Assignment 3.

Given Oct 17 2000, due Oct 27 2000.

NOTE: This homework is quite time consuming. You should start IMMEDIATELY and ask for help before it's too late.

(1) Programming:

(a)Do exercises 2.13 and 2.16 of the textbook.

(b)Apply the iterative refinement method in solving the system

$$\begin{pmatrix} 420 & 210 & 140 & 105 \\ 210 & 140 & 105 & 84 \\ 140 & 105 & 84 & 70 \\ 105 & 84 & 70 & 60 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 875 \\ 539 \\ 399 \\ 319 \end{pmatrix}.$$

The true solution is $(x_1, x_2, x_3, x_4) = (1, 1, 1, 1)$. Do five iterations to see how the computed solutions converges.

(2) We showed in class a (rare) situation where the relative error is really amplified by a factor $\kappa(A)$ in solving $A\boldsymbol{x} = \boldsymbol{b}$ with $A = \begin{pmatrix} 1 & 0 \\ 0 & \epsilon \end{pmatrix}$. Give an estimate for the amplification factor of relative error when $\Delta\boldsymbol{b} = \begin{pmatrix} \delta \\ \delta \end{pmatrix}$.

(3) We showed in class the derivation of the condition number inequality (2.22) for the special case $\Delta\boldsymbol{b} = \boldsymbol{0}$. We have implicitly used the matrix norm inequality

$$\|AB\| \leq \|A\| \, \|B\|$$

in the process.

(a) Prove this inequality and use it to show the condition number lower bound

$$\kappa(A) \geq 1$$

(b) (harder) Review the derivation for the special case mentioned above and prove the general case (2.22). (Hint: to produce the $\|\boldsymbol{b}\|$ term at the right hand side, use the trick

$$\frac{1}{\|A\|\|\boldsymbol{x}\|} \leq \frac{1}{\|\boldsymbol{b}\|}$$

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