

Final Exam

June 18, 2024:

1. (15 pts) Evaluate $\iiint_D z^2 dV$ where $D = \left\{ \frac{x^2}{4} + \frac{y^2}{9} + z^2 \leq 1 \right\}$.
2. (15 pts) Evaluate $I = \int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx$.
3. (15 pts) Rewrite $\int_0^1 \int_{-1}^0 \int_0^{y^2} dz dy dx$ in the order $dy dz dx$ and $dx dy dz$, respectively and evaluate the integral.

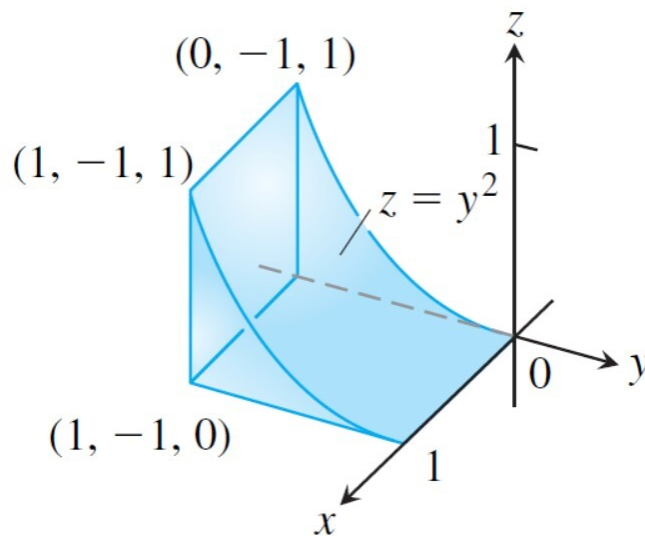


Figure 1: The domain in problem 3

4. (15 pts) Replace

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_0^{\sqrt{4-z^2}} r dr dz d\theta$$

by a triple integral in spherical coordinates and evaluate the integral in either coordinates of your choice.

5. (20 pts) State and prove the Fundamental Theorem of Line Integrals.
6. (20 pts) Let $\mathbf{F}(x, y) = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$. Is \mathbf{F} conservative on the domain where \mathbf{F} is defined? Explain.