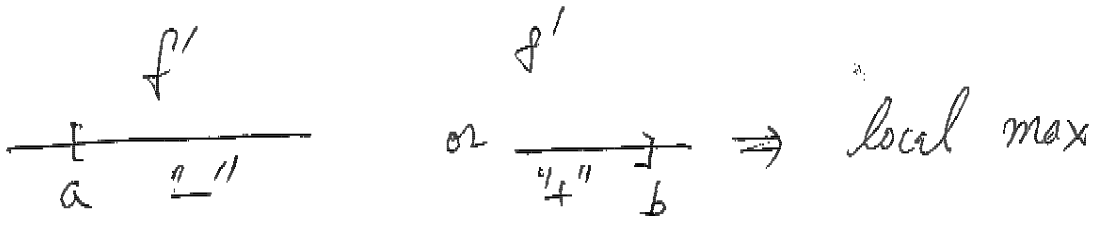
At critical point  $c \in$



$\Rightarrow$



End point

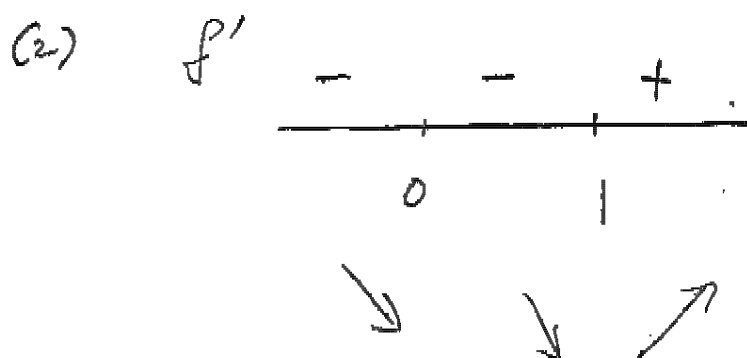


Example  $f(x) = x^{\frac{1}{3}}(x-4)$

find local extremes of  $f$

(1)  $f'(x) = \frac{4}{3} x^{-\frac{2}{3}}(x-1)$

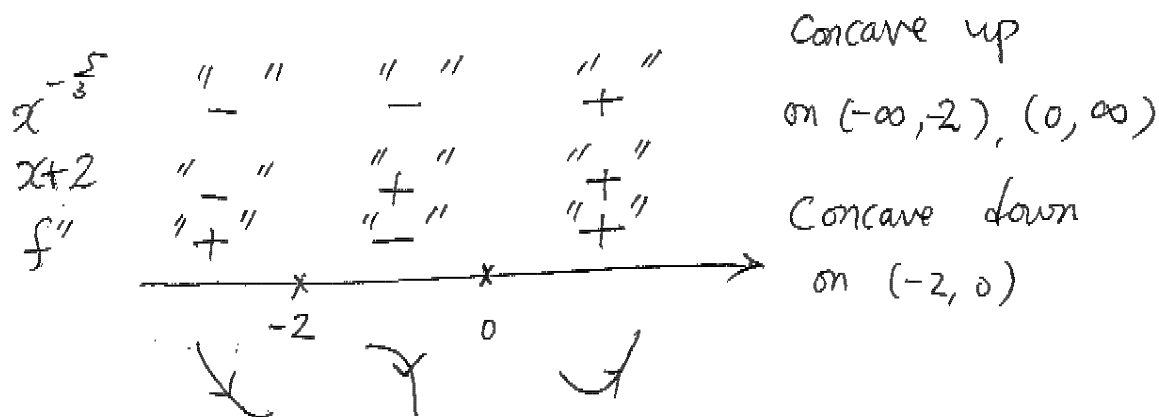
critical points:  $x=0, 1$



(3)  $f$  decreasing on  $(-\infty, 1)$   
and increasing on  $(1, \infty)$

Thus  $x=1$  is a local min and absolute min as well

(4)  $f''(x) = \frac{4}{9} x^{-\frac{5}{3}}(x+2)$



The Graph of  $f$  can be concluded from

(1), (2), (3), (4)

