

## Midterm 01

Oct 09, 2025. Show all details to get potential partial credits.

1. (6+10 pts)

(a) State the Sandwich Theorem.

(b) Prove it using precise definition of a limit ( $\varepsilon$  and  $\delta$  argument).

2. (8+8 pts)

(a) Evaluate  $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin^{-1} x}$ .

(b) Evaluate  $\lim_{z \rightarrow x} \frac{\tan^{-1} z - \tan^{-1} x}{e^z - e^x}$ .

3. (4+8 pts) True or False? Prove it if true, find a counter example if false.

If  $\lim_{x \rightarrow c} f(x) = L$ , and  $g(y)$  is continuous at  $y = L$ , then  $\lim_{x \rightarrow c} g(f(x)) = g(L)$ .

4. (12+6 pts) Find  $dy/dx$  for the following functions:

(a):  $y = a^{a^x} + a^{x^a} + x^{a^a}$ ,  $x > 0$ ,  $a > 0$ .

(b):  $y = \sin(\cos^2 x) \cos(\sin^2 x)$

5. (10 pts) State (need not prove) the Intermediate Value Theorem. Use it to prove that  $y = x^3 + 10x^2 + 100x + 2000$  has at least a real root.

6. (12 pts) Find  $y'(x)$  and  $y''(x)$  at  $(2, 1)$  if  $y(x)$  is implicitly defined by  $x^y = 2y^x$

7. (16 pts) Let  $f^{-1}$  be the inverse function of  $f$ . Evaluate  $\frac{d^2}{dy^2} f^{-1}(y)$  in terms of  $f'$  and  $f''$ . Show all details.

2. (a.) (b) 过程 + 答案  
4分 4分

3. T → 4分

①  $\lim_{x \rightarrow c} g(f(x)) = g(\lim_{x \rightarrow c} f(x)) = g(L)$

No reason → 不给分

② 誤用 f conti. at  $x=c$

5. State 5分

計算 5分

6.  $y' |_{(2,1)}$  6分

$\rightarrow 2 \cdot y^x$

$y'' |_{(2,1)}$  6分

$= e^{x \ln 2y}$   
 $\rightarrow x \ln 2y$  ) 不给分

Reviewed by Mr. Cheng (B09 TA)

1. (a) (6) 少寫 "except possibly at  $x=c$ " -1

(b) (10) 列出  $g(x)$  和  $h(x)$  極限 =  $L$  定義 +4

• 少寫  $g(x) \leq f(x) \leq h(x)$  的成立範圍

• 少寫  $\delta$  的取法. or  $0 < |x-c| < \delta$

-2

4. (a) (12) = 4 + 4 + 4  $\leftarrow (x^{a^a})'$

$(a^{a^x})'$   $(a^{x^a})'$

• 寫  $\ln y = \ln(a^{a^x}) + \ln(a^{x^a}) + \ln(x^{a^a})$

但  $\ln(a^{a^x})'$ ,  $\ln(a^{x^a})'$ ,  $\ln(x^{a^a})'$  正確  $\Rightarrow$  each +2

• 把  $a^{a^x}$  寫成  $a^{a^x}$  但微分正確  $\Rightarrow$  each +1.

(b) (6)

7. (16) 寫出  $\frac{d}{dy} f^{-1}(y)$  公式 +6 ~ +8

(depend on 直接寫出來 or 證明)

$(f^{-1})''(y)$ : 過程 + 答案 +

(不包含  $\frac{d}{dy} f^{-1}(y)$ )