Homework Assignment 9.

Given Nov 25, due Dec 07.

1. Section 5.6: Problems: 53, 54, 56, 58, 67, 74.
2. Section 6.4: Problems: 10, 19, 26, 41, 48, 70.
3. Section 6.9: Problems: 52, 54, 56, 58, 64, 68, 76, 80, 81.
4. Section 6.10: Problems: 47, 50, 67, 68.
5. Show by direct algebra that

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$$

6. (a) Show that $\lim_{x\to\infty} \int_x^{2x} \frac{1}{t^3+1} dt = 0$.

   (b) Evaluate $\lim_{x\to\infty} x^2 \int_x^{2x} \frac{1}{t^3+1} dt$

7. Challenge of the week, optional.

   The statement 'Continuous functions are integrable on $[a,b]$' is actually beyond this course. A simpler version is as follows:

   Suppose that for some constant $L > 0$, f satisfies

$$|f(x) - f(y)| \leq L|x-y| \quad \text{for all } x, y \in [a,b].$$

   (a) Show that f is continuous on $[a,b]$.

   (b) Show that for any choice of the partition P and any choice of $c_k \in [x_{k-1,}, x_k]$, there exist constants m_P and M_P, both depend on the partition P, such that

$$m_P \leq \sum_{k=1}^{n} f(c_k)\Delta x_k \leq M_P, \qquad M_P - m_P \leq L(b-a)\|P\|.$$

   This statement does not really prove that f is integrable on $[a,b]$, but is quite close though. (Why?)