Homework 04

1. Section 10.6:
   Show that $\int_{\pi}^{\infty} \frac{\sin t}{t}\,dt$ converges conditionally. That is, $\int_{\pi}^{\infty} \frac{\sin t}{t}\,dt$ converges and $\int_{\pi}^{\infty} \left|\frac{\sin t}{t}\right|\,dt$ diverges.
   Hint: $\int_{\pi}^{b} = \int_{\pi}^{[b]} + \int_{[b]}^{b}$ where $[b]$ denotes the largest integer n such that $n\pi \le b$.
   Remark (need not show it): The same conclusion holds for $\int_{0}^{\infty} \frac{\sin t}{t} dt$.
2. Section 10.7: problems 7, 11, 15, 19, 23, 29, 40, 43, 47.
   Hint for problem 40: $\frac{n}{n+1} = \left(1+\frac{1}{n}\right)^{-1}$.
3. Section 10.7: Find a power series that converges on $(1,3)$ and diverges elsewhere. Do the same for $(1,3]$, $[1,3)$ and $[1,3]$, respectively.
4. Section 10.7: Find the first three nonzero terms of the power series representation of

$$\frac{1 - x^2 + x^4 - \cdots}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots}$$

   by ”long division”.
5. Section 10.7: 51, 55, 57, 60.
6. Section 10.7: Use the power series representation of $\frac{1}{1 \pm x}$ to find the power series representation of $\ln(1 \pm x)$ on $|x| < 1$.
7. Section 10.8: Problems 5 ($n = 3$), 7 ($n = 3$), 15, 23, 29, 35.