Brief solutions to selected problems in homework 04

1. Section 10.8: Solutions, common mistakes and corrections:

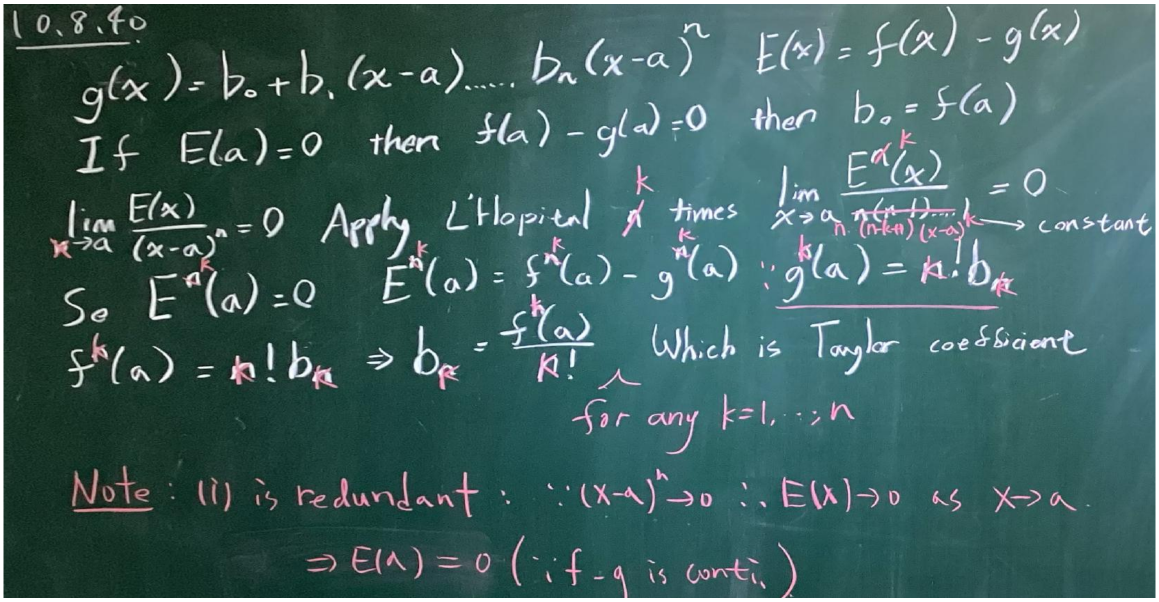

Figure 1: Solution to Section 10.8, problem 40

Remark:

The assumption "$f(x)$ is differentiable on $\cdots$" should be understood as "$f(x)$ is differentiable infinitely many times on $\cdots$".

Or, at least we need to assume

"$f^{(n)}(x)$ is continuous at $x = a$ (therefore so are $f^{(k)}(x)$, $0 \le k < n-1$)."

Note 1:

$\lim\limits_{x\to a}\dfrac{F(x)}{G(x)} = 0$ does not imply (from L'Hôpital's Rule) that $\lim\limits_{x\to a}\dfrac{F'(x)}{G'(x)} = 0$ unless you know in advance that $\lim\limits_{x\to a}\dfrac{F'(x)}{G'(x)}$ indeed exists (see counter example in Lecture 14 from last semester, 20251021). This is why we need

"$f^{(n)}(x)$ is continuous at $x = a$ (therefore so are $f^{(k)}(x)$, $0 \le k < n-1$)."

Note 2:

(i)+(ii)+"$f(x)$ is differentiable on $\cdots$" does not imply $f''(a)$ exists (counter example: $f(x) = |x|^{2.1}\sin\dfrac{1}{x}$, $a = 0$).

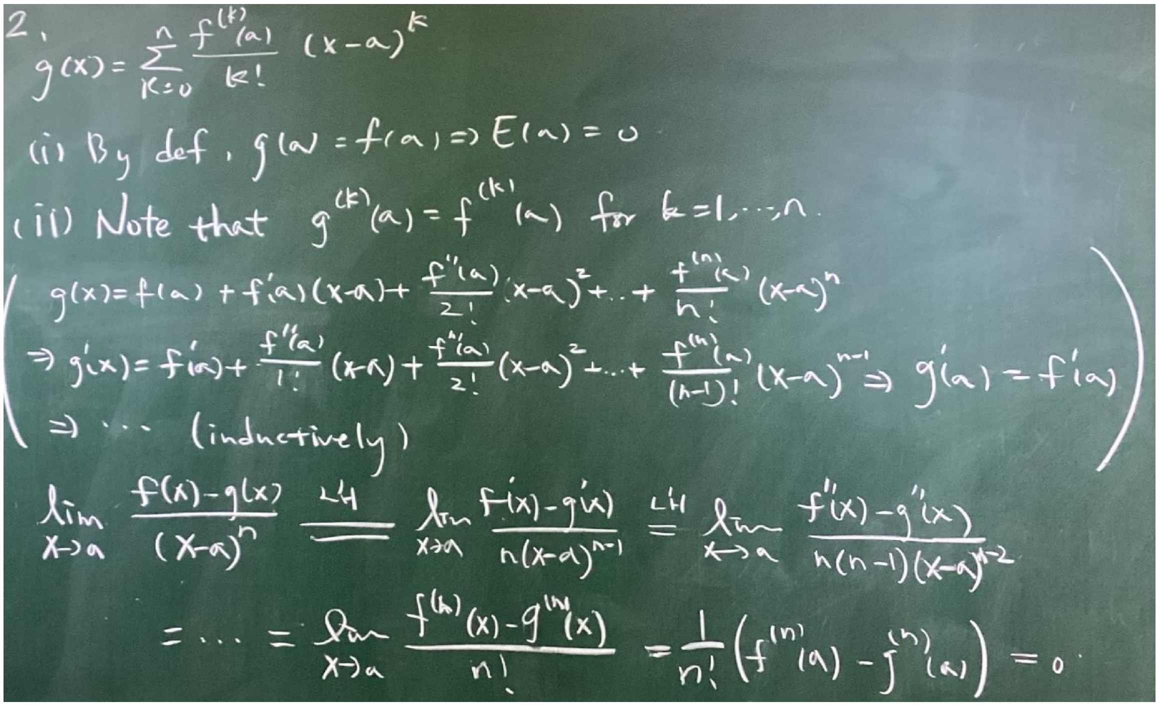

Figure 2: Solution to homework 05, problem 2

Remark: The assumption

”$f^{(n)}(x)$ is continuous at $x = a$ (therefore so are $f^{(k)}(x)$, $0 \le k < n-1$.”

weaker than the one given in this problem, but is sufficient for this problem.

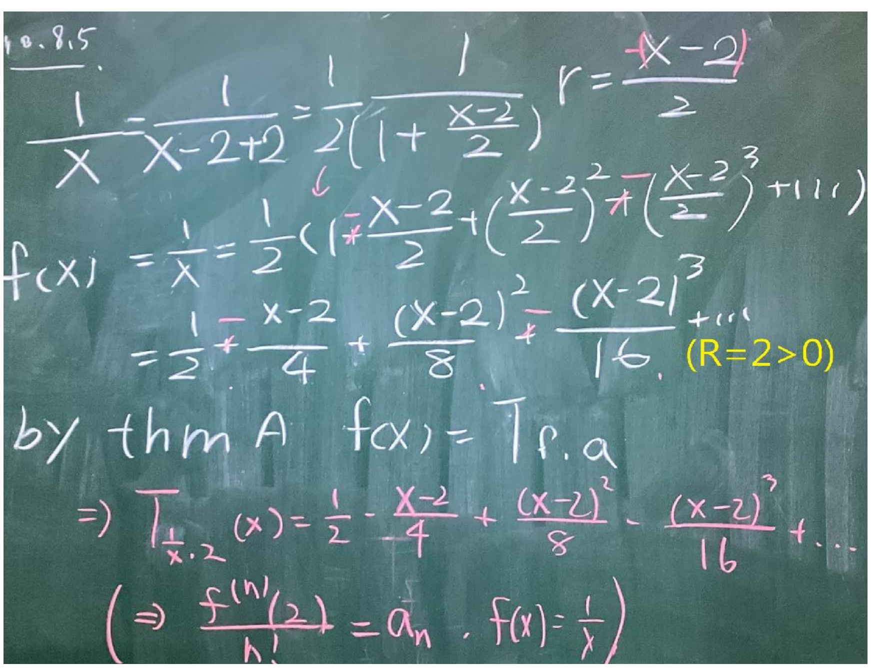

Figure 3: Solution to Section 10.8, problem 5, method 2

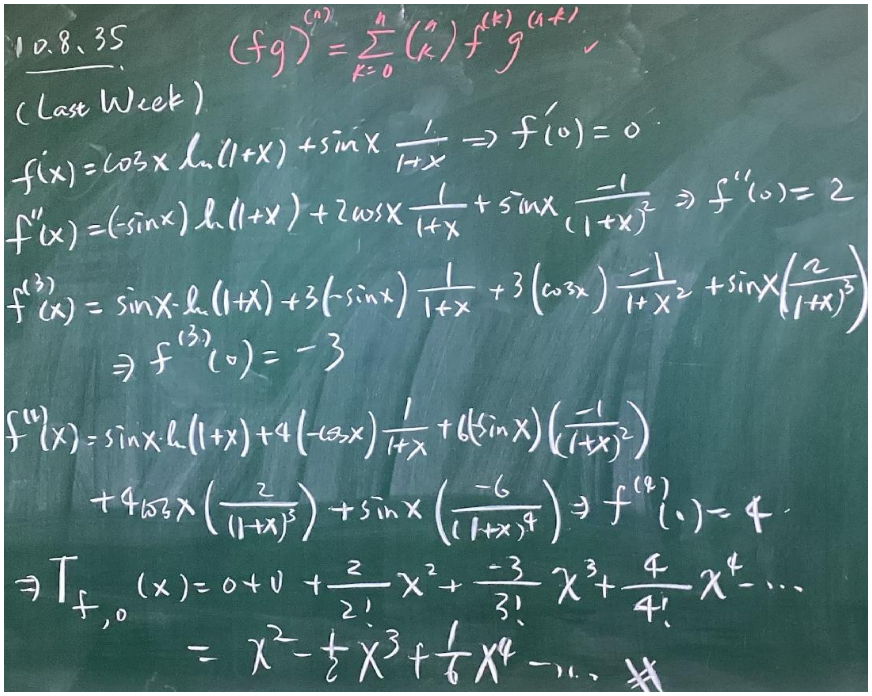

Figure 4: Solution to Section 10.8, problem 35, method 1

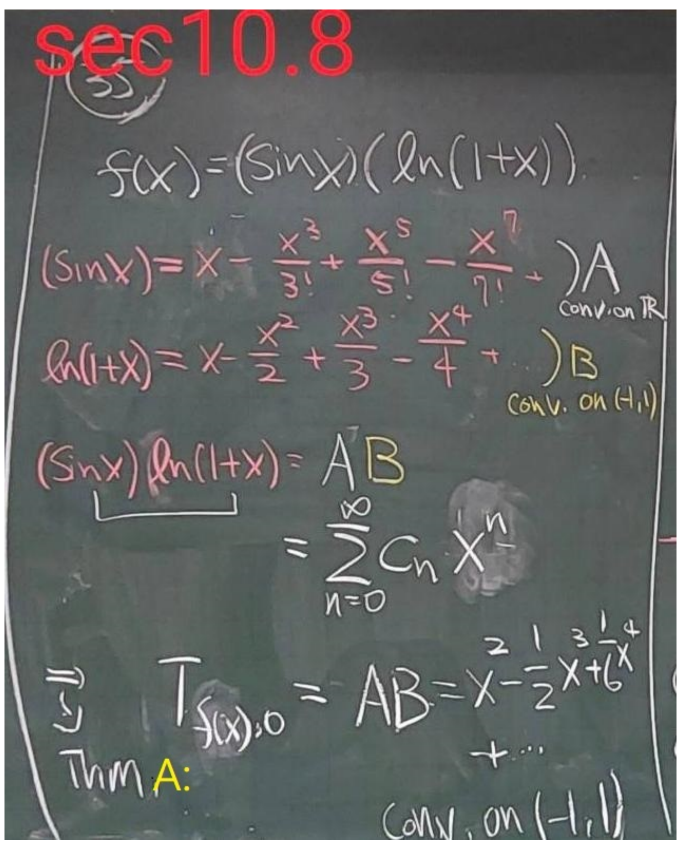

Figure 5: Solution to Section 10.8, problem 35, method 2

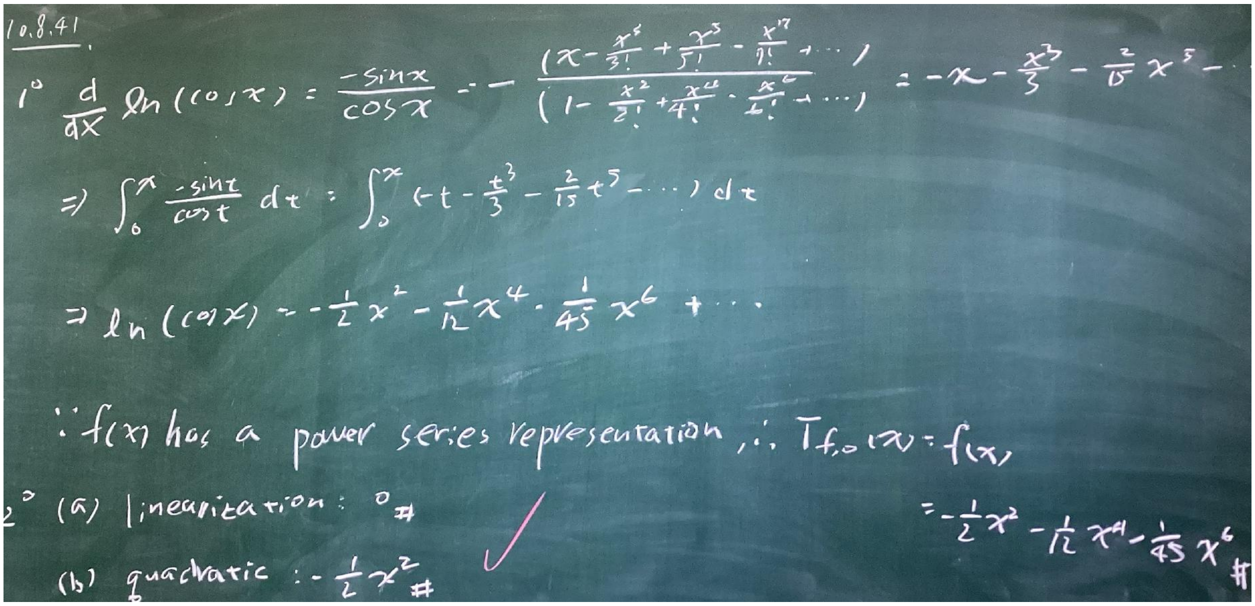

Figure 6: Solution to Section 10.8, problem 41

Method 2: $\ln(\cos x)$ is obviously infinitely many times differentiable near $x = 0$. Since we don't need $T_{\ln(\cos x),0}$ in this problem, it will be easier to find linear and quadratic approximation by differentiating $\ln(\cos x)$ twice at $x = 0$.

2. Section 10.9: Solutions, common mistakes and corrections:

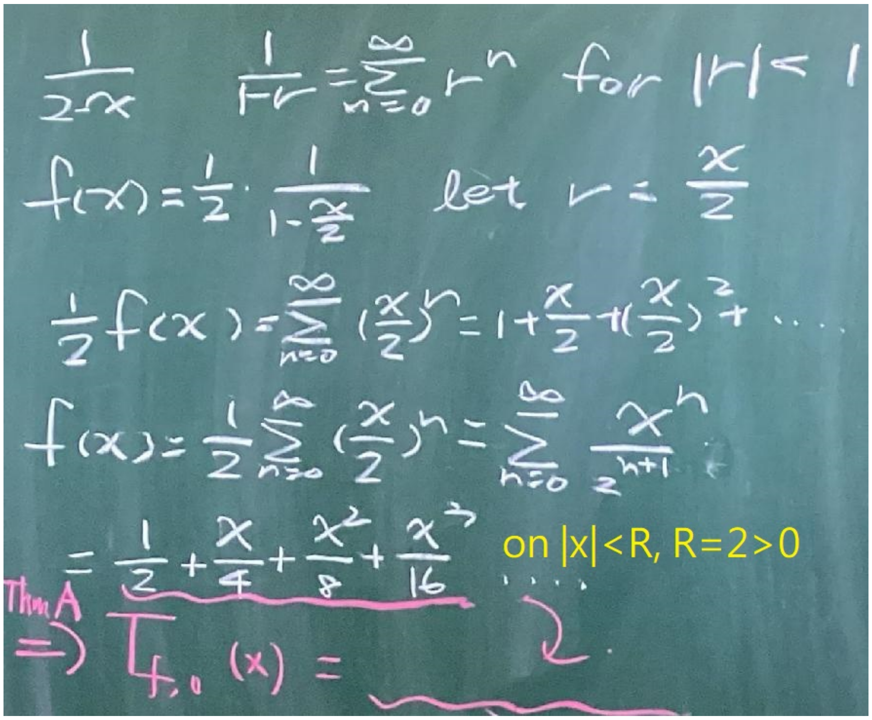

Figure 7: Solution to Section 10.9, problem 10

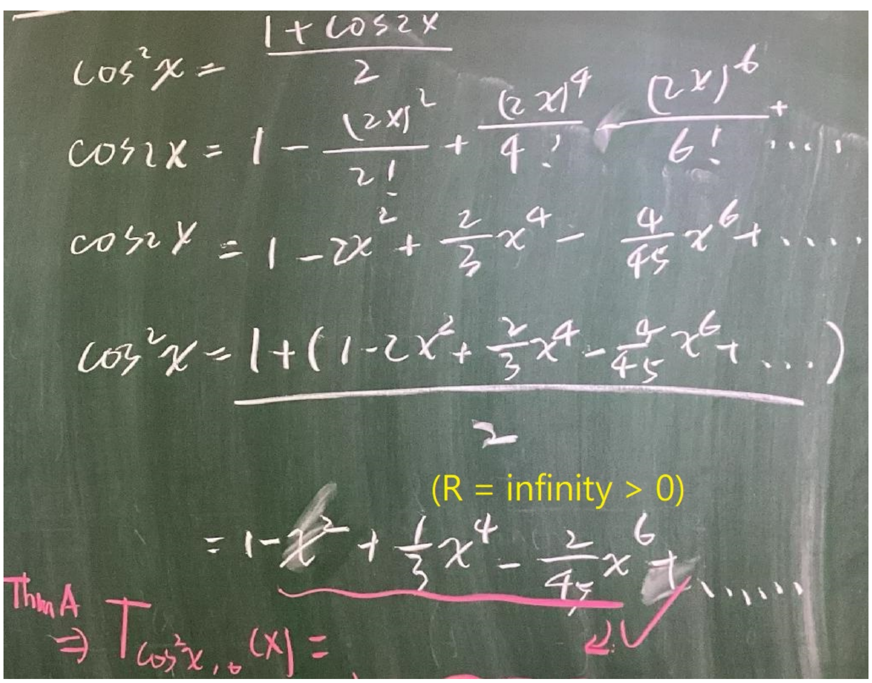

Figure 8: Solution to Section 10.9, problem 17

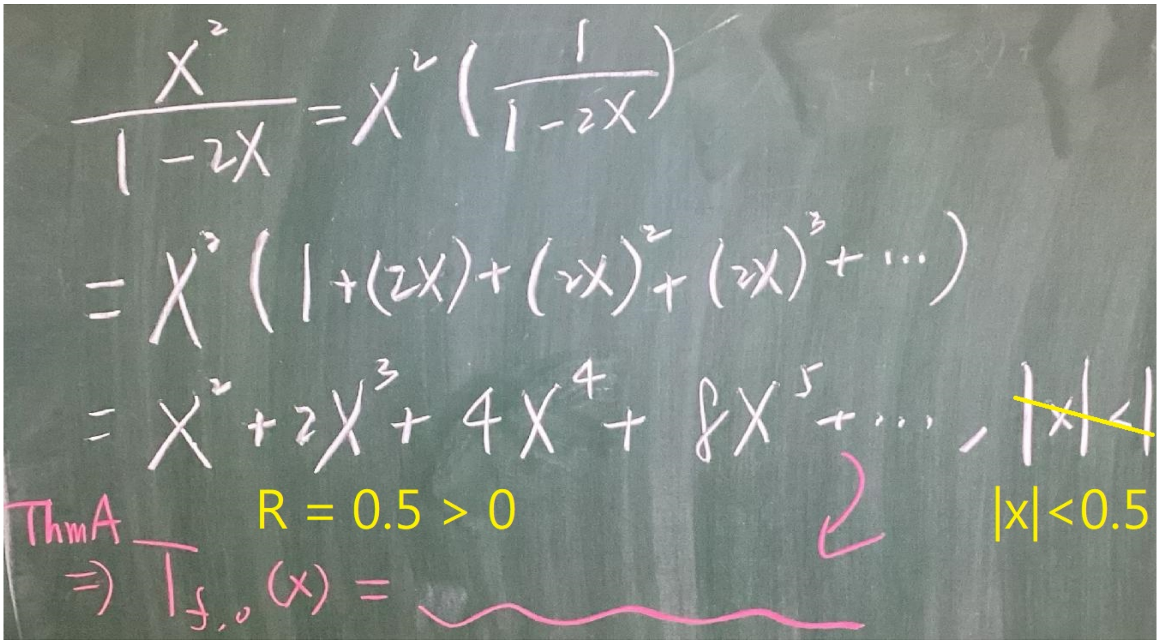

Figure 9: Solution to Section 10.9, problem 19

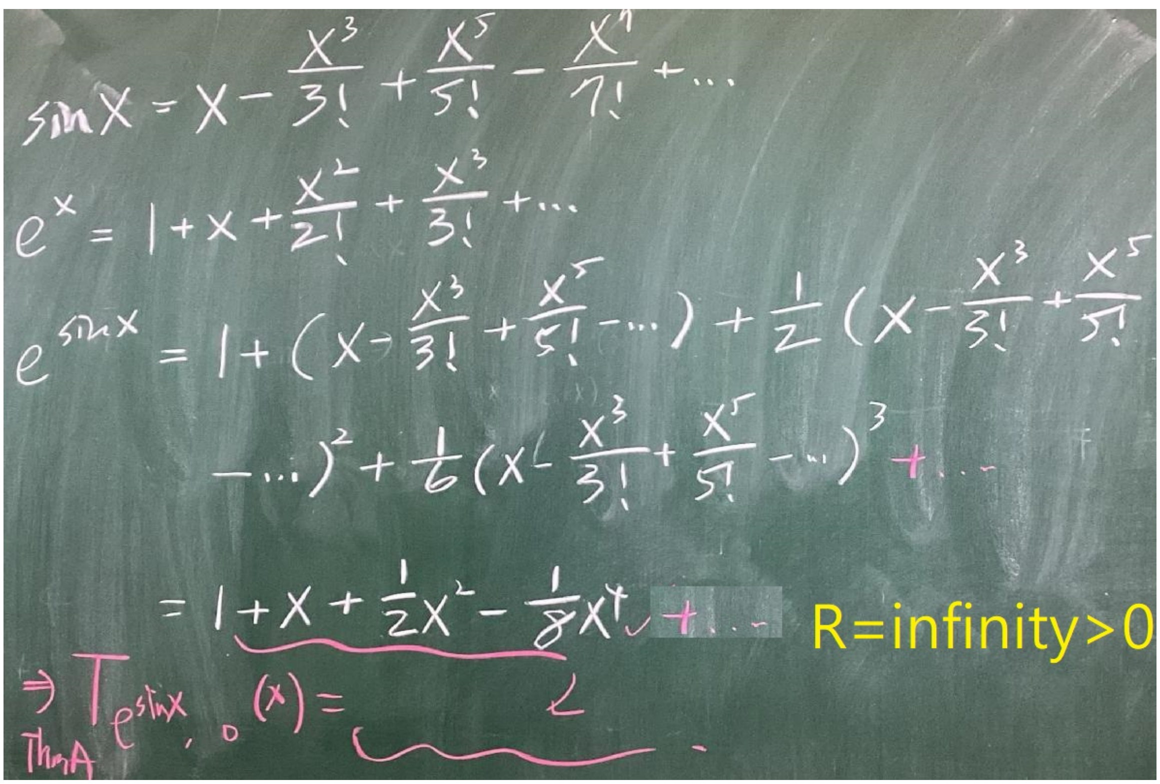

Figure 10: Solution to Section 10.9, problem 33

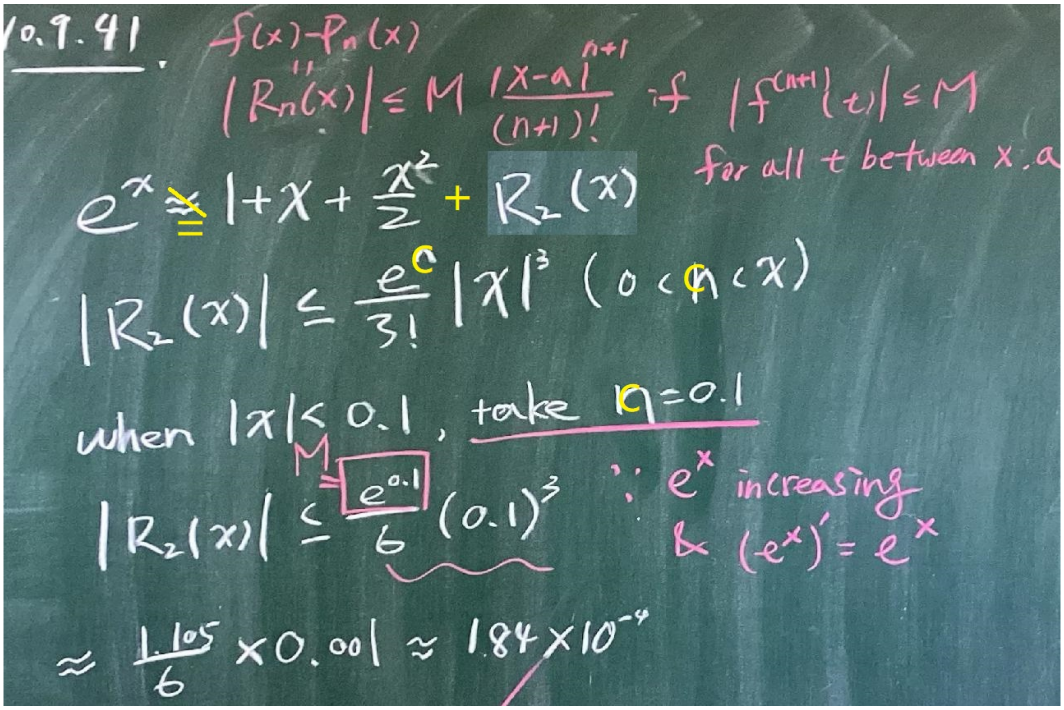

Figure 11: Solution to Section 10.9, problem 41

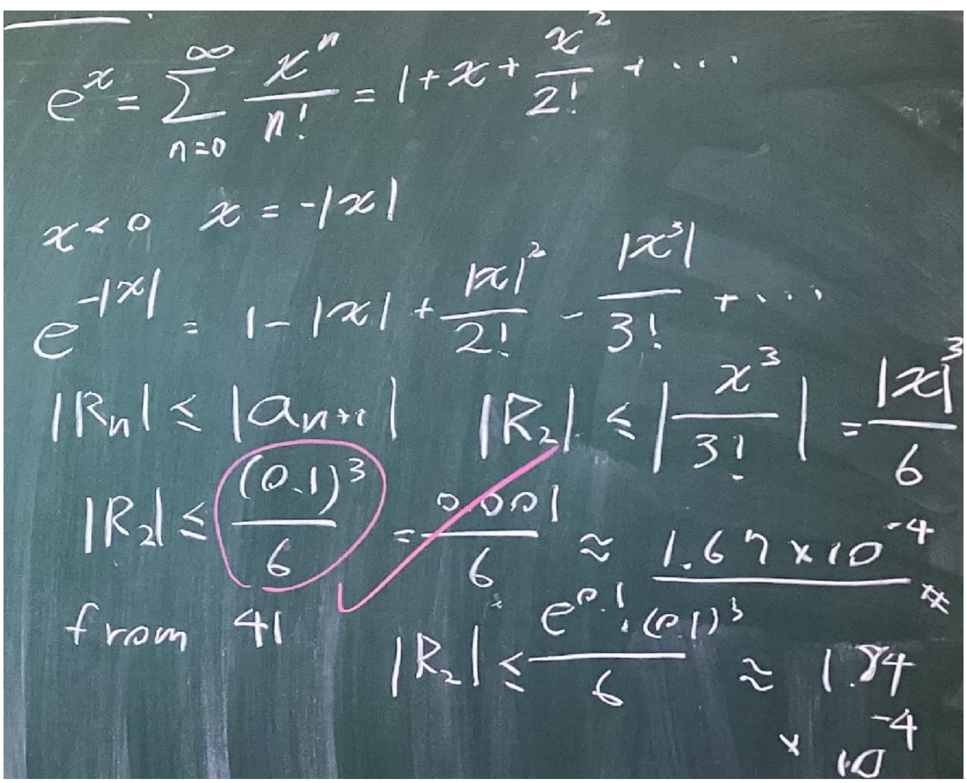

Figure 12: Solution to Section 10.9, problem 42

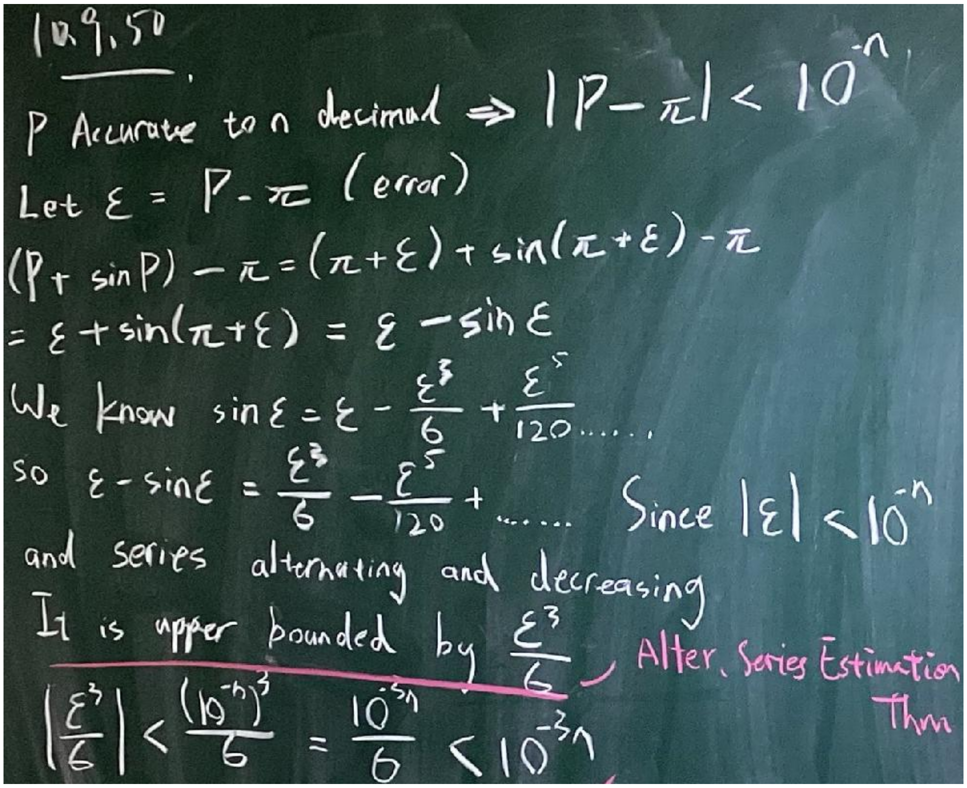

Figure 13: Solution to Section 10.9, problem 50

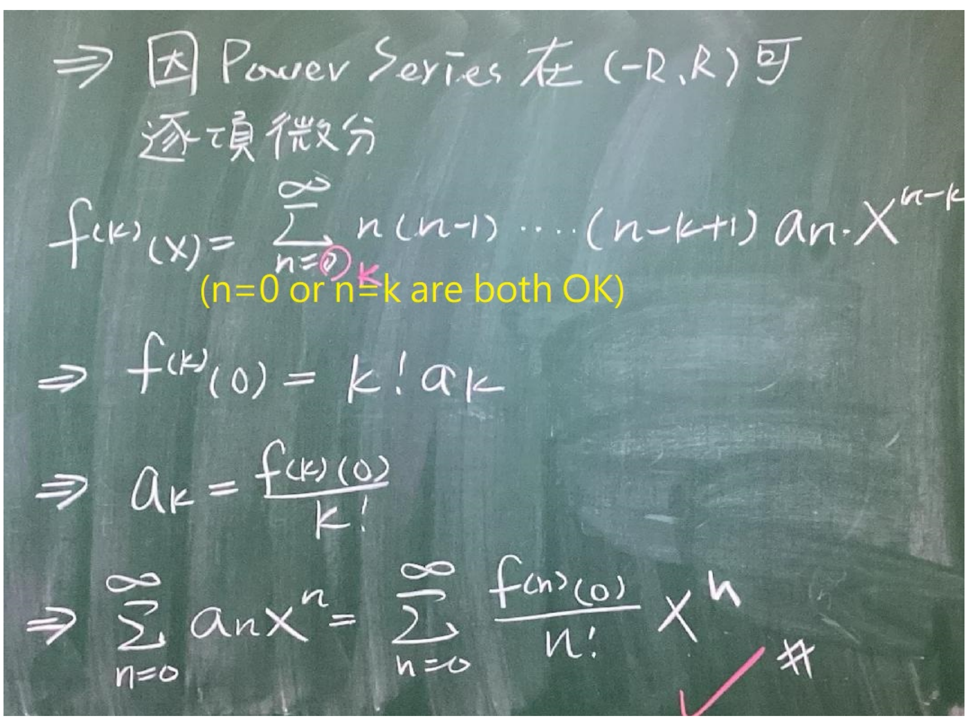

Figure 14: Solution to Section 10.9, problem 51

3. Section 10.10: Solutions, common mistakes and corrections:

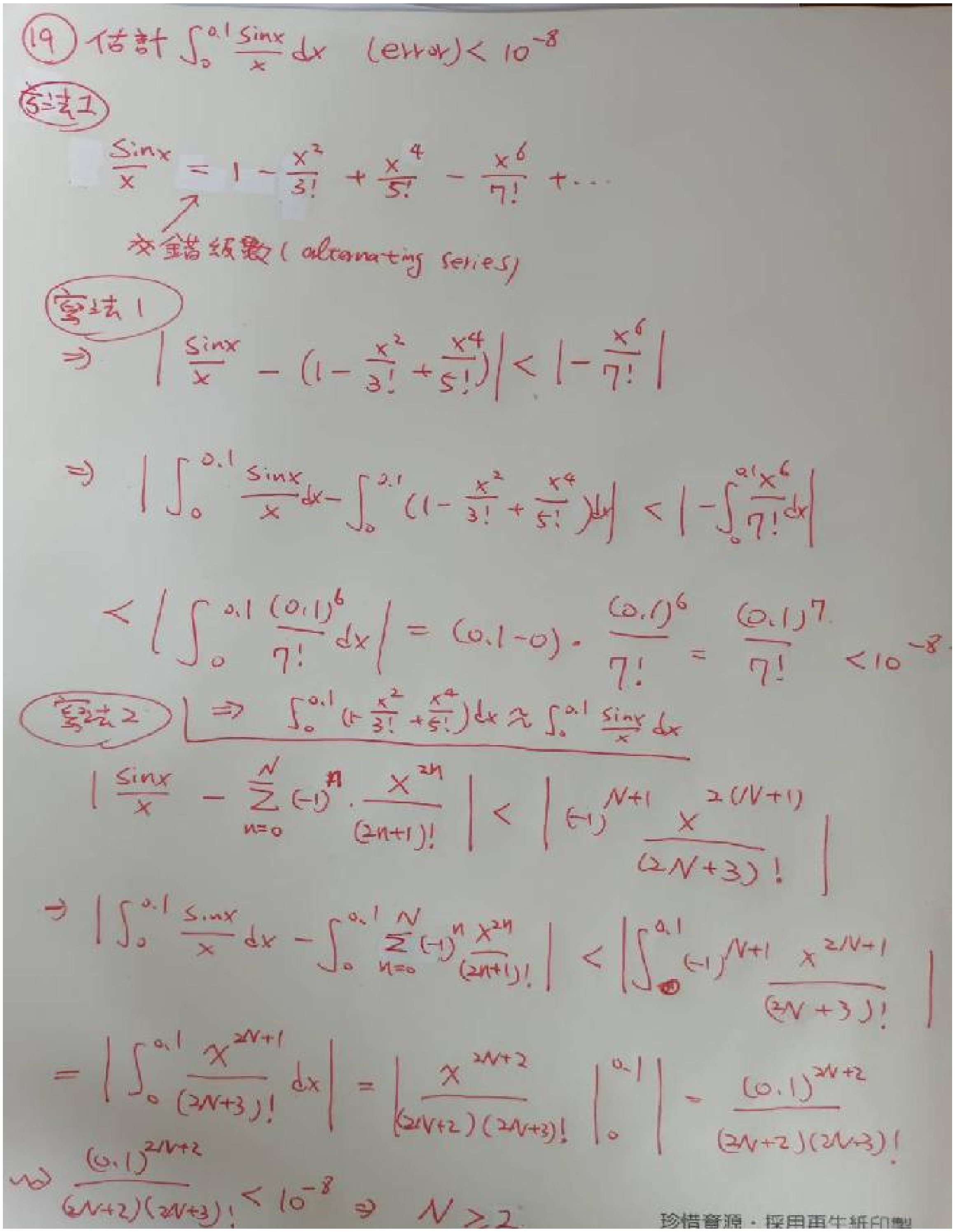

Figure 15: Solution to Section 10.10, problem 19

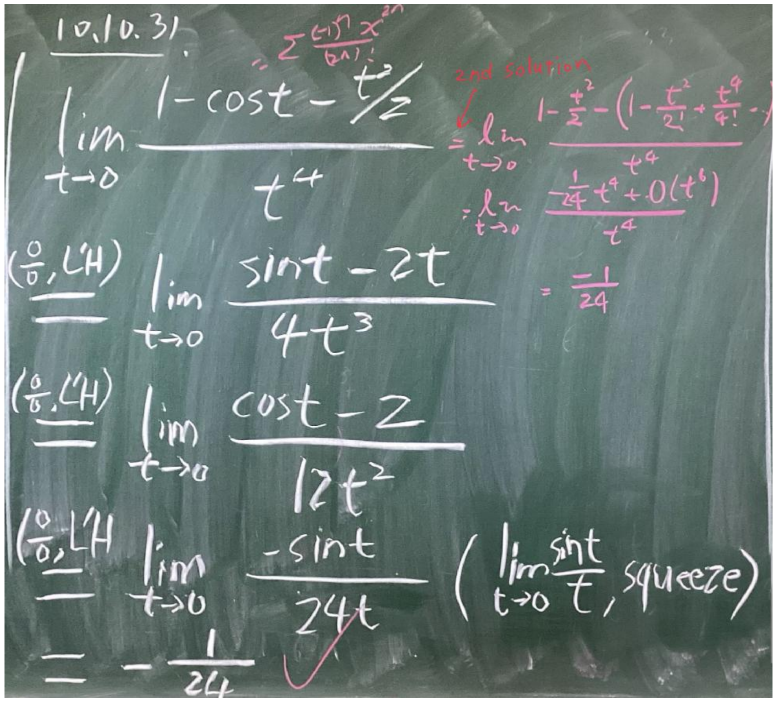

Figure 16: Solution to Section 10.10, problem 31

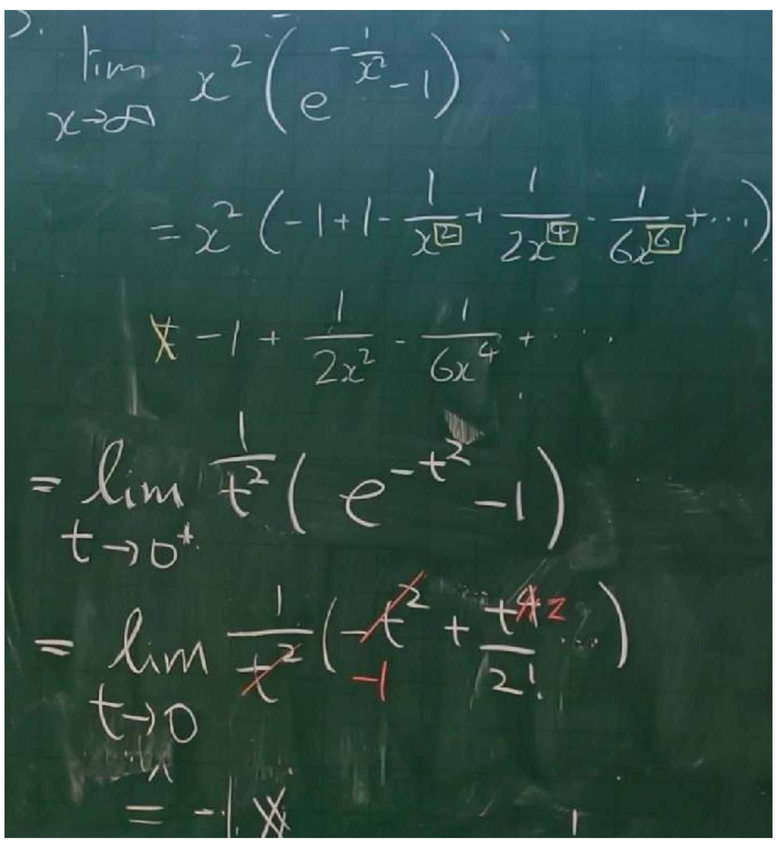

Figure 17: Solution to Section 10.10, problem 35

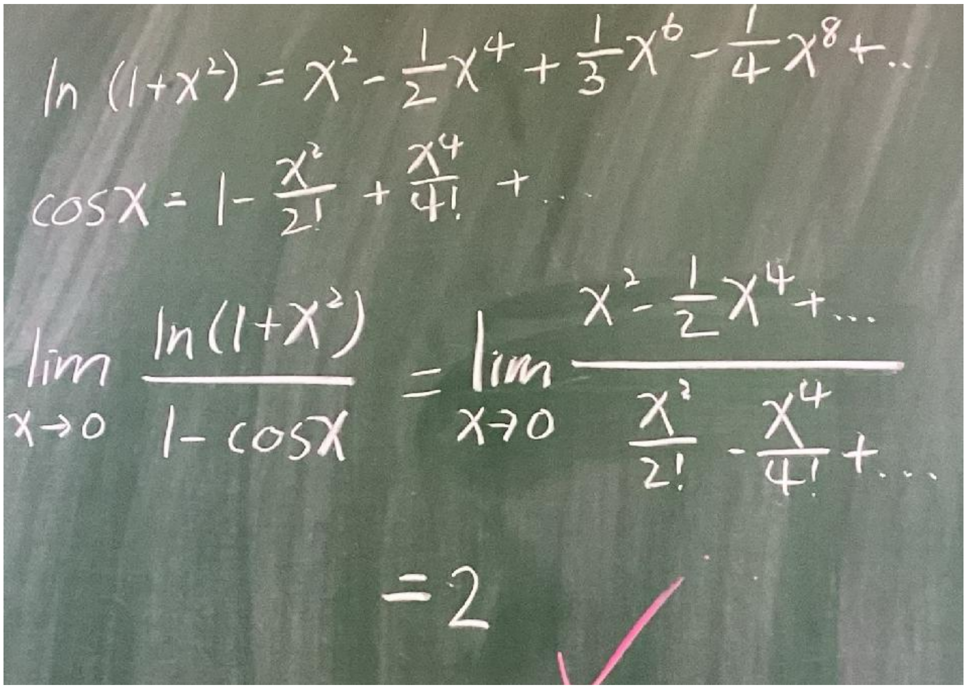

Figure 18: Solution to Section 10.10, problem 37

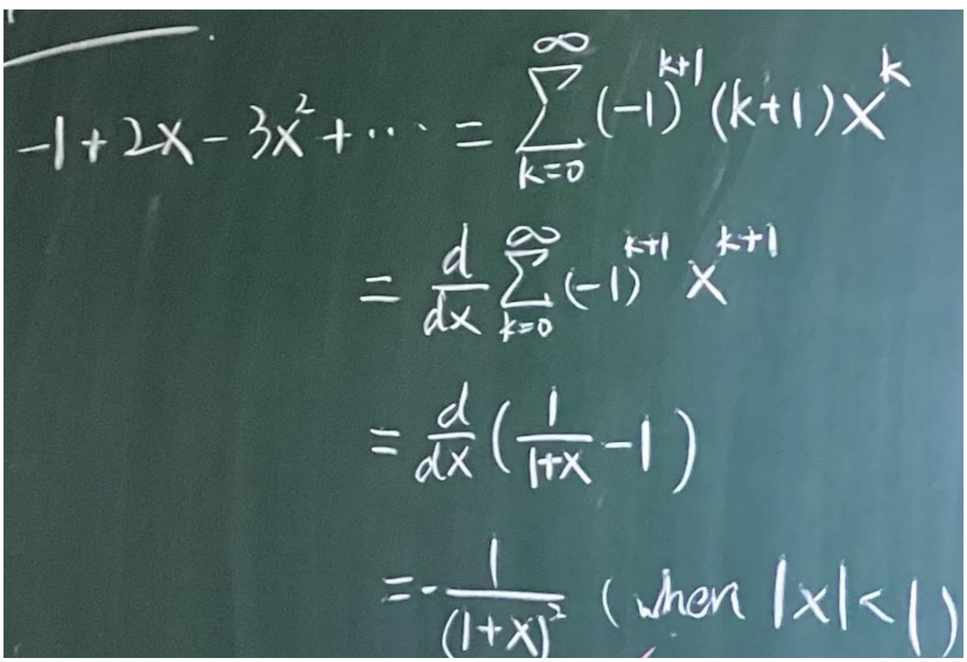

Figure 19: Solution to Section 10.10, problem 51

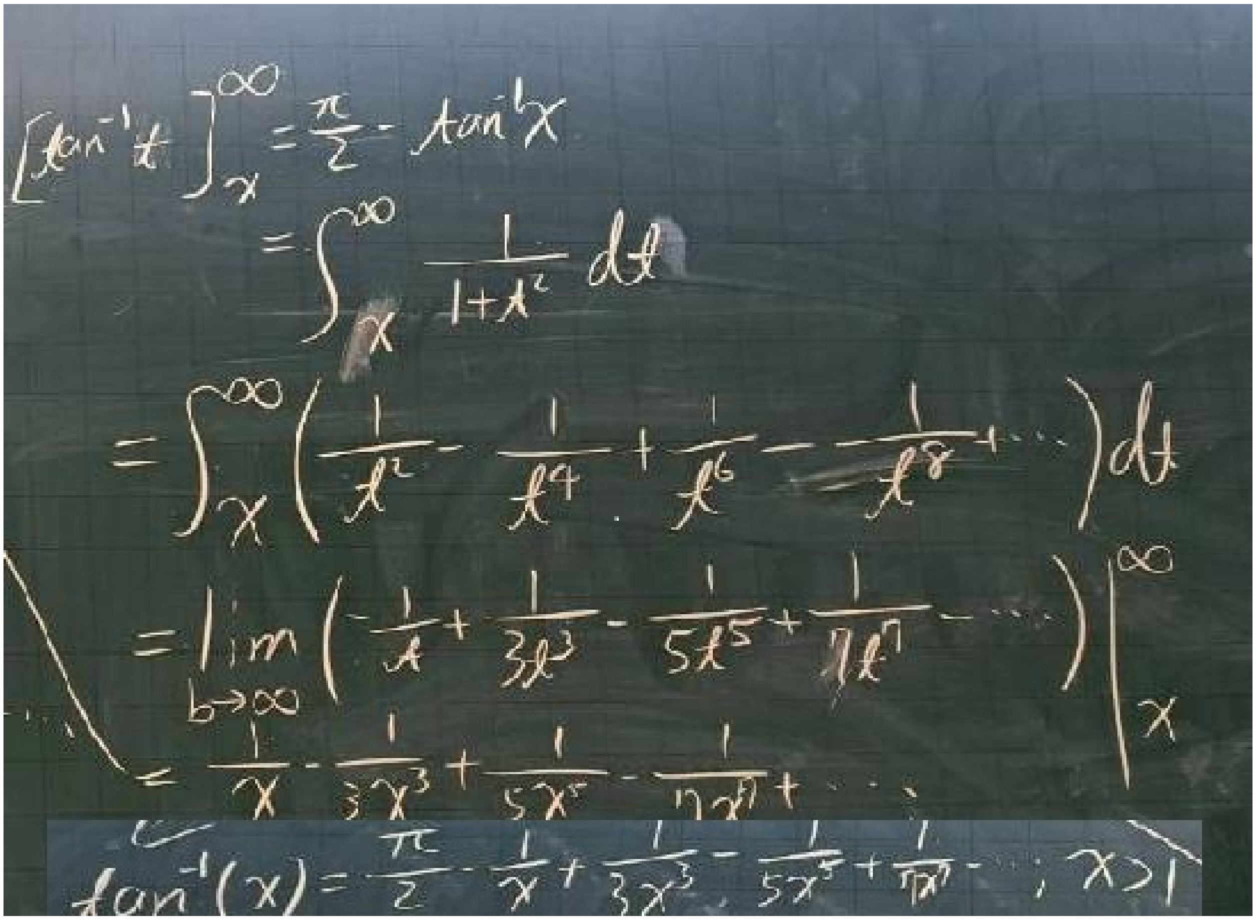

Figure 20: Solution to Section 10.10, problem 66

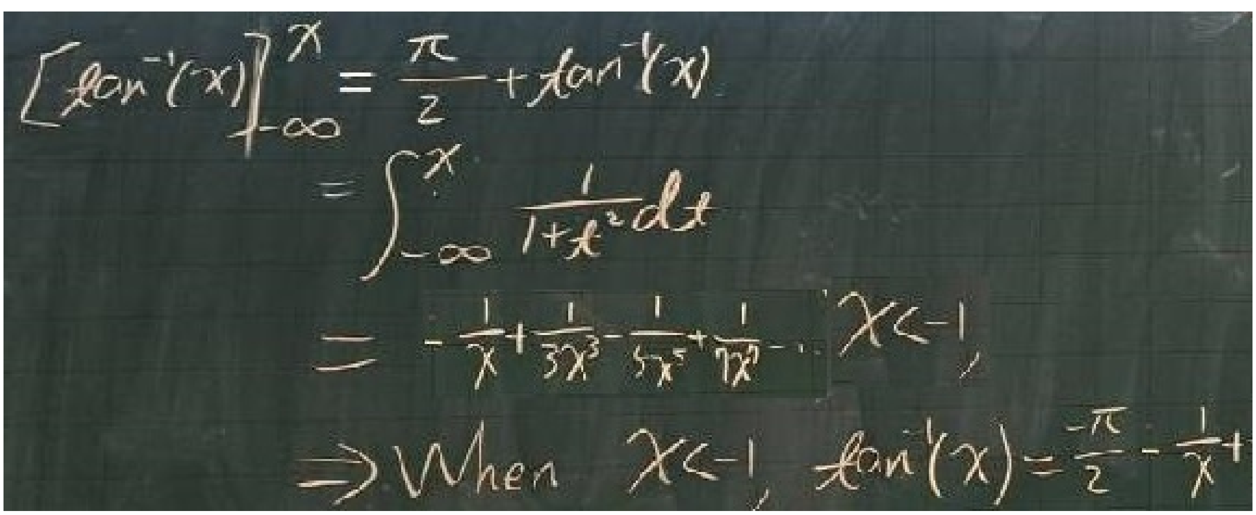

Figure 21: Solution to Section 10.10, problem 66, continued