Midterm 2

May 14, 2024: 10:10AM

All Theorems in the textbook can be used directly (unless you are asked precisely to prove that Theorem), just state clearly which Theorem you are using. All formula that you were asked to memorize, can be used directly unless you are asked explicitly to prove it or derive it.

1. (14 pts) True or False? Prove it if true and give an counter example if false.
   (a) If $f(x,y)$ is differentiable at $(0,0)$, then $f(x,y)$ is continuous at $(0,0)$.
   (b) If $f(x,y)$ is continuous at $(0,0)$, then $f(x,y)$ is differentiable at $(0,0)$.

2. (14 pts) Evaluate
$$\frac{d}{dx}\int_1^{\sqrt{x}} \sin\left(\frac{x}{t^2}\right)\,dt, \quad x>0.$$

3. (14 pts) Find the equation of plane normal to the following curve at $(1,1,3)$
$$\begin{cases} x^3+3x^2y^2+y^3+4xy-z^2=0 \\ x^2+y^2+z^2=11 \end{cases}$$

4. (14 pts) Find all critical points of $f(x,y)=\frac{x^5}{5}-\frac{x^4}{4}+\frac{y^3}{3}-\frac{y^2}{2}$ and determine whether each of them is a local minimum, local maximum or neither.

5. (14 pts) Find the point closest to origin on the intersection of the planes $y+2z=12$ and $x+y=6$. Solve this problem by method of Lagrange Multipliers only.

6. (14 pts) Use Taylor's formula to find the quadratic approximation of $f(x,y)=\cos x\cos y$ at the origin. Estimate the error of the approximation if $|x|<0.1$ and $|y|<0.1$.

7. (16 pts) Let $f(x,y)=\begin{cases} \frac{x^2y}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$.
   (a) Is f continuous at $(0,0)$? Explain.
   (b) Does f have a linear approximation at $(0,0)$? Explain.

8. (10 pts) Use any method to find $T_{\tan,0}(x)$ (up to and including x^5 term) and $T_{\tan^{-1},0}(x)$ (the full series), respectively. Show detailed derivation.

9. (10 pts) Evaluate $\sum_{n=0}^{\infty}\frac{n(\frac{1}{2})^{n+1}}{n+1}$. Show all details.