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Evaluating Ruin Probabilities: A Streamlined Approach*

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Abstract

This paper deals with the ruin probability evaluation in a classical risk theory model, under different hypotheses about claims distribution. Our approach is totally innovative, and is based on the application of the Mean-Value Theorem to solve the associated Volterra integral equation. The numerical experiments show that the procedure we are proposing works well in all circumstances, compared to other pre-existing methodologies.

1 Introduction and Preliminaries

Keeping abreast of an insurance company's solvency is an indispensable information. In this view, the risk theory has proffered numerous tools to manage the risk that the aforementioned health inexorably deteriorates for very long time, see e.g. [12] for a comprehensive survey. The literature refers to suitable stochastic processes to describe the evolution of the reserve over time, defining the so called *surplus process.* Traditionally, the reserve consists of incoming premia P and outgoing claims S such that

$$U(t) = U(0) + P(t) - S(t),\ t \geq 0, \tag{1}$$

where $U(0) := u \geq 0$ is the initial capital.

Depending on the choice for P and S, the resulting surplus process can be managed in an appropriate manner, from a mathematical point of view. In the *classical compound Poisson risk model* we set

$$S(t) = \sum_{i=1}^{N(t)} X_i,\ t \geq 0,$$

where $N(t)$ is a Poisson process with parameter $\lambda > 0$, and $\{X_1, X_2, \ldots, X_{N(t)}\}$ is a sequence of non-negative and i.i.d. random variables representing the claim sizes. Concerning the income process, we have

$$P(t) = ct,\ t \geq 0,$$

where c is the *premium rate.* Usually, c is set equal to $(1+\theta)\lambda p_1$, with $p_1 = \mathbb{E}[X_i]$, $i = 1, \ldots, N(t)$ the expected claim size, while $(1+\theta)$ represents the *loading factor* applied to the net premium rate.

The claim distribution represents a degree of freedom in the model: [13] involve to exponential random variables, thanks to the features of the latter, ensuring to attain closed-form solutions. In [22, 9, 10, 23] the authors refer to *heavy-tailed* distributions. Despite a higher adherence to reality, such a choice pays for the existence of exclusively asymptotic results. More recently, [6] addressed the same issue, by exploiting finite-moments distributed claims.

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However, once the evolution of the process has been established, we need to verify whether there exists a threshold representing a tipping point, thus marking the insolvency status. This can be accomplished in terms of *ruin probability,* defined as

$$\phi(u) := \mathbb{P}[U(t) < 0, \text{ for some } t > 0 \, | U(0) = u]. \tag{2}$$

In order to assign a value to $\phi(u)$, we would like to write the latter as explicitly as possible. Applying Taylor expansion, or even looking at the surplus process as a particular case of a renewal process, see e.g. [11], we can easily derive the following integro-differential equation for (2)

$$\phi'(u) = \frac{\lambda}{c}\phi(u) - \frac{\lambda}{c}\int_0^u \phi(u-z)\mathrm{d}F(z) \text{ a.e.,} \tag{3}$$

being ϕ' the derivative of function ϕ, and $F(x)$ the cumulative distribution function (CDF) for X_i, for all $i = 1, \ldots, N(t)$. Therefore, if the theoretical framework is formally defined, problems arise when we need to determine a numerical value to be assigned to $\phi(u)$. The literature suggests several alternatives, with strengths and weaknesses. In [14, 2, 16], Monte Carlo techniques are exploited, either in deterministic or stochastic frameworks, see also [8], [15] and [3] for a complete survey. On the other hand, several authors have highlighted the close relationship between Eq. (3) and a *Volterra equation of second type,* i.e.

$$\phi(u) = \frac{1}{1+\theta}\left(A(u) + \int_0^u K(u,t)\phi(t)\mathrm{d}t\right), \tag{4}$$

with

$$A(u) = \int_u^\infty \frac{1-F(t)}{p_1}\mathrm{d}t, \tag{5}$$

$$K(u,t) = \frac{1-F(u-t)}{p_1}, \tag{6}$$

as it is shown e.g. in [20]. Therefore, attention was shifted to the latter, and an intense scientific production has been carried out in this sense, see e.g. [17] for further details. The techniques mainly investigated concern the use of the *Laplace transform,* to reduce the Volterra equation into an algebraic equation, see e.g. [21] and references therein. The present paper would like to contribute to the last strand of the literature, providing a new approach to evaluate (4) by exploiting the *Mean-Value Theorem* (MVT), see e.g. [24] and references therein. The algorithm proposed is inspired by the technique introduced in [7]: under some regularity hypotheses, MVT allows to suitably disentangle the integrating function. Thus, the integral equation can be discretized and transformed into a system of linear equations, whose solution is obtained by means of simple and fast quadrature methods. Our findings show that the proposed procedure works well under any claim distribution hypothesis, and it is comparable to preexisting methods in the literature, see e.g. [6] and [18], for Gamma and Pareto distributions respectively, while refer to e.g. [4] and [19] for Exponential and Weibull-distributed claims, respectively.

This paper is structured as follows. Section 2 describes the algorithm, along with the main theoretical results. The numerical applications are displayed in Section 3. Section 4 concludes.

2 The Proposal

To describe the algorithm we would like to propose, we should start by recalling a couple of results. The first one is the well-known *Mean-Value Theorem* in the generalized version proposed in [24].

Theorem 1 *Let $\psi : [a,b] \to [0,\infty)$ be a monotonic function and $\phi : [a,b] \to \mathbb{R}$ a Lebesgue integrable function. Then, there exists $\xi \in [a,b]$ such that*

$$\int_a^b \psi(u)\phi(u)\,du = \phi(a+)\int_a^\xi \psi(u)\,du + \phi(b-)\int_\xi^b \psi(u)\,du,$$

where $\phi(a+) := \lim\limits_{u\to a^+}\phi(u)$ and $\phi(b-) := \lim\limits_{u\to b^-}\phi(u)$.

Moreover, we consider the following Volterra integral equation

$$\phi(u) = f(u) + \int_0^t k(u,s)\phi(s)\,ds, \tag{7}$$

where $u, s \in I = [0, U]$, f is a given continuous function, $k(u, s)$ is the *kernel* function, assumed to be continuous in $I \times I$. It is worth recalling that ϕ in (7) coincides with the ruin probability related to the surplus process (1), where f and k are given in (5) and (6), respectively. Furthermore, we assume a partition Γ of length n for the interval $[0, U]$, being $U < \infty$ a sufficiently large value for the initial reserve, i.e.

$$\Gamma = \{0 = u_0, u_1, \ldots, u_{n-1}, u_n = U\}. \tag{8}$$

Given the partition Γ, and exploiting the linearity property, Eq. (7) can be rewritten as

$$\phi(u_i) = f(u_i) + \sum_{m=1}^{i} \int_{u_{m-1}}^{u_m} k(u_i,s)\phi(s)\,ds, \text{ for } i = 1, \ldots, n. \tag{9}$$

Next, we define the operators K and $\widetilde{K}$ by

$$(K\phi)(u_i) := \sum_{m=1}^{i} \left[\phi(u_{m-1}) \int_{u_{m-1}}^{\xi_m} k(u_i,s)\,ds + \phi(u_m) \int_{\xi_m}^{u_m} k(u_i,s)ds \right],$$

$$(\widetilde{K\phi})(u_i) := \sum_{m=1}^{i} \left[\phi(u_{m-1}) \int_{u_{m-1}}^{\widetilde{\xi_m}} k(u_i,s)\,ds + \phi(u_m) \int_{\widetilde{\xi_m}}^{u_m} k(u_i,s)\,ds \right],$$

for $i = 1, \ldots, n$.

Remark 1 *In [7] the authors show that*

$$\xi_m = \xi_m(u_m, \phi(u_m)) \in [u_{m-1}, u_m], \;\; m = 1, \ldots, n,$$

and w.l.o.g. we can choose $\widetilde{\xi_m} \in [u_{m-1}, u_m]$ *as a constant, for all* m.

To complete the procedure, we should ensure the convergence of the operator K. In this respect, we evoke the following

Proposition 1 *Let the kernel function* $k(u,t)$ *be continuous in* $I \times I$. *Assume also that functions* $u \mapsto k(u,t)$ *and* $t \mapsto k(u,t)$ *are monotonic and non-negative, for any fixed* $t \in I$ *and* $u \in I$, *respectively. Let* $L > 0$ *be a constant such that* $|k(u,t)| \leq L$ *for each* $(u,t) \in I \times I$. *Then*

$$|(\widetilde{K\phi})(u) - (K\phi)(u)| \to 0, \;\; \textit{as } n \to \infty.$$

Prop. 1 was proved in [7] for a broad class of integral equations, being our Volterra equation (7) a special case.

As a consequence of Prop. 1, if we set ϕ (resp.,$\widetilde{\phi}$) the true (resp., approximated) solution to (7), it is straightforward to prove the following

Corollary 1 *Under the hypotheses of Proposition 1, then*

$$|\widetilde{\phi(u)} - \phi(u)| \to 0, \;\; \textit{as } n \to \infty.$$

2.1 The MVT-Based Algorithm

We provide the following algorithm in order to find a numerical solution to Eq. (7).

Step 1 Let n be a positive integer and consider the partition Γ in (8) of $n+1$ evenly spaced points, with $u_i - u_{i-1} = \frac{u_n - u_0}{n} = \frac{U}{n}$ for all $i = 1, \dots, n$. Thanks to Proposition 1, equation (7) can be written as follows

$$\phi(u_i) = f(u_i) + \left[\sum_{m=1}^{i} \phi(u_{m-1}) \int_{u_{m-1}}^{\xi_m} k(u_i, s) ds + \phi(u_m) \int_{\xi_m}^{u_m} k(u_i, s) ds \right], \tag{10}$$

where $\xi_m \in (u_{m-1}, u_m)$ for $m = 1, 2, \dots, i$.

Step 2 First, we observe that $\phi(u_0) = f(u_0)$. In view of Remark 1, $\xi_m = \tilde{\xi}_m$, for all m. In particular, we draw ξ_m, $m = 1, \dots, n$ from a uniformly distributed sample over (u_{m-1}, u_m). The n-dimensional random vector $\{\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_n\}$ can be replaced in (10), giving rise to the following linear system

$$\begin{cases} \phi(u_0) = f(u_0) \\ \phi(u_1) - \phi(u_0) \int_{u_0}^{\tilde{\xi}_1} k(u_1, s)\, ds - \phi(u_1) \int_{\tilde{\xi}_1}^{u_1} k(u_1, s)]\, ds = f(u_1), \\ \phi(u_2) - \left[\sum_{m=1}^{2} \phi(u_{m-1}) \int_{u_{m-1}}^{\tilde{\xi}_m} k(u_2, s)\, ds + \phi(u_m) \int_{\tilde{\xi}_m}^{u_m} k(u_2, u_m) ds \right] = f(u_2), \\ \vdots \\ \phi(u_n) - \left[\sum_{m=1}^{n} \phi(u_{m-1}) \int_{u_{m-1}}^{\tilde{\xi}_m} k(u_n, s)\, ds + \phi(u_m) \int_{\tilde{\xi}_m}^{u_m} k(u_n, s) ds \right] = f(u_n). \end{cases} \tag{11}$$

Step 3 We choose a positive integer q. We perform Step 2 q times, obtaining a $q \times (n+1)$-matrix, where each row represents an approximation of the solution to (7), namely

$$\begin{bmatrix} \tilde{\phi}_1(u_0) & \tilde{\phi}_1(u_1) & \cdots & \tilde{\phi}_1(u_n) \\ \phi_2(u_0) & \phi_2(u_1) & \cdots & \phi_2(u_n) \\ \vdots & \cdots & \ddots & \vdots \\ \tilde{\phi}_q(u_0) & \tilde{\phi}_q(u_1) & \cdots & \tilde{\phi}_q(u_n) \end{bmatrix}. \tag{12}$$

According to the weak law of large numbers, the approximated solution $\bar{\phi}$ is obtained computing the mean value on each column of the matrix (15), given by

$$\bar{\phi} = \frac{\sum_{j=1}^{q} \tilde{\phi}_j(u_i)}{q} \quad \text{for } i = 0, \dots, n. \tag{13}$$

Remark 2 *To further reduce the computational time, we notice that (11) in matrix form becomes*

$$A \cdot \Phi = G, \tag{14}$$

where $\Phi = [\phi(u_0), \phi(u_1), \dots, \phi(u_n)]^T$, $G = [f(u_0), f(u_1), \dots, f(u_n)]^T$ *and*

$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -\int_{u_0}^{\tilde{\xi}_1} k(u_1, s)\, ds & 1 - \int_{\tilde{\xi}_1}^{u_1} k(u_1, s)\, ds & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\int_{u_0}^{\tilde{\xi}_1} k(u_n, s)\, ds & -\int_{\tilde{\xi}_1}^{\tilde{\xi}_2} k(u_n, s)\, ds & -\int_{\tilde{\xi}_2}^{\tilde{\xi}_3} k(u_n, s)\, ds & \cdots & 1 - \int_{\tilde{\xi}_n}^{u_n} k(u_n, s)\, ds \end{bmatrix}. \tag{15}$$

3 Numerical Results

In this section, we present some examples to test the efficiency of our proposal. Calculations were made by exploiting MATLAB software. We run our algorithm on a MacBook Pro with processor 2.6 GHz Intel Core i7 with 16-GB RAM.

In the sequel, we will refer to the classical compound Poisson process

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i, \tag{16}$$

where the parameters involved have been described in Section 1.

3.1 Exponential Distribution for Claim Sizes

Consider a sequence $\{X_1, \dots, X_{N(t)}\}$ of independent and identically Exponential-distributed random variables to put into Eq. (16). The corresponding Cumulative Distribution Function (CDF) is

$$F(t) = 1 - e^{-\lambda t}, \tag{17}$$

with rate parameter $\lambda > 0$. In this case, the ruin probability can be analytically computed, so that

$$\phi(u) = \frac{1}{1+\theta} \exp\left(-\frac{\lambda\theta u}{1+\theta}\right), \quad \lambda > 0,\ u > 0,$$

see e.g. [12]. Hence, we might use the closed-form solution (17) as a benchmark for our numerical test. The results are displayed in Table 1.

U	$\theta = 0.10$		$\theta = 0.25$		$\theta = 0.50$		$\theta = 0.75$		$\theta = 1.00$	
	Exp	MVT	Exp	MVT	Exp	MVT	Exp	MVT	Exp	MVT
5	0.577033	0.576836	0.294303	0.294301	0.125917	0.126067	0.067039	0.067188	0.041042	0.041138
10	0.366263	0.366081	0.108268	0.108334	0.023782	0.02384	0.007865	0.007897	0.003368	0.003388
15	0.232481	0.232311	0.039829	0.039862	0.004491	0.004510	0.000922	0.000929	0.000276	0.000278
20	0.147564	0.147423	0.014652	0.014668	0.000848	0.000852	0.000108	0.000109	0.000022	0.000022
25	0.093664	0.093544	0.005390	0.005397	0.000160	0.000161	0.000012	0.000012	0.0000018	0.0000018
30	0.059452	0.059369	0.001983	0.001986	0.000030	0.000030	0.000001	0.000001	0.0000001	0.0000001
35	0.037736	0.037679	0.000729	0.000731	0.000005	0.000005	0.000001	0.000001	0.00000001	0.00000001
40	0.023952	0.023920	0.000268	0.000268	0.000001	0.000001	0.00000002	0.00000002	0.000000001	0.000000001
45	0.015203	0.015179	0.000098	0.000098	0.0000002	0.0000002	0.000000002	0.000000002	0.00000000008	0.00000000008
50	0.009650	0.009631	0.000036	0.000036	0.00000003	0.00000003	0.0000000002	0.0000000002	0.000000000006	0.000000000007

Table 1: Ruin probability for Exponential-distributed claims for different values of the maximum initial reserve U and several loading factors θ. Here, *Exp* refers to the closed-form solution when the claims are Exponential-distributed. MVT stands for our proposal. We set $\lambda = 1$, $n = 200$ and $q = 1000$.

3.2 Weibull Distribution for Claim Sizes

Consider a sequence $\{X_1, \dots, X_{N(t)}\}$ of independent and identically Weibull-distributed random variables to put into Eq. (16). The corresponding Cumulative Distribution Function (CDF) is

$$F(t) = 1 - e^{-(\alpha t)^r},$$

with parameters $\alpha, r > 0$. In this case, we liken our findings to the ones provided in [19], where the ruin probability is asymptotically related to Erlang-Mixture distributions. The results are given in Table 2.

U	$\theta = 0.10$		$\theta = 0.25$		$\theta = 0.50$		$\theta = 0.75$		$\theta = 1.00$	
	ErM17	MVT	ErM17	MVT	ErM17	MVT	ErM17	MVT	ErM17	MVT
10	0.7507	0.7509	0.5296	0.5299	0.3412	0.3414	0.2457	0.2459	0.1895	0.1898
20	0.6433	0.6433	0.3833	0.3834	0.2059	0.2062	0.1324	0.1325	0.0947	0.0949
30	0.5548	0.5547	0.2823	0.2824	0.1291	0.1292	0.0755	0.0756	0.0508	0.0509
40	0.4797	0.4795	0.2097	0.2098	0.0825	0.0826	0.0444	0.0445	0.0285	0.0285
50	0.4153	0.4151	0.1566	0.1565	0.0535	0.0535	0.0267	0.0268	0.0164	0.0165
100	0.2037	0.2035	0.0376	0.0376	0.0069	0.0069	0.0026	0.0026	0.0014	0.0014

Table 2: Ruin probability for Weibull-distributed claims for different values of the maximum initial reserve U and several loading factors θ. Here, *ErM17* refers to the approximation method proposed in [19], while *MVT* stands for our proposal. Conforming to [19], we set $r = 0.5$, $\alpha = 1$, $n = 100$ and $q = 1000$.

3.3 Pareto Distribution for Claim Sizes

Consider a sequence $\{X_1, \dots, X_{N(t)}\}$ of independent and identically Pareto-distributed random variables to put into Eq. (16). The corresponding Cumulative Distribution Function (CDF) is

$$F(t) = 1 - \left(\frac{\alpha}{t+\alpha}\right)^{\alpha+1},$$

with shape parameter $\alpha > 0$. We are going to compare our proposal described in Section 2 with the Laplace-transform approach introduced in [18]. The results are displayed in Table 3, where we set $\alpha = 1$ and $n = 320$. As regards parameter q in Eq. (13), in our numerical experiments we set $q = 100$.

U	$\theta = 0.10$		$\theta = 0.25$		$\theta = 0.50$		$\theta = 0.75$		$\theta = 1.00$	
	Ram03	MVT	Ram03	MVT	Ram03	MVT	Ram03	MVT	Ram03	MVT
10	0.627128	0.627128	0.372677	0.372677	0.206646	0.206646	0.138242	0.138242	0.102523	0.102523
20	0.498142	0.498143	0.245260	0.245262	0.119274	0.119274	0.075908	0.075909	0.055049	0.055049
30	0.411437	0.411438	0.178338	0.178339	0.081426	0.081426	0.051056	0.051056	0.036887	0.036887
40	0.347893	0.347893	0.137559	0.137560	0.060856	0.060856	0.038038	0.038039	0.027509	0.027509
50	0.299155	0.299152	0.110519	0.110520	0.048164	0.048164	0.030142	0.030142	0.021847	0.021847
60	0.260646	0.260639	0.091524	0.091524	0.039650	0.039650	0.024884	0.024884	0.018080	0.018080
70	0.229551	0.229541	0.077594	0.077594	0.033588	0.033588	0.021150	0.021150	0.015402	0.015402
80	0.204018	0.204004	0.067029	0.067029	0.029075	0.029075	0.018369	0.018369	0.013404	0.013404
90	0.182761	0.182743	0.058794	0.058792	0.025596	0.025596	0.016222	0.016222	0.011859	0.011859
100	0.164860	0.164839	0.052227	0.052225	0.022839	0.022839	0.014517	0.014517	0.010630	0.010630

Table 3: Ruin probability for Pareto-distributed claims for different values of the maximum initial reserve U and several loading factors θ. Here, *Ram03* refers to the Laplace transforms-based method introduced in [18], while *MVT* stands for our proposal. Following [18], we set $p_1 = 1$ and $q = 100$.

3.4 Gamma Distribution for Claim Sizes

Consider a sequence $\{X_1, \dots, X_{N(t)}\}$ of independent and identically Gamma-distributed random variables to put into Eq. (16). The corresponding Cumulative Distribution Function (CDF) is

$$F(t) = \frac{\gamma(r, \frac{t}{\alpha})}{\Gamma(\alpha)},$$

where $\gamma(r, \frac{t}{\alpha})$ is the *lower incomplete gamma function*. We can compare our findings with three procedures proposed in [6], where Mittag-Leffler functions and moments of the claims distribution are involved, and the

Laplace transform as a vehicle to manage the Volterra equation is presented. In this case, results are given in terms of *survival probability*

$$\psi(u) := 1 - \phi(u),\ u \in [0, U],$$

being ϕ the ruin probability. The aforementioned results are displayed in Table 4.

U	CSZ18–1 ($h = 0.1$)	CSZ18–1 ($h = 0.01$)	CSZ18–1 ($h = 0.001$)	CSZ18–2	CSZ18–3	MVT
0	0.167	0.167	0.167	0.167	0.167	0.167
1	0.340	0.350	0.352	0.352	0.352	0.352
2	0.480	0.503	0.505	0.506	0.506	0.506
3	0.588	0.620	0.623	0.623	0.623	0.623
4	0.674	0.709	0.713	0.713	0.713	0.713
5	0.742	0.777	0.781	0.782	0.782	0.782
6	0.796	0.830	0.833	0.834	0.834	0.834
7	0.839	0.870	0.873	0.873	0.873	0.873
8	0.874	0.900	0.903	0.903	0.903	0.903
9	0.900	0.923	0.925	0.926	0.926	0.926
10	0.920	0.939	0.941	0.944	0.944	0.944

Table 4: Survival probability for Gamma-distributed claims for different values of the maximum initial reserve U. $CSZ18-1$, $CSZ18-2$, $CSZ18-3$ indicate Methods 1, 2 and 3 introduced in [6] with different sizes for discretization step h, while MTV stands for our proposal. We set $r = 2$, $\alpha = 2.4$, $\theta = 0.2$. In Method 2, Method 3 and MTV we use $h = 0.05$. As regards parameter q in Eq. (13), in our numerical experiments we set $q = 100$.

4 Conclusion

In this paper we have implemented a numerical algorithm for the evaluation of the ruin probability, expressed in the form of a solution to a Volterra integral equation. For the first time in the literature, we propose a methodology based solely on the application of the Mean-Value-Theorem. Despite the structural difference with respect to the techniques already in use within the scientific community, our numerical experiments prove that the algorithm is competitive, and produces significant results.

We would like to further contribute to the existing literature by broadening the results illustrated in this paper. In particular, starting from the theoretical findings developed in [5], it would be interesting to numerically study the ruin probability in non-homogeneous compound Poisson risk models. Furthermore, we aim to extend the outreach of the proposed algorithm also to the case of non-Markovian inter-arrival times, see e.g. [1]. The aforementioned open problems are part of our ongoing research.

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