

Homework Assignment 1.

Given Sep 16, due Sep 30.

1. Section 2.3: problems 24, 34, 40, 46. Chap 2: problems 41, 42, 53.
2. State (need not prove) the ' $x \rightarrow c^+$ ' and ' $x \rightarrow \infty$ ' versions of the Sandwich Theorem.
3. Section 2.4: problems 54, 55, 56, 57.
4. Section 2.5: problems 41, 42. Chap 2: problems 74, 75.
5. How would you define the following limits formally using ϵ and δ ?

a.

$$\lim_{x \rightarrow c^+} f(x) = L$$

b.

$$\lim_{x \rightarrow c} f(x) = \infty$$

c.

$$\lim_{x \rightarrow -\infty} f(x) = L$$

(Hint: ' $x \rightarrow \infty$ ' can be written as 'there is a M such that for all $x > M$, ...')

6. Show that if $f(x)$ is continuous at $x = c$, then so is $3f(x)$ and $f(x)^2$.
7. Show that if $f(x)$ is continuous at $x = c$ and $g(y)$ is continuous at $y = f(c)$, then $g(f(x))$ is continuous at $x = c$.
8. (Challenge of the week, optional)

Let $f : (0, 1) \rightarrow R$ be defined as

$$f(x) = \begin{cases} 1/p & \text{if } x = q/p, \quad p, q \in N, \quad (p, q) = 1 \\ 0 & \text{otherwise} \end{cases}$$

For what values of $c \in (0, 1)$ is f continuous at c ?