Applied Mathematics E-Notes, 23(2023), 352-359 © ISSN 1607-2510
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Absolute Matrix Summability Of Factored Infinite Series And Fourier Series*

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Received 30 September 2022

Abstract

In [4], Bor has obtained two main theorems dealing with absolute summability factors of infinite series and Fourier series. In this paper, we have generalized these theorems for the absolute matrix summability method. Some new and known results have also been obtained.

1 Introduction

A positive sequence (b_n) is said to be an almost increasing sequence if there exist a positive increasing sequence (c_n) and two positive constants M and N such that $Mc_n \leq b_n \leq Nc_n$ (see [1]). A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in BV$, if $\sum_{n=1}^{\infty} \mid \Delta\lambda_n \mid < \infty$.

Let $\sum a_n$ be a given infinite series with partial sums (s_n) and let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad as \quad n \to \infty, \quad (P_{-i} = p_{-i} = 0, i \geq 1).$$

The sequence-to-sequence transformation

$$\sigma_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v$$

defines the sequence (σ_n) of the Riesz mean or simply the $(\bar{N}, p_n)$ mean of the sequence (s_n), generated by the sequence of coefficients (p_n) (see [9]). The series $\sum a_n$ is said to be summable $\mid \bar{N}, p_n \mid_k, k \geq 1$, if (see [2])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{k-1} \mid \sigma_n - \sigma_{n-1} \mid^k < \infty.$$

When $p_n = 1$ for all values of n, then we get $\mid C, 1 \mid_k$ summability. Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \ n = 0, 1, \dots \ .$$

The series $\sum a_n$ is said to be summable $\mid A, p_n; \delta \mid_k$, $k \geq 1$ and $\delta \geq 0$, if (see [11])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{\delta k + k - 1} \mid A_n(s) - A_{n-1}(s) \mid^k < \infty.$$

*Mathematics Subject Classifications: 26D15, 40D15, 40F05, 40G99.

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In the special case for $\delta = 0$, the $\mid A, p_n; \delta \mid_k$ summability reduces to $\mid A, p_n \mid_k$ summability (see [16]). If we set $\delta = 0$ and $p_n = 1$ for all n, then we obtain $\mid A \mid_k$ summability (see [17]). Also if we take $a_{nv} = \frac{p_v}{P_n}$, then we have $\mid \bar{N}, p_n; \delta \mid_k$ summability (see [3]). Finally if we take $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$, then we get $\mid \bar{N}, p_n \mid_k$ summability.

Given any sequences (u_n), (v_n), it is customary to write $v_n = O(u_n)$, if there exist η and N, for every $n > N$, $\mid \frac{v_n}{u_n} \mid \leq \eta$. For any matrix entry a_{nv}, we write that $\Delta_v a_{nv} = a_{nv} - a_{n,v+1}$. Now, we will introduce some necessary notations for our main theorems. Given a normal matrix $A = (a_{nv})$, we associate two lower semi-matrices $\overline{A} = (\overline{a}_{nv})$ and $\widehat{A} = (\widehat{a}_{nv})$ as follows:

$$\overline{a}_{nv} = \sum_{i=v}^{n} a_{ni}, n, v = 0, 1, ...$$

and

$$\widehat{a}_{00} = \overline{a}_{00} = a_{00}, \widehat{a}_{nv} = \overline{a}_{nv} - \overline{a}_{n-1,v}, n = 1, 2, ...$$

It may be noted that $\overline{A}$ and $\widehat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations, respectively. Then, we have

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \overline{a}_{nv} a_v, \tag{1}$$

and

$$\overline{\Delta} A_n(s) = \sum_{v=0}^{n} \widehat{a}_{nv} a_v. \tag{2}$$

2 Known Results

Recently, many authors have obtained some new theorems dealing with absolute summability factors of infinite series and Fourier series. ([4]–[7], [12]–[13], [16]–[18]). Among them, Bor has proved the following theorems about the $\mid \bar{N}, p_n \mid_k$ summability methods.

Theorem 1 *([4]) Let (X_n) be an almost increasing sequence. If the sequence $(X_n), (\lambda_n)$ and (p_n) satisfy the conditions*

$$\mid \lambda_n \mid X_n = O(1) \quad as\, n \to \infty, \tag{3}$$

$$\sum_{n=1}^{m} n X_n \mid \Delta^2 \lambda_n \mid = O(1) \quad as\, m \to \infty, \tag{4}$$

$$\sum_{n=1}^{m} \frac{\mid t_n \mid^k}{n X_n^{k-1}} = O(X_m) \quad as\, m \to \infty, \tag{5}$$

$$\sum_{n=1}^{m} \frac{p_n}{P_n} \frac{\mid t_n \mid^k}{X_n^{k-1}} = O(X_m) \quad as\, m \to \infty, \tag{6}$$

and

$$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m) \quad as\, m \to \infty, \tag{7}$$

then the series $\sum a_n \lambda_n$ is summable $\mid \bar{N}, p_n \mid_k$, $k \geq 1$.

3 Main Results

The aim of this paper is to generalize the above theorem for the general matrix summability methods. Before we state our main result, we show $A = (a_{nv})$ is said to be of class Ω if (see [15]) A is lower triangular

$$a_{nv} \geq 0, \quad n, v = 0, 1, ...;$$

$$a_{n-1,v} \geq a_{nv}, \quad \text{for } n \geq v+1,$$

$$\overline{a}_{n0} = 1, \quad n = 0, 1, ...$$

Notice that A given by

$$A_1(x) = x_1 \text{ and } A_n(x) = \frac{x_{n-1} + x_n}{2} \text{ for } n > 1$$

is an example of a matrix of class Ω. Now, we shall prove the following theorem.

Theorem 2 *Let (X_n) be an almost increasing sequence and A be of class Ω such that*

$$a_{nn} = O\left(\frac{p_n}{P_n}\right),$$

$$\sum_{v=1}^{n-1} \frac{\mid \widehat{a}_{n,v+1} \mid}{v} = O(a_{nn}),$$

$$\sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} \mid \Delta \widehat{a}_{nv} \mid = O\left(a_{vv}\left(\frac{P_v}{p_v}\right)^{\delta k}\right) \quad \text{as } m \to \infty,$$

$$\sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} \mid \widehat{a}_{n,v+1} \mid = O\left(\left(\frac{P_v}{p_v}\right)^{\delta k}\right) \quad \text{as } m \to \infty,$$

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k} \frac{\mid t_n \mid^k}{n X_n^{k-1}} = O(X_m) \quad \text{as } m \to \infty,$$

$$\sum_{n=1}^{m} a_{nn} \left(\frac{P_n}{p_n}\right)^{\delta k} \frac{\mid t_n \mid^k}{X_n^{k-1}} = O(X_m) \quad \text{as } m \to \infty.$$

If the conditions (3)–(4) of Theorem 1 are satisfied, then the series $\sum a_n \lambda_n$ is summable $\mid A, p_n; \delta \mid_k$, $k \geq 1$ and $0 \leq \delta < 1/k$.

We need the following lemma for the proof of our theorem.

Lemma 1 *([10]) Under the conditions of Theorem 1, we have the following*

$$n X_n \mid \Delta \lambda_n \mid = O(1) \text{ as } n \to \infty,$$

$$\sum_{n=1}^{\infty} X_n \mid \Delta \lambda_n \mid < \infty.$$

4 Proof of Theorem 2

Let (T_n) denote A-transform of the series $\sum a_n\lambda_n$. Then, by (1) and (2), we have

$$\bar{\Delta}T_n = \sum_{v=0}^{n} \widehat{a}_{nv} a_v \lambda_v.$$

Applying Abel's transformation, we have that

$$\begin{aligned}
\bar{\Delta}T_n &= \sum_{v=0}^{n} \frac{\widehat{a}_{nv}\lambda_v}{v} v a_v = \sum_{v=1}^{n-1} \Delta(\frac{\widehat{a}_{nv}\lambda_v}{v}) \sum_{r=1}^{v} r a_r + \frac{\widehat{a}_{nn}\lambda_n}{n} \sum_{v=1}^{n} v a_v \\
&= \sum_{v=1}^{n-1} \Delta(\frac{\widehat{a}_{nv}\lambda_v}{v})(v+1)t_v + \widehat{a}_{nn}\lambda_n \frac{n+1}{n} t_n \\
&= \sum_{v=1}^{n-1} \Delta_v(\widehat{a}_{nv})\lambda_v t_v \frac{v+1}{v} + \sum_{v=1}^{n-1} \widehat{a}_{n,v+1}\Delta\lambda_v t_v \frac{v+1}{v} \\
&+ \sum_{v=1}^{n-1} \widehat{a}_{n,v+1}\lambda_{v+1}\frac{t_v}{v} + a_{nn}\lambda_n t_n \frac{n+1}{n} \\
&= T_{n,1} + T_{n,2} + T_{n,3} + T_{n,4}.
\end{aligned}$$

Since

$$\mid T_{n,1} + T_{n,2} + T_{n,3} + T_{n,4} \mid^k \leq 4^k(\mid T_{n,1} \mid^k + \mid T_{n,2} \mid^k + \mid T_{n,3} \mid^k + \mid T_{n,4} \mid^k),$$

to complete the proof of Theorem 2, it is sufficient to show that

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \mid T_{n,r} \mid^k < \infty \quad \text{for } r = 1,2,3,4.$$

First, applying Hölder's inequality with indices k and k', where $k > 1$ and $\frac{1}{k} + \frac{1}{k'} = 1$, we get that

$$\begin{aligned}
&\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \mid T_{n,1} \mid^k \\
\leq\ & \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \left\{\sum_{v=1}^{n-1} \mid \frac{v+1}{v} \mid\mid \Delta_v(\widehat{a}_{nv}) \mid\mid \lambda_v \mid\mid t_v \mid\right\}^k \\
=\ & O(1)\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \left(\sum_{v=1}^{n-1} \mid \Delta_v(\widehat{a}_{nv}) \mid\mid \lambda_v \mid^k\mid t_v \mid^k\right) \\
&\times \left(\sum_{v=1}^{n-1} \mid \Delta_v(\widehat{a}_{nv}) \mid\right)^{k-1} \\
=\ & O(1)\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} a_{nn}^{k-1} \left(\sum_{v=1}^{n-1} \mid \Delta_v(\widehat{a}_{nv}) \mid\mid \lambda_v \mid^k\mid t_v \mid^k\right) \\
=\ & O(1)\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} \left(\sum_{v=1}^{n-1} \mid \Delta_v(\widehat{a}_{nv}) \mid\mid \lambda_v \mid^k\mid t_v \mid^k\right) \\
=\ & O(1)\sum_{v=1}^{m} \mid \lambda_v \mid^{k-1}\mid \lambda_v \mid\mid t_v \mid^k \sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} \mid \Delta_v(\widehat{a}_{nv}) \mid
\end{aligned}$$

$$\begin{aligned}
&= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k} a_{vv} \mid \lambda_v \mid \frac{\mid t_v \mid^k}{X_v^{k-1}}\\
&= O(1)\sum_{v=1}^{m-1}\Delta \mid \lambda_v \mid \sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\delta k} a_{rr}\frac{\mid t_r \mid^k}{X_r^{k-1}} + O(1) \mid \lambda_m \mid \sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k} a_{vv}\frac{\mid t_v \mid^k}{X_v^{k-1}}\\
&= O(1)\sum_{v=1}^{m} \mid \Delta\lambda_v \mid X_v + O(1) \mid \lambda_m \mid X_m\\
&= O(1) \text{ as } m\to\infty,
\end{aligned}$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Applying Hölder's inequality with the same indices above, we have

$$\begin{aligned}
&\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \mid T_{n,2} \mid^k\\
\leq &\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1} \mid \frac{v+1}{v} \mid\mid \widehat{a}_{n,v+1} \mid\mid \Delta\lambda_v \mid\mid t_v \mid\right\}^k\\
= &O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1} \mid \widehat{a}_{n,v+1} \mid\mid \Delta\lambda_v \mid\mid t_v \mid\right)^k\\
= &O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1} \mid \widehat{a}_{n,v+1} \mid (v \mid \Delta\lambda_v \mid)^k\frac{\mid t_v \mid^k}{v}\right)\left(\sum_{v=1}^{n-1}\frac{\mid \widehat{a}_{n,v+1} \mid}{v}\right)^{k-1}\\
= &O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1} a_{nn}^{k-1}\left(\sum_{v=1}^{n-1} \mid \widehat{a}_{n,v+1} \mid (v \mid \Delta\lambda_v \mid)^k\frac{\mid t_v \mid^k}{v}\right)\\
= &O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}\sum_{v=1}^{n-1} \mid \widehat{a}_{n,v+1} \mid (v \mid \Delta\lambda_v \mid)^{k-1} v \mid \Delta\lambda_v \mid \frac{\mid t_v \mid^k}{v}\\
= &O(1)\sum_{v=1}^{m} v \mid \Delta\lambda_v \mid \frac{\mid t_v \mid^k}{v}\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k} \mid \widehat{a}_{n,v+1} \mid\\
= &O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\frac{\mid t_v \mid^k}{vX_v^{k-1}} v \mid \Delta\lambda_v \mid\\
= &O(1)\sum_{v=1}^{m-1}\Delta(v \mid \Delta\lambda_v \mid)\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\delta k}\frac{\mid t_r \mid^k}{rX_r^{k-1}} + O(1)m \mid \Delta\lambda_m \mid \sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\frac{\mid t_v \mid^k}{vX_v^{k-1}}\\
= &O(1)\sum_{v=1}^{m-1} \mid \Delta(v \mid \Delta\lambda_v \mid) \mid X_v + O(1)m \mid \Delta\lambda_m \mid X_m\\
= &O(1)\sum_{v=1}^{m-1} v \mid \Delta^2\lambda_v \mid X_v + O(1)\sum_{v=1}^{m-1} \mid \Delta\lambda_v \mid X_v + O(1)m \mid \Delta\lambda_m \mid X_m\\
= &O(1) \text{ as } m\to\infty,
\end{aligned}$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Again, we have that

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \mid T_{n,3} \mid^k \quad\leq\quad \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1} \mid \widehat{a}_{n,v+1} \mid\mid \lambda_{v+1} \mid \frac{\mid t_v \mid}{v}\right\}^k$$

$$
\begin{aligned}
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}\mid \widehat{a}_{n,v+1}\mid\mid \lambda_{v+1}\mid^k \frac{\mid t_v\mid^k}{v}\right)\\
&\times \left(\sum_{v=1}^{n-1}\frac{\mid \widehat{a}_{n,v+1}\mid}{v}\right)^{k-1}\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1} a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}\mid \widehat{a}_{n,v+1}\mid\mid \lambda_{v+1}\mid \frac{\mid t_v\mid^k}{v}\right)\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}\sum_{v=1}^{n-1}\mid \widehat{a}_{n,v+1}\mid \lambda_{v+1}\mid \frac{\mid t_v\mid^k}{v}\\
&= O(1)\sum_{v=1}^{m}\mid \lambda_{v+1}\mid \frac{\mid t_v\mid^k}{v}\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}\mid \widehat{a}_{n,v+1}\mid\\
&= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\mid \lambda_{v+1}\mid \frac{\mid t_v\mid^k}{vX_v^{k-1}}\\
&= O(1)\sum_{v=1}^{m-1}\Delta\mid \lambda_{v+1}\mid\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\delta k}\frac{\mid t_r\mid^k}{rX_r^{k-1}}\\
&+ O(1)\mid \lambda_{m+1}\mid\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\frac{\mid t_v\mid^k}{vX_v^{k-1}}\\
&= O(1)\sum_{v=1}^{m-1}\mid \Delta(v\mid \Delta\lambda_v\mid)\mid X_v + O(1)m\mid \Delta\lambda_m\mid X_m\\
&= O(1)\sum_{v=1}^{m-1}\mid \Delta\lambda_v\mid X_{v+1} + O(1)\mid \lambda_{m+1}\mid X_{m+1}\\
&= O(1) \text{ as } m\to\infty,
\end{aligned}
$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Finally, by the similar process in $T_{n,1}$, we have that

$$
\begin{aligned}
\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\mid T_{n,4}\mid^k &= O(1)\sum_{n=}^{m}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1} a_{nn}^k\mid \lambda_n\mid^k\mid t_n\mid^k\\
&= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\delta k} a_{nn}\mid \lambda_n\mid^{k-1}\mid \lambda_n\mid\mid t_n\mid^k\\
&= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\delta k} a_{nn}\mid \lambda_n\mid \frac{\mid t_n\mid^k}{X_n^{k-1}}\\
&= O(1) \quad \text{as } m\to\infty.
\end{aligned}
$$

So we get

$$
\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\mid T_{n,r}\mid^k<\infty, \quad \text{for } r=1,2,3,4.
$$

This completes the proof of Theorem 2.

If we take $p_n = 1$ for all values of n, then we have a new result dealing with $\mid A,\delta\mid_k$ summability factors of infinite and Fourier series. Also, if we take $\delta = 0$, then we get the result due to Yıldız [18]. Finally, if we take $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$, then we obtain the result of Bor [4].

5 An Application to Fourier Series

Let f be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. Without any loss of generality the constant term in the Fourier series of f can be taken to be 0, so that

$$f(t) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} C_n(t),$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)dt, a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos ntdt, b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin ntdt.$$

We write

$$\phi(t) = \frac{1}{2}f(x+t) + f(x-t) \;\; \text{and} \;\; \phi_\alpha(t) = \frac{\alpha}{t^\alpha}\int_0^t (t-u)^{\alpha-1}\phi(u)du, \;(\alpha > 0).$$

It is well known that if $\phi_1(t) \in BV(0,\pi)$, then $t_n(x) = O(1)$, where $t_n(x)$ is the $(C,1)$ mean of the sequence $(nC_n(x))$ (see [8]). The following theorem is known dealing with $\mid \bar{N}, p_n \mid_k$ summability factors of Fourier series.

Theorem 3 *([4]) Let (X_n) be an almost increasing sequence. If $\phi_1(t) \in BV(0,\pi)$ and the sequences $(p_n), (\lambda_n)$ and (X_n) satisfy the conditions of Theorem 1, then the series $\sum C_n(x)\lambda_n$ is summable $\mid \bar{N}, p_n \mid_k$, $k \geq 1$.*

Now, we generalize Theorem 3 for $\mid A, p_n; \delta \mid_k$ summability method in the following form.

Theorem 4 *Let (X_n) be an almost increasing sequence and A be a matrix as in Theorem 2. If $\phi_1(t) \in BV(0,\pi)$ and the sequences $(p_n), (\lambda_n)$ and (X_n) satisfy the conditions of Theorem 2, then $\sum C_n(x)\lambda_n$ is summable $\mid A, p_n; \delta \mid_k$, $k \geq 1$ and $0 \leq \delta < 1/k$.*

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