Range-Valued Fuzzy Colouring Of Intuitionistic Fuzzy Graphs With Application*

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Abstract

Intuitionistic fuzzy graph, belonging to fuzzy graphs family has good capabilities when facing with problems that cannot be expressed by fuzzy graphs. Intuitionistic fuzzy graph can handle the vagueness connected with the incompatible and determinate information of any real-world problem, where fuzzy graphs may not succeed to bear satisfactory results. The previous restrictions in fuzzy graphs have led us to propose new definitions in intuitionistic fuzzy graphs. Colouring problem in graphs theory is one of the key issues that has many applications in computer science and social networks. Today, many researchers are trying to prove its application in medical sciences and psychology. Hence, in this paper, new concept of colouring of intuitionistic fuzzy graphs has been introduced. Also, some important terms like power cut graph of intuitionistic fuzzy graphs, range-valued fuzzy colour, chromatic number of an intuitionistic fuzzy graph colouring have been described. Some relevant results are proved. This technique is used to colour world political map mentioning the strength of relationship among the countries. Also, a new kind of traffic signal system has been proposed.

1 Introduction

In 1736, Euler first introduced the concept of graph theory. A graph is a convenient way of interpreting information involving the relationship between objects. In the history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem was considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as logic, geometry, algebra, topology, analysis, number theory, information theory, artificial intelligence, operations research, optimization, neural networks, planning, computer science [18, 25], etc .

Colouring of graph is one of the most important research area of combinatorial optimization due to its wide applications in real life, viz. management sciences, wiring printed circuits [24], resource allocation [16], scheduling problems [7, 17, 23], etc. These problems are modelled by appropriate crisp graphs and solved by colouring of these graphs. In the conventional graph colouring problem, minimum number of colours is given to the vertices of the graphs such that no two adjacent vertices have the same colours. Several works have been done in this topic [19, 20, 28, 42]. Intuitionistic Fuzzy graph representation is more appropriate to reality than crisp graph representation. Every event in real world can be represented by intuitionistic fuzzy graphs appropriately. Akram et al. [1, 2, 3, 4, 5, 6] investigated several concepts in fuzzy graphs.

Presently, science and technology are featured with complex processes and phenomena for which complete information is not always available. For such cases, mathematical models are developed to handle types of systems containing elements of uncertainty. A large number of these models are based on an extension

of the ordinary set theory, namely, fuzzy sets. The notion of fuzzy sets, introduced by Zadeh in 1965, is a mathematical tool for handling uncertainties like vagueness, ambiguity and imprecision in linguistic variables [51]. Research on theory of fuzzy sets has been witnessing an exponential growth; both mathematics and its application. Fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest.

In 1975, Rosenfeld [36] introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, Bhattacharya gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng [25]. Rashmanlou et al. [30, 31, 32, 33, 34, 35] studied new concepts in fuzzy graphs and intuitionistic fuzzy graphs. Fuzzy graphs are designed to represent structures of relationships between objects such that the existence of a concrete object (vertex) and relationship between two objects (edge) are matters of degree. A fuzzy graph can well describe the uncertainty of all kinds of networks. Now, fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim of reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems.

In 1983, Atanassov [8] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [51]. Atanassov added a new component (which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the nonmembership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [9].

Early in the literature, one of useful problems, the traffic signal problem was solved by using crisp graph colouring technique. But, in traffic signal problem, some paths are busy compared to the other paths and some still deal with the traffic signal problem. Also, sometimes, two routes can be opened simultaneously with some caution. Here "crowder" and "warning" are fuzzy terms. Munoz et al. [27] designed the traffic signal problem by fuzzy graphs and introduced the method of colouring in fuzzy graphs. In that paper, the fuzzy graphs are considered with crisp nodes and fuzzy edges. Then, α-cut [26] of these fuzzy graphs are coloured by the technique of crisp graph colouring. For distinct values of α, we have dissimilar crisp graphs, and these crisp graphs are coloured. Therefore, the chromatic number changes for similar fuzzy graphs according to distinct values of α. Moreover, Bershtein, and Bozhenuk [10] planned a method to colour fuzzy graphs. In that paper, a term separation degree of fuzzy graphs has been defined and based on the value of separation degree, number of minimum colour is found.

Over the years, many extensions of fuzzy graphs and their properties have been deeply studied by several researchers, such as intuitionistic fuzzy graphs, interval valued fuzzy graphs, interval valued intuitionistic fuzzy graphs, bipolar fuzzy graphs and etc [11, 12, 13, 14, 15, 21, 22, 29, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 52].

In the present paper, a new idea to colour an intuitionistic fuzzy graph is presented. Here, "range-valued fuzzy colour" is defined. An intuitionistic fuzzy graph is coloured by the range-valued fuzzy colour depending on the power of an edge incident to a node. This latest colouring idea is used to colour the political map and solve the latest type of traffic signal colouring problem. Likewise, the "power cut graph" of intuitionistic fuzzy graphs is measured. Some interesting theorems on this type of graph are considered, then, "range-valued fuzzy colouring of intuitionistic fuzzy graphs" is proposed.

2   Preliminaries

Definition 1 *A graph is an ordered pair $G^* = (V, E)$, where V is the set of nodes of G^* and E is the set of edges of G^*. A graph G^* is finite if its node set and edge set are finite.*

Definition 2 *The node colouring of a graph G is the consignment of labels or colours to each node of a graph such that no edge links two similarly coloured nodes. The general type of node colouring search minimizes the number of colours for a graph. This type of colouring is known as least node colouring, and the lowest number of colours with which the nodes of a graph G can be coloured is called the chromatic number. The chromatic number of a graph G is denoted by $\chi(G)$.*

Definition 3 *Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x) \rangle : x \in X\}$, where $\mu_A(x) : X \longrightarrow [0,1]$ is the membership function of the fuzzy set. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.*

Definition 4 *A fuzzy graph $G = (V, \sigma, \mu)$ with the node set $\sigma : V \longrightarrow [0,1]$ and the edge set $\mu : V \times V \longrightarrow [0,1]$ such that for all $x, y \in V$, $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$. $\sigma(x)$ is the membership value of the node x and $\mu(x,y)$ is the membership value of the edge (x,y). Here μ is a symmetric fuzzy relation on σ.*

Definition 5 *A path in a fuzzy graph is an arrangement of different nodes $v_1, v_2, v_3, ..., v_n$, such that $\mu(v_{i-1}, v_i) > 0$, $1 \leq i \leq n$. The fuzzy path is said to be a fuzzy cycle if v_0 and $v_n 0$ overlap. The original crisp graph of the fuzzy graph $G = (V, \sigma, \mu)$ is denoted as $G^* = (V, \sigma^*, \mu^*)$, where $\sigma^* = \{x \in V \mid \sigma(x) > 0\}$ and $\mu^* = \{(x,y) \in V \times V \mid \mu(x,y) > 0\}$. For the original fuzzy graph, $\sigma^* = V$.*

Definition 6 *A fuzzy graph $G = (V, \sigma, \mu)$ is said to be complete if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$, where (x,y) denotes the edge between nodes x and y.*

Definition 7 *The fuzzy graph $H = (V, \sigma_H, \mu_H)$ is called a fuzzy subgraph of $G = (V, \sigma, \mu)$ if $\sigma_H(x) \leq \sigma(x)$ for all $x \in V$ and $\mu_H(x,y) \leq \mu(x,y)$ for all edges $(x,y) \in E$.*

Definition 8 *Let X be a nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$, where the functions $\mu_A(x)$, $\nu_A(x) : X \longrightarrow [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.*

Definition 9 *An intuitionistic fuzzy graph (IFG) is of the form $IFG : (V, \sigma, \mu)$, where $\sigma = (\sigma_1, \sigma_2)$ and $\mu = (\mu_1, \mu_2)$ such that*

(i) *The functions $\sigma_1 : V \longrightarrow [0,1]$ and $\sigma_2 : V \longrightarrow [0,1]$ denote the degree of membership and nonmembership of the element $x \in V$, respectively and $0 \leq \sigma_1(x) + \sigma_2(x) \leq 1$ for every $x \in V$;*

(ii) *The functions $\mu_1 : V \times V \longrightarrow [0,1]$ and $\mu_2 : V \times V \longrightarrow [0,1]$ are the degree of membership and nonmembership of the edge $(x,y) \in E$, respectively, such that $\mu_1(x,y) \leq \min\{\sigma_1(x), \sigma_1(y)\}$ and $\mu_2(x,y) \leq \max\{\sigma_2(x), \sigma_2(y)\}$ and $0 \leq \mu_1(xy) + \mu_2(xy) \leq 1$ for every $(x,y) \in E$.*

Definition 10 *An intuitionistic fuzzy graph G is said to be complete intuitionistic fuzzy graph if it is strong. i.e., $\mu_1(x,y) = \min\{\sigma_1(x), \sigma_1(y)\}$, $\mu_2(x,y) = \max\{\sigma_2(x), \sigma_2(y)\}$, for all $x, y \in V$.*

Definition 11 *For $0 \leq \alpha \leq 1$, α - cut graph of intuitionistic fuzzy graph $IFG : (V, \sigma, \mu)$ is a crisp graph $IFG_\alpha^* : (V_\alpha, E_\alpha)$ such that $V_\alpha = \{x \in V \mid \sigma_1 \geq \alpha, \sigma_2 \geq \alpha\}$ and $E_\alpha = \{(x,y) \mid \mu_1 \geq \alpha, \mu_2 \geq \alpha\}$.*

Definition 12 *Intuitionistic fuzzy neighbourhood of a node x of an intuitionistic fuzzy graph $IFG = (V, \sigma, \mu)$ is an intuitionistic fuzzy set $N(x) = (V_x, m_x)$, where $V_x = \{y | (\mu_1(x,y), \mu_2(x,y) > (0,0))$ and $m_x : V_x \longrightarrow [0,1]$ is defined by $m_x(y) = \mu(x,y)$.*

Definition 13 *A fuzzy graph $G = (V, \sigma, \mu)$ is said to be a fuzzy star if every vertex of G has exactly one strong neighbour x (say) in $V(G)$.*

Definition 14 *A power neighbourhood of a node x is a node y such that (x,y) is a power full edge. An intuitionistic fuzzy graph IFG is said to be the intuitionistic fuzzy star if each node of IFG has precisely one power neighbour in IFG.*

3 Power Cut Graph of Intuitionistic Fuzzy Graphs

In this part, the α-power cut graph of IFG is defined with an example.

For an intuitionistic fuzzy graph $IFG = (V, \sigma, \mu)$, an edge (x, y); $x, y \in V$ is said to be independently powerful if $(0.5) \min\{\sigma_1(x), \sigma_2(x), \sigma_1(y), \sigma_2(y)\} \leq \mu_1(x, y)$ and $(0.5) \min\{\sigma_1(x), \sigma_2(x), \sigma_1(y), \sigma_2(y)\} \leq \mu_2(x, y)$. Otherwise, it is independent and powerless. The power of an edge (x, y) in an intuitionistic fuzzy graph $IFG = (V, \sigma, \mu)$ is denoted by $\tau_{(x,y)}$ and is defined as $\tau_{(x,y)} = (\tau_{1(x,y)}, \tau_{2(x,y)})$, where

$$\tau_{1(x,y)} = \frac{\mu_1(x, y)}{\min\{\psi_2(x), \psi_2(y)\}}, \quad \tau_{2(x,y)} = \frac{\mu_2(x, y)}{\min\{\psi_2(x), \psi_2(y)\}},$$

$$\psi_1(x) = \min\{\sigma_1(x), \sigma_2(x)\} \quad \text{and} \quad \psi_2(x) = \max\{\sigma_1(x), \sigma_2(x)\}.$$

Again, the power of a node x is denoted by τ_x and defined as $\tau_x = (\tau_{1x}, \tau_{2x})$, where τ_{1x} is the maximum value along its membership value $\psi_1(x)$, and the powers $\tau_{1(x,y)}$ of edges (x, y) incident to x and τ_{1x} are the maximum values along its membership value $\psi_2(x)$ and powers $\tau_{2(x,y)}$ of edges (x, y) incident to x.

Definition 15 *Let $IFG = (V, \sigma, \mu)$ be an intuitionistic fuzzy graph. For $0 \leq \alpha \leq 1$, the α-power cut graph of IFG is defined to be the crisp graph $IFG^\alpha = (V^\alpha, E^\alpha)$ such that $V^\alpha = \{x \in V \mid \tau_x \geq (\alpha, \alpha)\}$ and $E^\alpha = \{(x, y) \mid x, y \in V, \ \tau_{(x,y)} \geq (\alpha, \alpha)\}$.*

Example 1 *Let $IFG = (V, \sigma, \mu)$ be an intuitionistic fuzzy graph, where $V = \{u, v, w, x, y\}$, as shown in Figure 1. The node membership values are considered to be $(u, (0.4, 0.6))$, $(v, (0.2, 0.3))$, $(w, (0.5, 0.49))$, $(x, (0.4, 0.5))$, $(y, (0.55, 0.45))$. The edge membership values are considered to be $((u, v), (0.2, 0.29))$, $((u, w), (0.3, 0.35))$, $((u, x), (0.4, 0.45))$, $((u, y), (0.4, 0.5))$, $((v, w), (0.1, 0.2))$, $((v, x), (0.15, 0.25))$, $((v, y), (0.17, 0.23))$, $((w, x), (0.42, 0.47))$, $((w, y), (0.48, 0.42))$, $((x, y), (0.4, 0.42))$.*
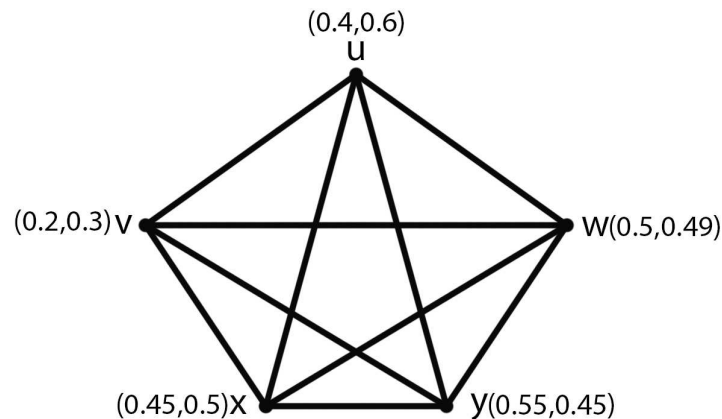

Figure 1: An intuitionistic fuzzy graph IFG.

Now the resultant 0.4-cut graph and 0.4-power cut graph are shown in Figure 2 and Figure 3, respectively.

The relation among dissimilar α-power cut graphs of an intuitionistic fuzzy graph is demonstrated using the following theorems.

Theorem 1 *Let IFG be an intuitionistic fuzzy graph. If $0 \leq \alpha \leq \beta \leq 1$, then $IFG^\beta \subseteq IFG^\alpha$.*
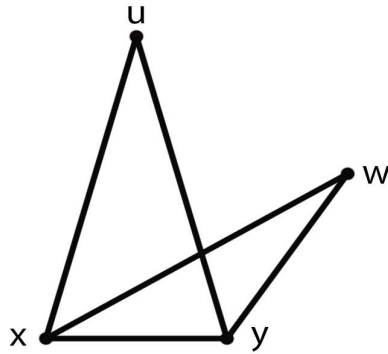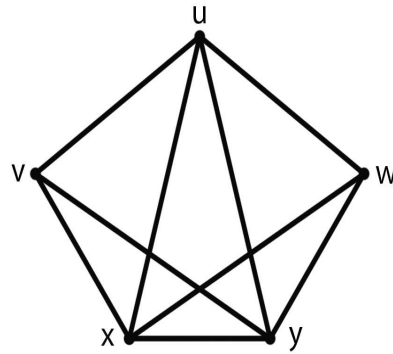
Figure 2: 0.4-cut graph of IFG.

Figure 3: 0.4-power cut graph of IFG.

Proof. Suppose $IFG = (V, \sigma, \mu)$ is an intuitionistic fuzzy graph and $0 \leq \alpha \leq \beta \leq 1$. Now, $IFG^\alpha = (V^\alpha, E^\alpha)$ such that $V^\alpha = \{x \in V \mid \tau_x \geq (\alpha, \alpha)\}$ and $E^\alpha = \{(x, y) \mid x, y \in V, \ \tau_{(x,y)} \geq (\alpha, \alpha)\}$. Additionally, $IFG^\beta = (V^\beta, E^\beta)$, where

$$V^\beta = \{x \in V \mid \tau_x \geq (\beta, \beta)\} \quad \text{and} \quad E^\beta = \{(x, y) \mid x, y \in V, \ \tau_{(x,y)} \geq (\beta, \beta)\}.$$

Let $p \in V^\beta$, then $\tau_p \geq \beta \geq \alpha$ and we have $p \in V^\alpha$. Similarly, for any element $(p, q) \in E^\beta$ gives $(p, q) \in E^\alpha$. Therefore, $IFG^\beta \subseteq IFG^\alpha$. ∎

The connection between α-cut graph IFG_α of an intuitionistic fuzzy graph IFG and power cut graph IFG^α of an intuitionistic fuzzy graph IFG is provided in the subsequent theorem.

Theorem 2 *Let IFG be an intuitionistic fuzzy graph. If $0 \leq \alpha \leq 1$, then $IFG_\alpha \subseteq IFG^\alpha$.*

Proof. Suppose $IFG = (V, \sigma, \mu)$ is an intuitionistic fuzzy graph, it follows $IFG_\alpha = (V_\alpha, E_\alpha)$, where $V_\alpha = \{x \in V \mid \sigma_1 \geq \alpha, \ \sigma_2 \geq \alpha\}$ and $E_\alpha = \{(x, y) \mid \mu_1 \geq \alpha, \ \mu_2 \geq \alpha\}$. Again, $IFG^\alpha = (V^\alpha, E^\alpha)$ such that $V^\alpha = \{x \in V \mid \tau_x \geq (\alpha, \alpha)\}$ and $E^\alpha = \{(x, y) \mid x, y \in V, \ \tau_{(x,y)} \geq (\alpha, \alpha)\}$. Let $x, y \in V_\alpha$ and $(x, y) \in E_\alpha$, therefore $(\sigma_1(x) \geq \alpha, \ \sigma_2(x) \geq \alpha$ and $\mu_1(x, y) \geq \alpha, \ \mu_2(x, y) \geq \alpha$. These results along with $\alpha \leq 1$. Give

$$\left(\frac{\mu_1(x, y)}{\min\{\psi_2(x), \psi_2(y)\}}, \frac{\mu_2(x, y)}{\min\{\psi_2(x), \psi_2(y)\}}\right) \geq (\alpha, \alpha) \quad \text{and} \quad \tau_{(x,y)} \geq \alpha,$$

so $(x, y) \in E^\alpha$. It shows that for every edge IFG_α, there is an edge IFG^α. Clearly, from the definition of power of nodes, we have $V_\alpha \subseteq V^\alpha$. It follows, the result $IFG_\alpha \subseteq IFG^\alpha$ is true. ∎

4   Range-Valued Fuzzy Colouring of an Intuitionistic Fuzzy Graph

In our latest plan, the colouring method of intuitionistic fuzzy graph is coloured by the range-valued fuzzy colour. The range-valued fuzzy colour is defined below.

4.1   Range-Valued Fuzzy Colour

Definition 16 *The combination of two dissimilar colours makes a new colour. However, the combination of any colour with a white colour decreases the brightness of that colour. Now, brightness is a fuzzy term. Assume that $T(\leq 1)$ units of a colour C_m is combined with $1 - T$ units of white colour, that combination is called a standard mixture of the colour C_m. The consequential colour is called a fuzzy colour of the colour C_m with membership value T, whereas C_m is called the fundamental colour. Yellow, Red, Black, Green, and so on are examples of fundamental colours.*

Definition 17 *Suppose that $C = \{C_1, C_2, ..., C_n\}, n \geq 1$ are the set of fundamental colours. The intuitionistic fuzzy set $(C, (g_1, g_2))$ is called the set of range- valued fuzzy colours if $(g_1 : C \longrightarrow [0,1]$ and $(g_2 : C \longrightarrow [0,1]$; with $(g_1(C_k), g_2(C_k))$, the membership value of the colour C_k is the amount per unit of standard mixture. The colour $\check{C}_k = (C_k, (g_1(C_k), g_2(C_k)))$ is called the range-valued fuzzy colour, which corresponds to the fundamental colour C_k. In view of range-valued fuzzy colour $(1,1)$, is the membership value for a range-valued fuzzy fundamental colour.*

From Definition 17, range-valued fuzzy colour, we can make different range-valued fuzzy colours beginning from a fundamental colour. For example, Green is a fundamental colour. A "range-valued fuzzy Green" colour may be designed from Green by combining $(0.7, 0.23)$ units of Green with $(1-0.7, 1-0.23) = (0.3, 0.77)$ units of white. This "range-valued fuzzy Green" colour is denoted by (Green, $(0.7, 0.23)$). Similarly, another range-valued fuzzy Green colour (Green, $(0.6, 0.3)$ may be formed by combining $(0.6, 0.3)$ Green units with $(0.4, 0.7)$ white units, and others. As a reminder, the range-valued fundamental Green colour is represented by $(Green, (1, 1))$.

Now, the range-valued fuzzy colouring of intuitionistic fuzzy graphs can be given below.

4.2 Working Rule of an Intuitionistic Fuzzy Graph Colouring

In crisp graph colouring, if any two nodes are adjacent, these two nodes receive distinct colours; if not, the colour is the same. Here, an intuitionistic fuzzy graph is coloured by the range-valued fuzzy colour. In this way, two nodes have distinct fundamental colours if they are adjacent to an independent power full branch. If not, they have distinct range-valued fuzzy colours.

Suppose $IFG = (V, \sigma, \mu)$ is a connected intuitionistic fuzzy graph and $C = \{C_1, C_2, ..., C_n\}$ is a set of fundamental colours. We observed that there are two types of intuitionistic fuzzy edges in intuitionistic fuzzy graphs, Viz. independent power engineering edges and independent non power engineering edges. The independent power less edge is less significant than the independent power full edge. As a result, the relationship between the corresponding nodes is independent power less. Our latest colouring idea is like conventional graph colouring, which depends on independent power full and independent power less edges. The planned intuitionistic fuzzy graph colouring can be divided into three groups that depend on the power of the edges.

Viz. (1) All edges are independent power full, (2) some edges are independent power full and (3) all edges are independent power less.

Case 1 *The intuitionistic fuzzy graph contains all independent power full edges.*

If an intuitionistic fuzzy graph contains all independent power full edges, the colouring of this intuitionistic fuzzy graph is identical to the colouring of crisp graphs, such that two nodes can be coloured by two different range-valued fuzzy fundamental colours if there is an independent power full edge between the nodes.

Case 2 *The intuitionistic fuzzy graph contains some independent power full edges.*

Suppose x is a node and the set of all neighbourhoods of node x is $N(x) = \{y_i, \ i = 1, 2, ..., n\}$. For simplicity, we assume that y_1, y_2 are two nodes such that $(x, y_1), (x, y_2)$ are only the two independent power full edges incident on x and all remaining edges $(x, y_i), \ i = 3, 4, ..., n$ are independent power less edges. In this case, we colour x by the colour $(C, (1, 1))$ and y_1 by a different colour $(C_2, (1, 1))$. Similarly, y_2 will achieve a distinct colour other than the colour of x.

Now, we consider the node y_3 for colouring. Notice that (x, y_3) is an independent power less edge.

Subcase 2.1: None of the adjacent nodes of y_3 are coloured.

As (x, y_3) is an independent power less edge, y_3 has a range-valued fuzzy colour corresponding to the colour of x. If the colour of x is $(C, (1, 1))$, the intuitionistic fuzzy colour of y_3 is $(C, (g_1(C), g_2(C)))$, where $(g_1(C), g_2(C))$ can be calculated as $(g_1(C), g_2(C)) = (1 - \tau_{1(x,y_3)}, 1 - \tau_{2(x,y_3)})$.

Subcase 2.2: All adjacent nodes of y_3 are coloured.

If an edge (y_3, z) incident to y_3 is independent power full, then y_3 cannot be coloured by the colour of z. That is, if the colour of z is $(C_z, (g_1(C_z), g_2(C_z)))$, y_3 cannot be coloured by any range-valued fuzzy colour of C_z. Suppose that y_3 has some independent power less incident edges, then (y_3, z_i), $i = 1, 2, ..., k$. Without a loss of generality, we assume that the colour of z_i is $(y_i, (g_1(y_i), g_2(y_i)))$, $i = 1, 2, ..., k$, where $(g_1(y_i), g_2(y_i))$, $i = 1, 2, ..., k$ are the membership values of the colour y_i, $i = 1, 2, ..., k$ and y_i, $i = 1, 2, ..., k$ may be the different or same. Now, to determine the colour of y_3, calculate $|\tau_{2(y_3,z_1)} - \tau_{1(y_3,z_1)}|$, $|\tau_{2(y_3,z_2)} - \tau_{1(y_3,z_2)}|$, ..., $|\tau_{2(y_3,z_k)} - \tau_{1(y_3,z_k)}|$ and $|\tau_{2(x,y_3)} - \tau_{1(x,y_3)}|$, to determine the maximum amount and then take the ordered pair as L, corresponding to the maximum amount.

Let L contribute to the edge (y_3, z_p), i.e., $L = (1 - \tau_{1(y_3,z_p)}, 1 - \tau_{2(y_3,z_p)})$. If the colour of z_p is $(y_p, g(y_p))$, then y_3 receives the colour(y_p, L). If the amounts are the same, an arbitrary choice can be made.

Subcase 2.3: Some of the adjacent nodes of y_3 are coloured.

The adjacent nodes that are not coloured, do not affect the colouring of y_3. Then, the adjacent nodes that are coloured will be considered for the colouring of y_3. The process of colouring y_3 is comparable to the subcase 2.2.

After colouring y_3, all of the other nodes are coloured in the same manner.

Case 3 *Intuitionistic fuzzy graph contains all independent power less edges.*

If IFG contains all independent power less edges, only one fundamental colour is needed to colour the nodes of IFG. Arbitrarily, consider any node (say x) and assign the fundamental colour to this node $(C_x, (1, 1))$, for the remaining nodes, assign the range-valued fuzzy colours corresponding to $(C_x, (1, 1))$ similar to the above subcase 2.2.

Notation 1 *If an intuitionistic fuzzy graph has more than one component, then each component is coloured by the above described method.*

5   Chromatic Number of an Intuitionistic Fuzzy Graph Colouring

The lowest number of fundamental colours used to colour an intuitionistic fuzzy graph is known as a fuzzy chromatic number. A fuzzy chromatic number of an intuitionistic fuzzy graph IFG is denoted by $\gamma(IFG)$. Here, we provide two examples for various fuzzy chromatic numbers.

Suppose, we consider two intuitionistic fuzzy graphs in which Figure 4(a) contains all independent power less edges. As all of the edges are independent power less, one fundamental colour is sufficient to colour all of the nodes. Here, the central node is coloured by $(R, (1, 1))$ and the other nodes are coloured by different range-valued fuzzy colours $(R, (0.p_1, 0.p_2))$, $(R, (0.q_1, 0.q_2))$, $(R, (0.r_1, 0.r_2))$ and $(R, (0.s_1, 0.s_2))$ of G, where p_1, p_2, q_1, q_2, r_1, r_2, s_1 and s_2 are natural numbers. Therefore, the fuzzy chromatic number of this intuitionistic fuzzy graph is one. In Figure 4(b), all of the edges are independent power full. Here, the fuzzy chromatic number is two, which is equal to the chromatic number of the original crisp graph. In the Figure 5.1, range-valued fuzzy colours with membership values are shown in parentheses.

Assume that we take four intuitionistic fuzzy graphs whose original crisp graph is K_4. In Figure 5(a), an intuitionistic fuzzy graph is measured such that all of its edges are independent power full. At that point, the intuitionistic fuzzy graph is four chromatic, as for K_4. In Figure 5(b), an intuitionistic fuzzy graph is considered where edges incident to exactly one node (say u) are independent power less and the remaining are independent power full. Again, colouring of one node depends on this type of independent power less incident edges whose other end nodes are coloured. That node is coloured by a range-valued fuzzy colour that is determined by measuring the powers of such independent power less edges. Let the minimum of the powers be (m, n) (here $m = 0.3, n = 0.6$, corresponding to the branch (u, v), and v is coloured by $(G, (1, 1))$. Then, node u will be coloured by the range-valued fuzzy colour $(G, (1 - m, 1 - n))$, i.e., $(G, (0.7, 0.4))$. Hence, it is triple-chromatic. Similarly, two other intuitionistic fuzzy graphs are coloured (see Figure 5(c) and Figure 5(d)).
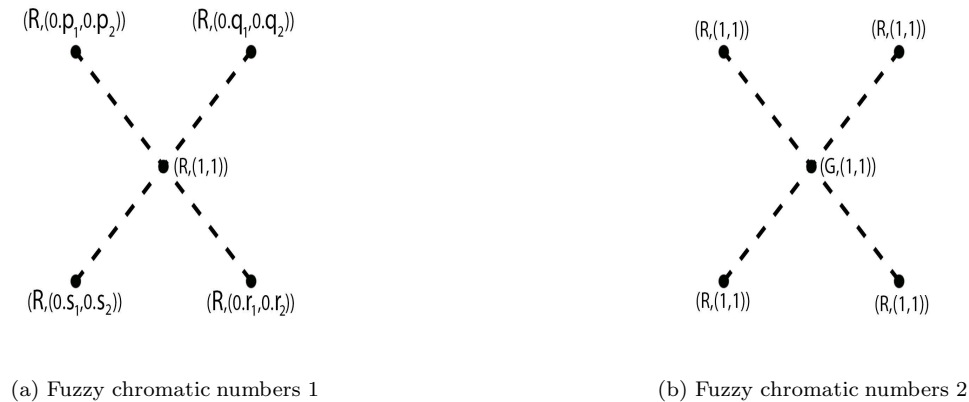
(a) Fuzzy chromatic numbers 1   (b) Fuzzy chromatic numbers 2

Figure 4: Fuzzy chromatic numbers graphs.

Proof. Suppose $IFG = (V, \sigma, \mu)$ is an intuitionistic fuzzy graph. Let the original crisp graph of IFG be $G = (V, E)$. Hence, $E = \{(x, y) \mid (\mu_1(x, y), \mu_2(x, y)) \geq (0, 0)\}$. Suppose G contains p edges and $\chi(G) = q$, where $q \leq p$. Therefore, IFG has at most p independent power full edges.

It is easy to consider whether all of the edges of IFG are independent power full, then the fuzzy chromatic number of IFG is the same as the chromatic number of its equal original crisp graph. In addition, we observe that two end nodes of an edge can be coloured by two range-valued fuzzy colours that are equivalent to the same fundamental colour if the edge is independent power less. As a result, if IFG has a lower number of independent power full edges than $p, \gamma(IFG)$ may be less than or equal to p. $\gamma(IFG) \leq p$. Hence, $\gamma(IFG) \leq \chi(G)$. ■

Remark 1 *If we know the fuzzy chromatic number of an intuitionistic fuzzy graph and chromatic number of an original crisp graph, we can easily calculate the minimum number of independent power less edges in the intuitionistic fuzzy graph. Above statement supports the following theorem.*

Theorem 3 *Let IFG be an intuitionistic fuzzy graph and G is its original crisp graph, $\chi(G) - \gamma(IFG)$ represents the minimum number of independent power less edges in IFG.*

Proof. Let $m = \chi(G) - \gamma(IFG)$. Here two cases arise.

Case 1. If $m = 0$, $\chi(G) = \gamma(IFG)$. This suggests that all of the edges of IFG are independent power full. As a result, the graph has no independent power less edge.

Case 2; $m > 0$. Let (x, y) be an edge in G and $(\acute{x}, \acute{y})$ be the corresponding edge in IFG. In G, two distinct colours are needed to colour the nodes x, y. In IFG, if $(\acute{x}, \acute{y})$ is an independent power full edge, $\acute{x}, \acute{y}$ are also coloured by two different fundamental colours. If the edge is not independent power full, $\acute{x}$ and $\acute{y}$ can be coloured by range-valued fuzzy colours of same fundamental colour; otherwise, any independent power full edge is related to $\acute{x}$. Therefore, $\acute{x}$ and $\acute{y}$ are coloured by range-valued fuzzy colours of the same fundamental colour if all edges incident to $\acute{x}$ or $\acute{y}$ are not independent power full edges. Hence, two end nodes in an intuitionistic fuzzy graph are coloured by range-valued fuzzy colours of the same fundamental colour if, at least, the corresponding edge of the end nodes is independent power less. This implies that IFG has at least $m = \chi(G) - \gamma(IFG)$ independent power less edges. As a result, $\chi(G) - \gamma(IFG)$ is the minimum number of independent power less edges. ■

The connection along chromatic numbers of dissimilar power cut graphs of an intuitionistic fuzzy graph is built up below.

Theorem 4 *Let IFG be an intuitionistic fuzzy graph. If $0 \leq \alpha \leq \beta \leq 1$, then $\chi(IFG^\beta) \leq \chi(IFG^\alpha)$.*
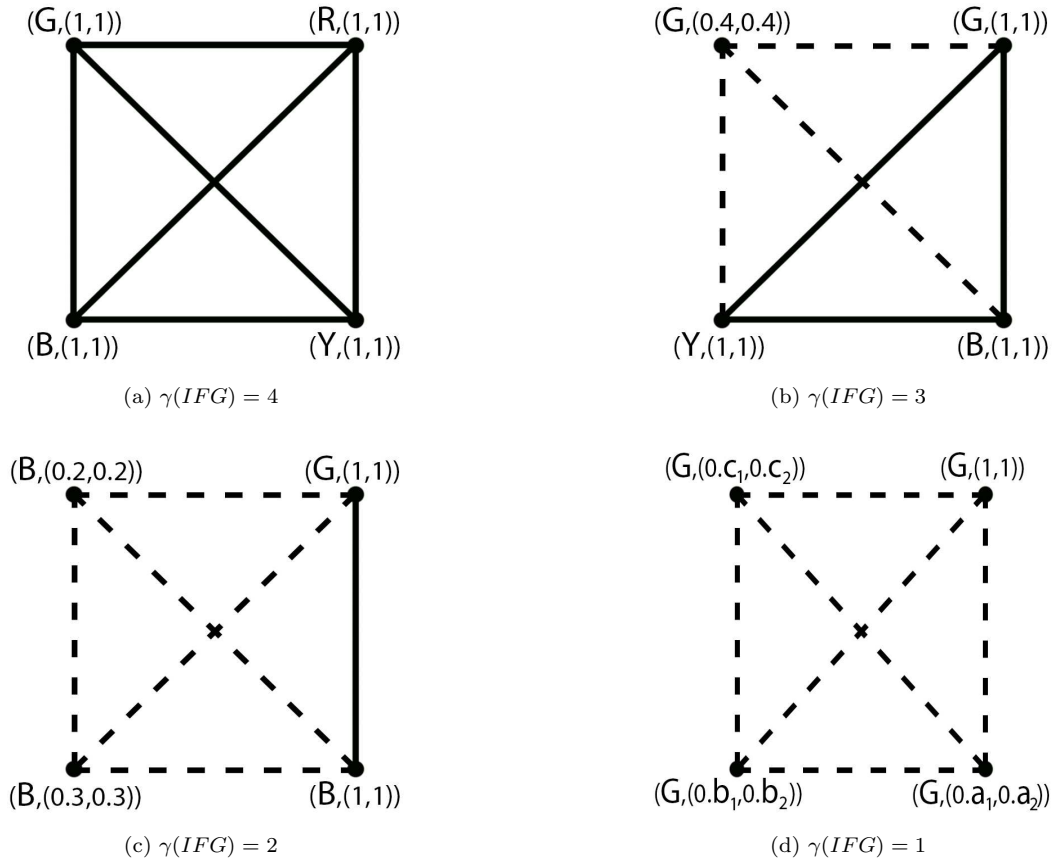
Figure 5: Intuitionistic fuzzy graph

Proof. It is evident that the chromatic number of a crisp graph is greater than or equal to the chromatic number of any subgraph. Additionally, from Theorem 1, if $0 \leq \alpha \leq \beta \leq 1$, then $IFG^{\beta} \subseteq IFG^{\alpha}$, so the theorem demonstrates that $\chi(IFG^{\beta}) \leq \chi(IFG^{\alpha})$ for $0 \leq \alpha \leq \beta \leq 1$. ■

The connection between the chromatic number of power cut graphs and intuitionistic fuzzy graphs can be found from the accompanying hypothesis. It is not probable to create a connection between the chromatic number of cut graphs and corresponding interval-valued fuzzy graphs. This is additionally outlined in note taking after the hypothesis.

Theorem 5 *If IFG is an intuitionistic fuzzy graph, then* $\chi(IFG^{0.5}) = \gamma(IFG)$.

Proof. Let $IFG = (V, \sigma, \mu)$ be an intuitionistic fuzzy graph. Now, $IFG^{0.5} = (V^{0.5}, E^{0.5})$, where $V^{0.5} = \{x \in V \mid \tau_x \geq (0.5, 0.5)\}$ and $E^{0.5} = \{(x, y) \mid x, y \in V, \ \tau_{(x,y)} \geq (0.5, 0.5)\}$. All independent power full edges of IFG are edges of $IFG^{0.5}$. Thus, there is a one to one connection between the independent power full edges in IFG and the edges in $IFG^{0.5}$, i.e., there is one edge in $IFG^{0.5}$ for all independent power full edge in IFG. Thus, $\chi(IFG^{0.5}) = \gamma(IFG)$. ■

Notation 2 *The chromatic number of* 0.5-*cut graph of an intuitionistic fuzzy graph may not equal that of an intuitionistic fuzzy graph. Theorem 2 implies that* $IFG_{\alpha} \subseteq IFG^{\alpha}$. *Moreover, Theorem 5 implies that* $\chi(IFG^{0.5}) = \gamma(IFG)$. *Thus* $\chi(IFG_{0.5}) \leq \gamma(IFG)$.

Notation 3 *We know that for complete crisp graph K_n; $\chi(K_n)$ is the number of its nodes. This result gives the following statement. If all of the edges of a complete intuitionistic fuzzy graph $IFG = (V, \sigma, \mu)$ are independent power full, then $\gamma(IFG)$ is the number of nodes of IFG.*

6   Application of a Range-Valued Fuzzy Colouring of the Intuitionistic Fuzzy Graph

A few applications are accessible in determining the intuitionistic fuzzy graph colouring. Here, two applications are depicted. Political map colouring and traffic signal problems are portrayed in the accompanying subsections.

6.1   Colouring of a Political Map

Political maps are intended to indicate legislative limits of nations, states and the area of rural and urban areas, and they typically incorporate critical water ways. To evaluate a certain region of nations, it will be more important to colour it as a fundamental of political boundaries and political relationships along such nations. In general, it is seen that the "relationships" are indefinite. As a result, the range-valued fuzzy colouring of an intuitionistic fuzzy graph is an essential problem.

Here, we model an intuitionistic fuzzy graph by taking nations $P, Q, R, S, T, U, V, W, X, Y$ and W as nodes, and there is an edge if they share a border (see Figure 6). Here, the edges were (P, R), (P, S), (P, T), (P, U), (P, V), (P, W), (Q, R), (Q, Y), (R, S), (S, T), (U, V), (U, X) and (V, W). The membership values of the nodes are the power of a nation (with respect to wealth, weapon, teaching, cooking facility, known branch, etc.), and the membership value of an edge is the degree of political connection in a confident approach. Because, we only concentrated on the model, we have to consider nodes with membership values of $(P, (0.p_1, 0.p_2))$, $(Q, (0.q_1, 0.q_2))$, $(R, (0.r_1, 0.r_2))$, $(S, (0.s_1, 0.s_2))$, $(T, (0.t_1, 0.t_2))$, $(U, (0.u_1, 0.u_2))$, $(V, (0.v_1, 0.v_2))$, $(W, (0.w_1, 0.w_2))$, $(X, (0.x_1, 0.x_2))$ and $(Y, (0.y_1, 0.y_2))$. Let $(P, R), (P, S), (P, U), (Q, Y)$ and (V, W) be independent power full edges, while (P, T), (P, V), (P, W), (Q, R), (R, S), (S, T), (U, V) and (U, X) are power less edges. The graphical representation is shown in Figure 7.
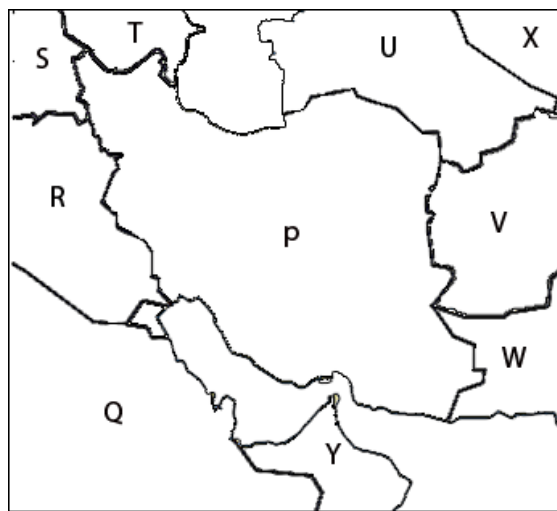

Figure 6: Iran political map of some nations.

Now the intuitionistic fuzzy graph IFG has both independent power full and independent power less edges. As a result, the colouring plan of Case 2 of Section 4.2 is applied. First, p is coloured with an arbitrary fundamental colour, such as Green. Because the edges $(P, R), (P, S)$ and (P, U) are independent
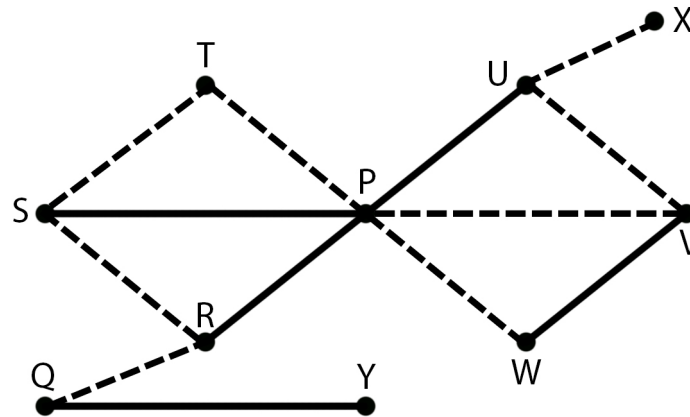
Figure 7: Intuitionistic fuzzy graph of IFG.

power full edges incident to P, R, S and U, they are coloured by different fundamental colours. However, (R, S) is an independent power less edge. As a result, we first colour any of R and S, node R coloured by the fundamental colour, namely Red. As the edge (R, S) is independent power less, the node S coloured by range-valued fuzzy colour $(Red, (0.a_1, 0.a_2))$ is similar and the other nodes Q, T, U, V, W, X and Y are coloured by $(Green, (1, 1))$, $(Green, (0.b_1, 0.b_2))$, $(Red, (1, 1))$, $(Green, (0.c_1, 0.c_2))$, $(Red, (1, 1))$, $(Green, (1, 1))$ and $(Red, (1, 1))$, respectively. Finally, IFG becomes coloured by range-valued fuzzy colours (Figure 8). This colouring is not only used to be visually pleasing; it has an important meaning regarding the political relationship. Here,
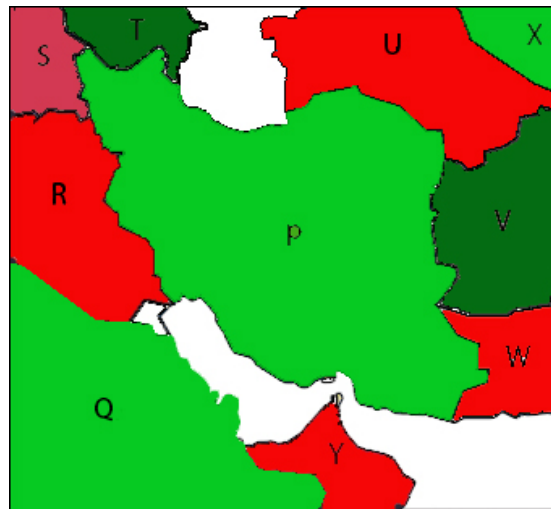

Figure 8: Iran political colour map.

the graph IFG is 2-chromatic because we utilized two fundamental colours (Red and Green) alone.

6.2   Traffic Signal Problem

The traffic signalling framework is at the junction and uses bright signals, mainly red and green. They twinkle in an organized pattern at the intersection of the paths for the safety of the people. Red means stop, and Green means go. Occasionally, a third colour is utilized to open some paths with a warming. Yet, this

third colour is unable to analyse the amount of threat on the paths.

These traffic signal frameworks can be visualized as a graph. In this graph, every path is considered to be nodes of the graph and there is an edge if paths intersect. This graph model can account for impacts at the intersections of paths. Furthermore, the issue strikes some paths, which are crowder contrasted with different paths. At some locations, a large number of individuals need to wait for long times because of traffic signals. Likewise, some paths impact hazards in contrast with different paths. Here, the expressions "crowder", "warning", and so forth do not represent the crisp graphs or intuitionistic fuzzy graphs with crisp nodes. As a result, straightforward graphs or intuitionistic fuzzy graphs with crisp nodes do not completely encompass the traffic signal framework. In this part, we acquainted another arrangement for the traffic signal issue.

As a traffic signal uses only red and green colours, we chose a slightly different approach to colour the corresponding intuitionistic fuzzy graph of the traffic signal system. Here, between any two nodes, if an independent power full edge exists, two nodes will be coloured red and green. The intuitionistic fuzzy graph is constructed so that if two paths have a slight chance to crash into one another, there is an edge between two paths, i.e., two nodes. As a result, the existence of the independent power less edge implies that there is some chance of a crash. Hence, two end nodes of an independent power less edge are given two range-valued fuzzy colours of two different fundamental colours.

We considered every path in terms of intuitionistic fuzzy nodes. Depending on the crowdedness on the path, we assign membership values to the nodes.

At the same time, opening some paths can contribute to accidents. Calculating the probability of accidents, an edge between two nodes (paths) is drawn if the paths collide. The membership values of the edges are calculated from the probability of accidents if the paths are opened. Then, the traffic signal system indicates an intuitionistic fuzzy graph. Here, red indicates the full stoppage of the path, and green indicates the full opening of the path. Range-valued fuzzy red will indicate certain danger compared to the safety of the path, and range-valued fuzzy green will represent certain safety over the danger of the path. In such intuitionistic fuzzy graphs, independent power full edges indicate that two paths, corresponding to the end nodes, are not opened at the same time, i.e., the nodes will have to be given distinct colours, i.e., red and green. Therefore, one path will be opened and the other will be closed. Independent power less edges in the intuitionistic fuzzy graph indicate that some danger is present between the paths of the corresponding nodes. As a result, if one end node is given red (or green), the other end node will be given range-valued fuzzy green (or range-valued fuzzy red). Now, we describe a specific traffic signal problem as follows.

Let some courses assemble at a particular point. We denote the north fixed path as N, south fixed path as S, east as E and west as W.

For simplicity, let E be a fixed path, i.e., the journey is imaginable from path E to other paths and traveling is not possible between any other paths and path E. We denote a path from north to south as NS, likewise for the other directions. As a result, the number of likely paths is NS, NW, WN, WS, SN, SW, EN, ES and EW (see Figure 9(a)).

For simplicity, we use $D = NS$, $F = NW$, $G = WN$, $H = WS$, $I = SN$, $J = SW$, $K = EN$, $L = ES$ and $M = EW$.

These are taken as the nodes of the proposed intuitionistic fuzzy graph. Suppose that in a specific time, course W is not crowded relative to the other paths. Determining the on crowd, let the colours of the nodes be given as $(D, (0.d_1, 0.d_2))$, $(F, (0.f_1, 0.f_2))$, $(G, (0.g_1, 0.g_2))$, $(H, (0.h_1, 0.h_2))$, $(I, (0.i_1, 0.i_2))$, $(J, (0.j_1, 0.j_2))$, $(K, (0.k_1, 0.k_2))$, $(L, (0.l_1, 0.l_2))$ and $(M, (0.m_1, 0.m_2))$. Observing the probability of accidents along crowds, the nodes (paths) are linked by edges. The edges (K, D), (M, D), (M, I), (L, I), (D, J), (F, I), (F, G) and (H, J) are calculated with membership values.

After calculating the powers of the paths, suppose that only the (M, D) and (M, I) paths are independent power full edges. The equivalent intuitionistic fuzzy graph is shown in Figure 9(b).

Now, the colouring of this intuitionistic fuzzy graph is completed by only two fundamental colours, viz. red and green. Here, M is coloured with a fundamental colour green. As a result, course M is opened. Now, we colour D and I. Here, (M, D) and (M, I) are independent power full edges. Therefore, D and I each receive a fundamental colour red. Now, K neighbors D, which is coloured red. Then, K receives the range-

(a) Fuzzy chromatic numbers 1   (b) Fuzzy chromatic numbers 2
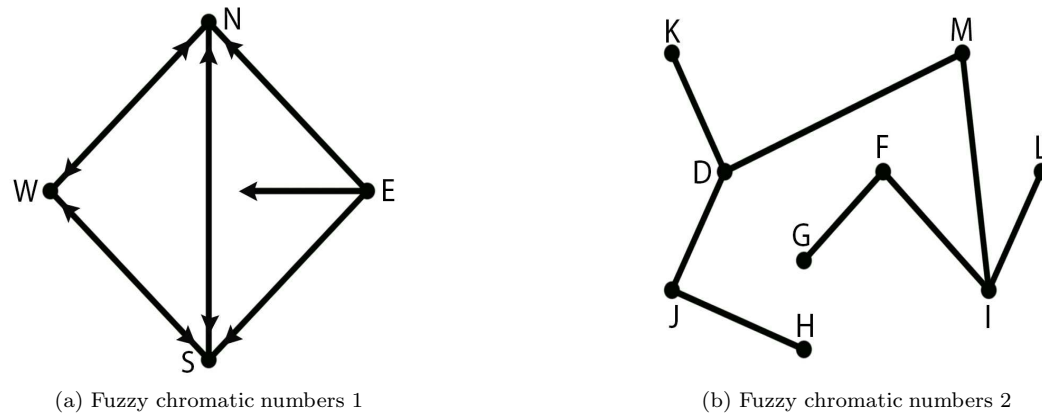
Figure 9: Traffic signal fuzzy graph.

valued fuzzy colour of Green. Suppose that K receives a range-valued fuzzy colour $(G, (0.k_1', 0.k_2'))$, similarly, other nodes J, F, and L receive the colours $(G, (0.j_1', 0.j_2'))$, $(G, (0.f_1', 0.f_2'))$, and $(G, (0.l_1', 0.l_2'))$, respectively. Finally, G receives the range-valued fuzzy red colour $(R, (0.g_1', 0.g_2'))$ and H receives range-valued fuzzy red $(R, (0.h_1', 0.h_2'))$.

7   Conclusion

Intuitionistic fuzzy models give more precision, flexibility, and compatibility to the system as compared to the fuzzy models. There are several different ideas regarding the quality of an edge in an intuitionistic fuzzy graph. We connected one of those notations. Cut label graphs are vital for intuitionistic fuzzy graphs. Here, power cut label graphs are presented. Connections between the power cut label graphs and cut label graphs are established. Additionally, few fundamental outcomes of graph theory have been demonstrated. The connection between the chromatic number of intuitionistic fuzzy graphs and its original graphs as well as connection between the chromatic number of intuitionistic fuzzy graphs and its power cut graphs are also recognized. Intuitionistic fuzzy graphs appropriately signify a few public problems. One of them is world political map. The current maps do not give information on the political connection among neighbouring nations. However, the intuitionistic fuzzy graph gives a genuine picture of the political connections. This introduced the colouring of intuitionistic fuzzy graphs and showed the political connection among the nations. In addition, straight traffic signals, red, green, and so on, do not suitably represent traffic systems. Hence, range-valued fuzzy colours are used to simplify the available systems. Thus, the traffic signal problem is explained here by colouring intuitionistic fuzzy graphs. Edge colouring is also significant for some genuine events. We are working on the edge colouring and total colouring of intuitionistic fuzzy graphs as an added layer of this subject. Also, since the concept of colouring fuzzy graphs has useful and wide applications in various sciences and life today, so, in our future work, we will study the concepts of colouring single valued neutrosophic graphs, colouring interval-valued neutrosophic graphs, and colouring neutrosophic cubic graphs with several examples.

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