Assignment 7.

Given Dec 12 2000, due Dec 26 2000 at beginning of class.

NOTE: You are encouraged to discuss together, but in the end, you have to write own version alone based on your understanding. Copied version will not get any credit at all. Notice the change in the nonlinear term in problem 2.

(1) Do exercises 4.6, 4.7, 4.8 and 4.26.

(2) (extra credit)
Solve the nonlinear problem

$$-u'' + u^3 = \pi^2 \sin(\pi x) + \sin^3(\pi x) \quad \text{on } [0,1]$$

with $u(0) = u(1) = 0$. Discretize the equation with finite difference method and solve the resulting nonlinear systems with Newton's iteration. The exact solution is given by $u_e(x) = \sin(\pi x)$. Check the accuracy by comparing the results of $N = 10$, $N = 20$ and $N = 40$ in terms of e_1, e_2 and e_∞. You should see the error become roughly one quarter when you half the mesh size (i.e. when N doubles). You should hand in some paper-pencil work together with your program.

(3) (extra credit, and a lot of it. Due later, after you finish (2) with no problems)
Do the same for

$$-\triangle u + u^3 = 2\pi^2 \sin(\pi x)\sin(\pi y) + \sin^3(\pi x)\sin(\pi y) \quad \text{on } \Omega = [0,1]^2$$

with $u = 0$ on $\partial\Omega$. The exact solution is given by $u_e(x,y) = \sin(\pi x)\sin(\pi y)$. You will need a working code (iterative method) for the linear problem from previous homework, so find your bug first. For this problem, there are two iterations (one from G-S or Jacobi, one from Newton), you should think about how to balance the computational load.