(1) (a) find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$ (5%)

(b) find all values of x for which $\sum_{n=1}^{\infty} \frac{2^n}{n x^n}$ converges (5%)

(c) find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where $F(x,y,z) = x^3 + y^3 + z^3 - 3xyz = 0$ (5%)

(2) (a) What is the radius of convergence of the Taylor series of $\sin x$ around $x = x_0$? (5%)

(b) For what values of $x_0 \in [0, 2\pi)$ is $\sum_{n=1}^{\infty} \left(\sin^{-1}(x_0 + \frac{1}{n}) - \sin^{-1}(x_0) \right)$ convergent ? Show your work. (15%)

(3) Find the ... distance between origin and the line $L: \begin{cases} x + y + z = 1 \\ 2x - y + z = 2 \end{cases}$ by formulating this problem as a constrained minimization problem. (10%)

(4) (a) Can $f(x,y) = \frac{\sin xy}{x^2 + y^2}$, $(x,y) \neq (0,0)$ be extended as a continuous function on $\mathbb{R}^2$? why ? (5%)

(b) , $g(x,y) = \begin{cases} 2\sqrt{xy}, & \text{if } xy \geq 0 \\ 0, & \text{if } xy < 0 \end{cases}$

Show that $D_1 g(0,0)$ and $D_2 g(0,0)$ exist, but g is not differentiable at $(0,0)$. (10%)

(5) $f(x) = \int_0^{\frac{1}{x}} \frac{1 - e^{-tx}}{t} \, dt$, $x > 0$

(a) $\frac{d}{dx} f(x) = ?$ (5%)

(b) find approximate value of $f(1)$ up to 2 decimals.

(You may leave your answer as, for example, $1 + \frac{1}{2} + \frac{1}{16}$ without finding the numerical value of $1 + \frac{1}{2} + \frac{1}{16}$, but you need not show the error is $\leq 5 \times 10^{-3}$) (10%)

(6) Suppose $f(x,y)$ and all its partial derivatives of any order are continuous in a open ball containing $(0,0)$

(a) write down the Taylor polynomial of f around $(0,0)$ up to 3rd order. (5%)