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Analysis Of The Impact Of Media In Spreading The Blue Whale Game *

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Abstract

Media through its awareness program play an important role in communicating any crucial information like spread of any epidemic, terror attacks or any disease to the society. It helps the unaware population to gain awareness regarding the information which the media telecasts. But in many cases, it is seen that even though a lot of information is being transmitted through the print or electronic media for a very long period of time, many youngsters, although aware about the negative aspects of the information, act as unaware and sometimes face a fatal result for example the Blue Whale Game.

In this paper, we have proposed a mathematical model dealing with the effect of telecast of a media driven awareness program for a very long period of time, particularly the "Blue Whale Game". With the help some simple calculations and figures, we have predicted that sometimes, the media driven awareness programs may cause a negative impact on the society.

1 Introduction

Recently the Blue Whale game [12] also known as Blue Whale Challenge was in news due to its fatal results among the students of very young age. Blue whale challenge is a social network phenomenon that reportedly consists of a series of tasks assigned to the players by the administrators over a 50-days period with the final challenge requiring the player to commit suicide. Blue Whale Game came to prominence in May 2016 through an article in Russian newspaper, NOVAYA GAZETA that linked many unrelated child suicides to membership of group F57 on Russian based VKONTAKTA social network. A wave of moral panic swept Russia. But, due to its infectious nature, most of the developed and developing countries are effected from the challenges associated with this game.

In this paper, we have formulated a mathematical model to study the impact of media in popularizing the Blue whale game among the society. The effect of awareness programs by media have been the direction of study for many authors [9, 13, 4, 7, 3, 10]. Here we have considered the role of media on the three core classes of the population which we have taken as basic variables viz., (i) susceptible class of population i.e., the class of population which is using INTERNET (ii) infected class of population i.e. the class population which is involved in playing the game (iii) susceptible aware class of population i.e., the class of population which knows about the challenge of the game but still gets involved in it.

Although the role of media is to spread the awareness about the challenge so that the susceptible becomes alert and isolate themselves not only from the infective population but also from the irrelevant links received from the social media. However, due to repeated transmission of information regarding the game by the media, curiousness usually gets generated among the young generation and they try to know more and more about the game, which in turn makes them follow the game as designed by the game developer.

Here we have assumed that due to the curiousness some population lying in the aware susceptible class also move to infective class. This particular nature of some of the human beings, in our opinion, is erratic due

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to which, they do not consider the information [6] delivered by the media as awareness and try to interact with infective by any chance.

2 Mathematical Model

We consider that in the region under consideration the total population using internet is $N(t)$ at time t. The total population is divided into three classes the susceptible population $S(t)$, the infective population $I(t)$ and aware susceptible population $X_m(t)$. Also $M(t)$ is the cumulative density of the awareness created due to the information transmitted by the media in that region at time t. Here we have assumed that any type of addiction spreads not only due to direct contact between susceptible and infective but also due to curiosity which gets generated due to the information transmitted. The growth rate of density of the awareness transmitted by media is assumed to be proportional to the number of infective individuals. Further, it is assumed that due to the awareness transmitted by media susceptible individuals form a different class avoid contact with the infective individuals but some aware susceptible individuals move to infective class due to their curiousness.

Keeping above fact in mind the dynamics of model is governed by the following system of non-linear ordinary differential equations:

$$\begin{cases} \dfrac{dS}{dt} = A - \beta SI - dS - \mu SM + rI + \lambda_0 X_m, \\ \dfrac{dI}{dt} = \beta SI - dI - rI - \alpha I + aX_m, \\ \dfrac{dX_m}{dt} = \mu SM - dX_m - \lambda_0 X_m - aX_m, \\ \dfrac{dM}{dt} = \mu_0 I - \gamma M, \end{cases} \tag{1}$$

where $S(0) > 0$, $I(0) \geq 0$, $X_m(0) \geq 0$, $M(0) \geq 0$. In the above model (1), A is the immigration of population to susceptible class, β is the contact rate of susceptible with infective, d is the rate of natural death, μ is the rate at which susceptible class move to aware susceptible class due to awareness created through media driven awareness programs, r is the recovery rate, λ_0 is the rate of transfer of aware individuals to susceptible class, α is the disease induced death rate, a is the rate of curiousness due to which, aware susceptible population becomes infective class. μ_0 represents rate with which awareness program are being implemented and γ is the rate at which, the aware individuals get curious, due to excessive transmission of a particular awareness program.

Using the fact that $N = S + I + X_m$, the above system reduces to the following system,

$$\begin{cases} \dfrac{dI}{dt} = \beta(N - I - X_m)I - dI - rI - \alpha I + aX_m, \\ \dfrac{dX_m}{dt} = \mu(N - I - X_m)M - dX_m - \lambda_0 X_m - aX_m, \\ \dfrac{dN}{dt} = A - d(S + I + X_m) - \alpha I = A - dN - \alpha I \\ \dfrac{dM}{dt} = \mu_0 I - \gamma M. \end{cases} \tag{2}$$

Now it is sufficient to study model system (2). The set

$$\Omega = \left\{ (I, X_m, N, M) : 0 \leq I, X_m \leq N \leq \frac{A}{d}, 0 \leq M \leq \frac{\mu_0 A}{\gamma d} \right\}$$

is the region of attraction for the system and it attracts all solutions initiating in the interior of the positive octant.

3 Equilibrium Analysis

The above system (2) has two non-negative equilibrium points as follows:

1. $E_1(I_0, X_{m_0}, N_0, M_0)$.
2. $E_2(I^*, X_m^*, N^*, M^*)$.

The equilibrium point E_1 is given by the following.

$$I_0 = 0, X_{m_0} = 0, N_0 = \frac{A}{d}, M_0 = 0.$$

The equilibrium point $E_2(I^*, X_m^*, N^*, M^*)$ is obtained by equating to zero the right hand side of the system of equation (2), that is,

$$\beta(N - I - X_m)I - (d + r + \alpha)I + aX_m = 0, \tag{3}$$

$$\mu(N - I - X_m)M - (d + \lambda_0 + a)X_m = 0, \tag{4}$$

$$A - dN - \alpha I = 0, \tag{5}$$

$$\mu_0 I - \gamma M = 0, \tag{6}$$

which, on solving gives

$$I^* = \frac{[(d + \lambda_0 + a)\gamma\{\beta A - d(d + r + \alpha)\} + aA\mu\mu_0]}{[(\alpha + d)\{\beta\gamma(d + \lambda_0 + a) + a\mu\mu_0\} + d\mu\mu_0(d + r + \alpha)]},$$

$$X_m^* = \frac{(d + r + \alpha)\mu\mu_0}{[\beta\gamma(d + \lambda_0 + a) + a\mu\mu_0]} I^*,$$

$$N^* = \frac{[A - \alpha I^*]}{d} \quad \text{where } A > \alpha I^*,$$

$$M^* = \frac{\mu_0}{\gamma} I^*.$$

The equilibrium E_2 will exist only when I^* is positive, i.e.

$$\frac{\beta A}{d} - (d + r + \alpha) > 0,$$

$$R_0 := \frac{\beta A}{d(d + r + \alpha)} > 1.$$

R_0 is the basic reproduction number for the system (2). Thus equilibrium E_2 exists for $R_0 > 1$.

Remark 1 *From I^*, it is easy to note that $\frac{dI^*}{d\gamma} > 0$, which shows that the number of infective individuals increases with an increase in the rate at which, the aware individuals get curious, due to excessive telecast of particular awareness program.*

4 Stability Analysis

4.1 Local Stability Analysis

The local stability of the system (2) around each of the equilibrium point is obtained by computing the variational matrix $V(E)$, where E is an equilibrium point [11], [8].

The stability conditions for the equilibrium points E_1 and E_2 are stated in the following cases:

Case-I: The variational matrix $V(E_1)$ around equilibrium point $E_1(I_0, X_{m_0}, N_0, M_0)$ is given by

$$V(E_1) = \begin{bmatrix} \beta\frac{A}{d} - (d+r+\alpha) & a & 0 & 0 \\ 0 & -(d+\lambda_0+a) & 0 & \mu\frac{A}{d} \\ -\alpha & 0 & -d & 0 \\ \mu_0 & 0 & 0 & -\gamma \end{bmatrix}.$$

Lemma 1 *The system (2) around the equilibrium point $E_1(0,0,\frac{A}{d},0)$ is locally asymptotically stable if $\frac{\beta A}{d} - (d+r+\alpha) < 0$, that is, if $R_0 < 1$ and becomes unstable if $R_0 > 1$.*

Case-II: The variational matrix $V(E_2)$ around equilibrium point $E_2(I^*, X_m^*, N^*, M^*)$ is given by

$$V(E_2) = \begin{bmatrix} -V_1 & -V_2 & V_3 & 0 \\ V_4 & -V_5 & V_6 & V_7 \\ -V_8 & 0 & -V_9 & 0 \\ V_{10} & 0 & 0 & -V_{11} \end{bmatrix}$$

where $V_1 = \frac{aX_m}{I} + 2\beta I$, $V_2 = \beta I - a$, $V_3 = \beta I$, $V_4 = -\mu M$, $V_5 = \mu M + (d+\lambda_0+a)$, $V_6 = \mu M$, $V_7 = \mu(N - I - X_m)$, $V_8 = \alpha$, $V_9 = d$, $V_{10} = \mu_0$ and $V_{11} = \gamma$. Therefore corresponding characteristic equation is

$$D(\lambda) = \lambda^4 + \sigma_1\lambda^3 + \sigma_2\lambda^2 + \sigma_3\lambda + \sigma_4 = 0, \tag{7}$$

where λ is the eigen value and

$$\sigma_1 = V_1 + V_5 + V_9 + V_{11},$$

$$\sigma_2 = V_2V_4 + V_1V_5 + V_3V_8 + V_1V_9 + V_5V_9 + V_1V_{11} + V_5V_{11} + V_9V_{11},$$

$$\begin{aligned} \sigma_3 \;\; = \;\; & V_3V_5V_8 - V_2V_6V_8 + V_2V_4V_9 + V_1V_5V_9 + V_2V_7V_{10} + V_2V_4V_{11} \\ & + V_1V_5V_{11} + V_3V_8V_{11} + V_1V_9V_{11} + V_5V_9V_{11}, \end{aligned}$$

$$\sigma_4 = V_2V_7V_9V_{10} + V_3V_5V_8V_{11} - V_2V_6V_8V_{11} + V_2V_4V_9V_{11} + V_1V_5V_9V_{11}.$$

Since $\sigma_1 > 0$, $\sigma_3 > 0$, $\sigma_4 > 0$ and $\sigma_1\sigma_2\sigma_3 > \sigma_3^2 + \sigma_1^2\sigma_4$ thus by Routh-Hurwitz criteria all roots of (7) are either negative or have negative real parts. Thus the system will be locally asymptotically stable.

4.2 Global Stability Analysis

Lemma 2 *The equilibrium point $E_2(I^*, X_m^*, N^*, M^*)$ is globally asymptotically stable if the following inequalities hold true:*

$$\begin{aligned} & \left(a - \beta I^* - \frac{I^*\beta d(\mu M^* + d + \lambda_0 + a)}{2k\alpha\mu M^*}\right)^2 \\ & < \frac{9I^*\beta d(\mu M^* + d + \lambda_0 + a)^2}{2\alpha\mu^2(M^*)^2}(-\beta N^* + 2\beta I^* + \beta X_m^* + d + \alpha + r) \end{aligned}$$

and

$$\frac{I^*\beta d(N^* - I^* - X_m^*)^2}{k\alpha(M^*)^2\gamma} < \frac{\gamma}{3\mu_0^2}(2\beta I^* + \beta X_m^* - \beta N^* + d + r + \alpha).$$

Proof. To check global stability of equilibrium point E_2 we use Liapunov's method [[5], [14]]. Consider a positive definite function U such that

$$U = \frac{1}{2}i^2 + \frac{p_1}{2}x_m^2 + \frac{p_2}{2}n^2 + \frac{p_3}{2}m^2$$

where p_1, p_2 and p_3 are positive constants, and i, x_m, n and m are small perturbation from the equilibrium point E_2,that is, $I = I^* + i, X_m = X_m^* + x_m, N = N^* + n$ and $M = M^* + m$. On differentiating U with respect to t, we get

$$\frac{dU}{dt} = i\frac{dI}{dt} + p_1 x_m \frac{dX_m}{dt} + p_2 n \frac{dN}{dt} + p_3 m \frac{dM}{dt}$$

which due to (2), (3), (4), (5) and (6) gives

$$\begin{aligned}
\frac{dU}{dt} &= i[\beta(N - I - X_m)I - (d + \alpha + r)I + aX_m] + p_1 x_m[\mu(N - I - X_m)M \\
&\quad -(d + \lambda_0 + a)X_m] + p_2 n[A - dN - \alpha I] + p_3 m[\mu_0 I - \gamma M] \\
&= i[\beta(N^* - I^* - X_m^*)I^* - (d + \alpha + r)I^* + aX_m^* + \beta(N^* - I^* - X_m^*)i \\
&\quad -(d + \alpha + r)i + ax_m + \beta(n - i - x_m)i + \beta(n - i - x_m)I^*] \\
&\quad +p_1 x_m[\mu(N^* - I^* - X_m^*)M^* - (d + \lambda_0 + a)X_m^* + \mu(n - i - x_m)m \\
&\quad -(d + \lambda_0 + a)x_m + \mu(N^* - I^* - X_m^*)m + \mu(n - i - x_m)M^*] \\
&\quad +p_2 n[A - dN^* - \alpha I^* - dn - \alpha i] + p_3 m[\mu_0 I^* - \gamma M^* + \mu_0 i - \gamma m].
\end{aligned}$$

On linearizing the above equation, we get

$$\begin{aligned}
\frac{dU}{dt} &= i[\beta(N^* - I^* - X_m^*)i - (d + \alpha + r)i + ax_m + \beta(n - i - x_m)i \\
&\quad +\beta(n - i - x_m)I^*] + p_1 x_m[\mu(n - i - x_m)m - (d + \lambda_0 + a)x_m + \\
&\quad \mu(N^* - I^* - X_m^*)m + \mu(n - i - x_m)M^*] + p_2 n[dn - \alpha i] + p_3 m[\mu_0 i - \gamma m] \\
&= -i^2[-\beta(N^* - I^* - X_m^*) + \beta I^* + (d + \alpha + r)] - x_m^2 p_1[(d + \lambda_0 + a) + \mu M^*] \\
&\quad -n^2[p_2 d] - m^2[p_3\gamma] + ix_m[a - \beta I^* - p_1\mu M^*] + in[\beta I^* - p_2\alpha] + im[-p_3\mu_0] \\
&\quad +x_m m[p_1\mu(N^* - I^* - X_m^*)] + x_m n[p_1\mu M^*].
\end{aligned}$$

$\frac{dU}{dt}$ will be negative definite provided, for $k > 1$,

$$\{a - \beta I^* - p_1\mu(M^*)\}^2 < 9k(-\beta N^* + 2\beta I^* + \beta X_m^* + d + \alpha + r)p_1(\mu M^* + d + \lambda_0 + a),$$

$$(\beta I^* - p_2\alpha)^2 < \frac{1}{2}(-\beta N^* + 2\beta I^* + \beta X_m^* + d + \alpha + r)p_2 d,$$

$$p_3\mu_0^2 < \frac{1}{3}(-\beta N^* + 2\beta I^* + \beta X_m^* + d + \alpha + r)\gamma,$$

$$p_1\mu^2(N^* - I^* - X_m^*)^2 < \frac{1}{2}p_3(\mu M^* + d + \lambda_0 + a)\gamma$$

and

$$p_1\mu^2(M^*)^2 < \frac{1}{2}p_2(\mu M^* + d + \lambda_0 + a)d.$$

On choosing

$$p_2 = \frac{\beta I^*}{\alpha}$$

and

$$p_1 = \frac{I^*\beta d}{2k\alpha\mu^2(M^*)^2}(\mu M^* + d + \lambda_0 + a)$$

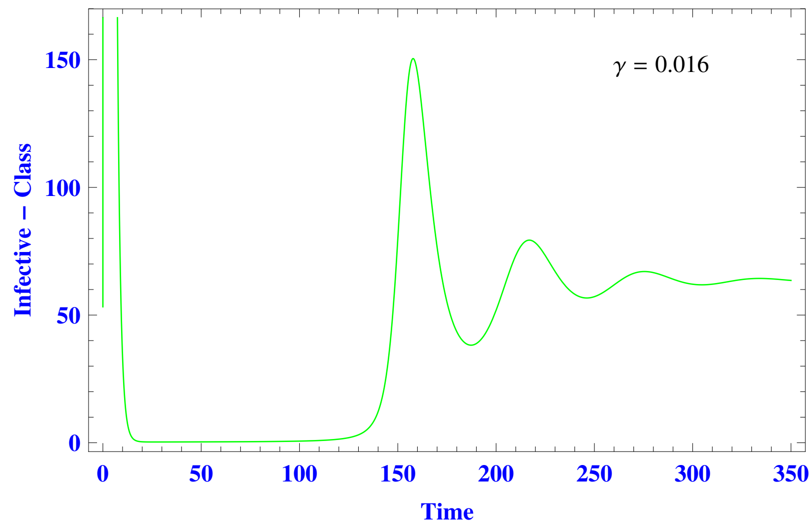

Figure 1: Variation in Infective Class with respect to time, when value of $\gamma = 0.016$.

the above inequalities may be combined from which the Lemma follows. ■

From above inequality conditions it appears that the γ rate at which, the aware individuals get curious, due to excessive telecast of particular awareness program, have a destabilizing effect on our society.

5 Numerical Simulation

In this section, a numerical verification is provided for the existence of the equilibrium points E_2 and its stability properties. The model is simulated using the different set of parameter values.

To check the feasibility of our analysis regarding stability conditions, we have conducted some numerical computation using MATHEMATICA by choosing following set of parameter values in model (2):

$A = 1100$, $\beta = 0.445$, $d = 0.32$, $r = 0.0035$, $\alpha = 0.40$, $a = 0.00005$, $\mu = 0.65$, $\lambda_0 = 0.3020$, $\mu_0 = 0.490$, $\gamma = 0.016$.

For the above set of parameter, the condition of existence of equilibrium E_2 ($i.e. R_0 > 1$) and the stability condition in Lemma 2 are satisfied. The equilibrium components are found as follows for different values of γ:

1. For $\gamma = 0.016$,
$$I^* = 63.51, X_m^* = 3292.96, N^* = 3358.1, M^* = 1945.27.$$

2. For $\gamma = 0.245$,
$$I^* = 608.443, X_m^* = 2066.88, N^* = 2676.95, M^* = 1261.89.$$

The eigenvalues of the Variational matrix corresponding to the equilibrium point E_1 for the model system (2) are

$$1529.07, -0.621991, -0.32, -0.160591$$

and that for equilibrium point E_2 for the model system (2) are

$$-1293.92, -27.6514, -0.320097, -0.0164102.$$

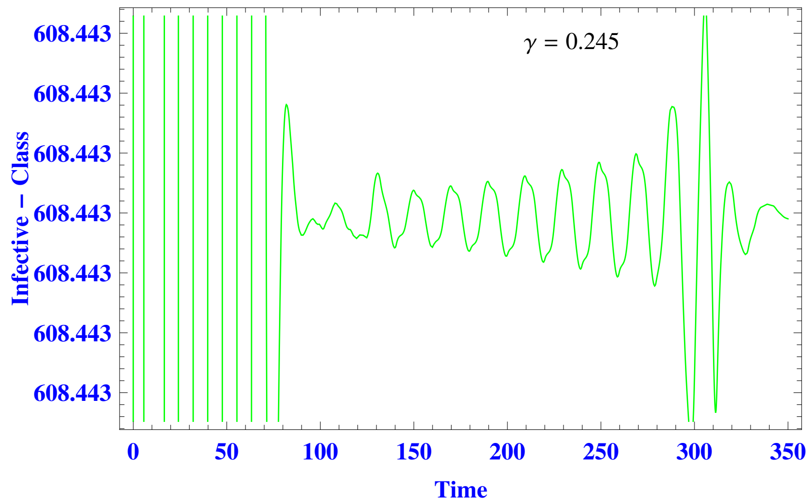

Figure 2: Variation in Infective Class with respect to time, when value of $\gamma = 0.245$.

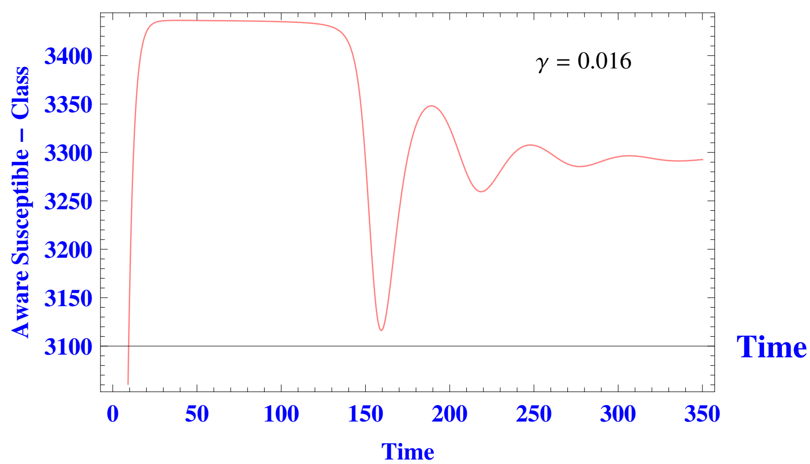

Figure 3: Variation in Aware Susceptible-Class with respect to time, when value of $\gamma = 0.016$.

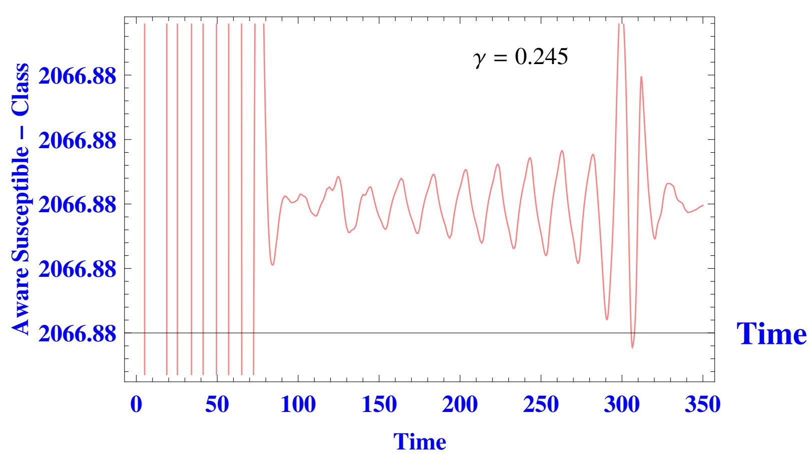

Figure 4: Variation in Aware Susceptible-Class with respect to time, when value of $\gamma = 0.245$.

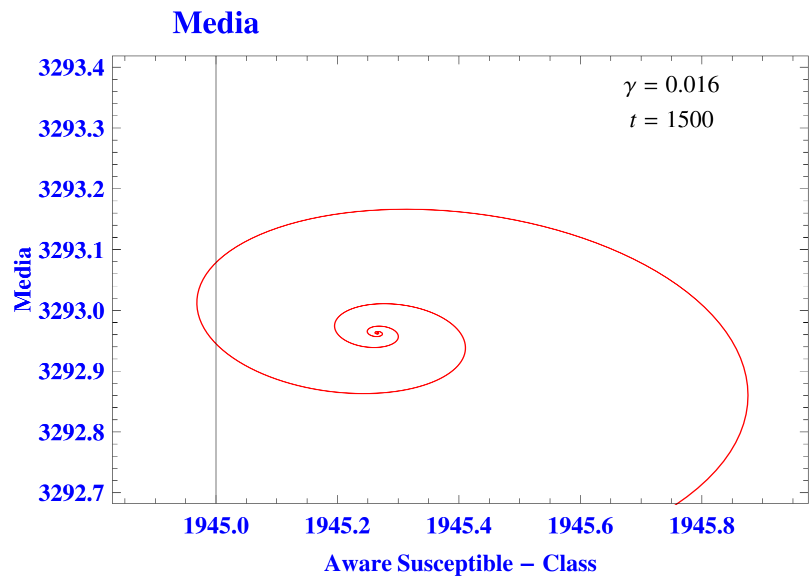

Figure 5: Phase portrait corresponding to stability of equilibrium point E_2 in $M - X_m$ plane.

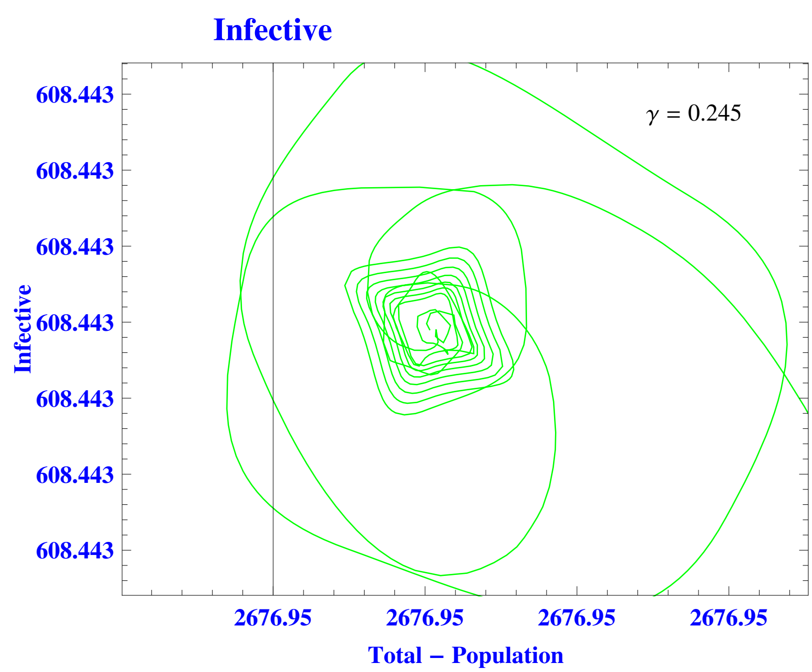

Figure 6: Phase portrait corresponding to stability of equilibrium point E_2 in $I - N$ plane.

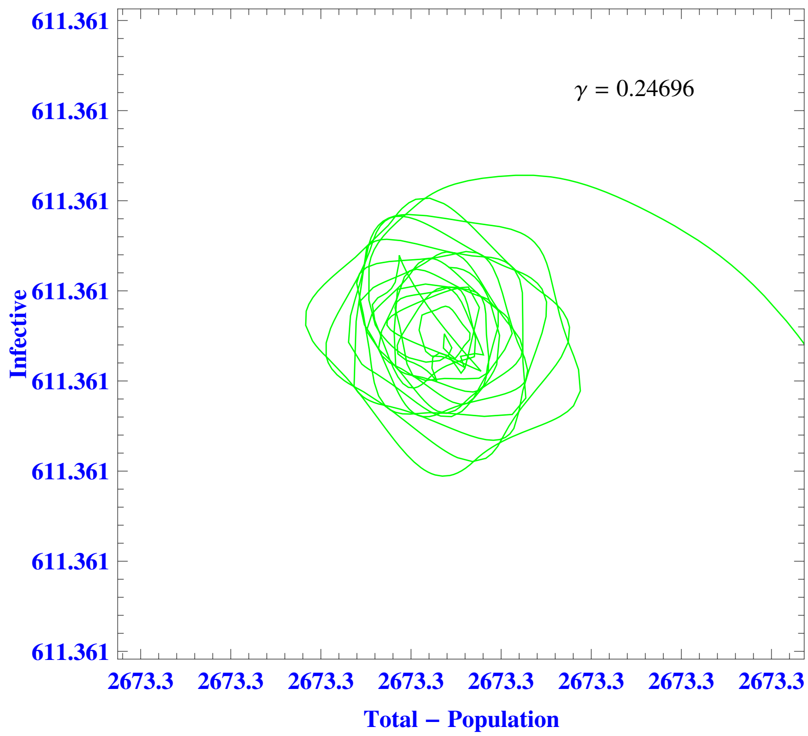

Figure 7: Phase portrait corresponding to stability of equilibrium point E_2 in $I - N$ plane.

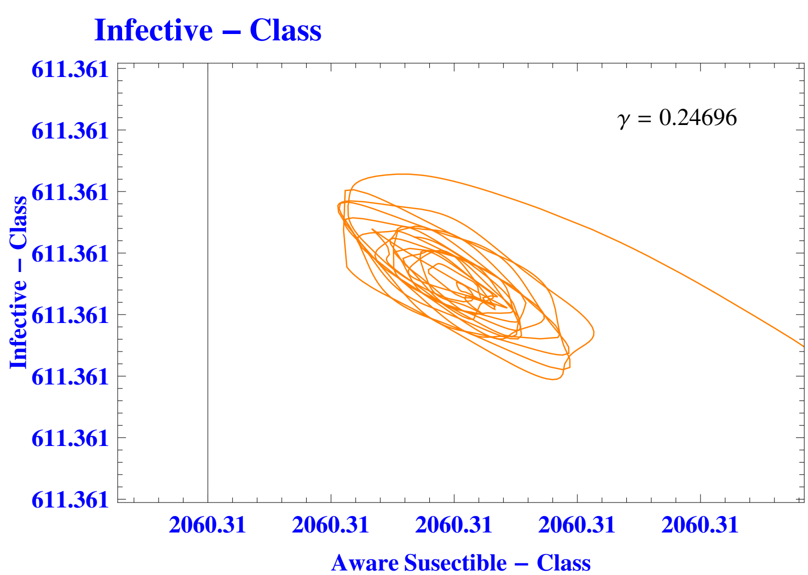

Figure 8: Phase portrait corresponding to stability of equilibrium point E_2 in $I - X_m$ plane.

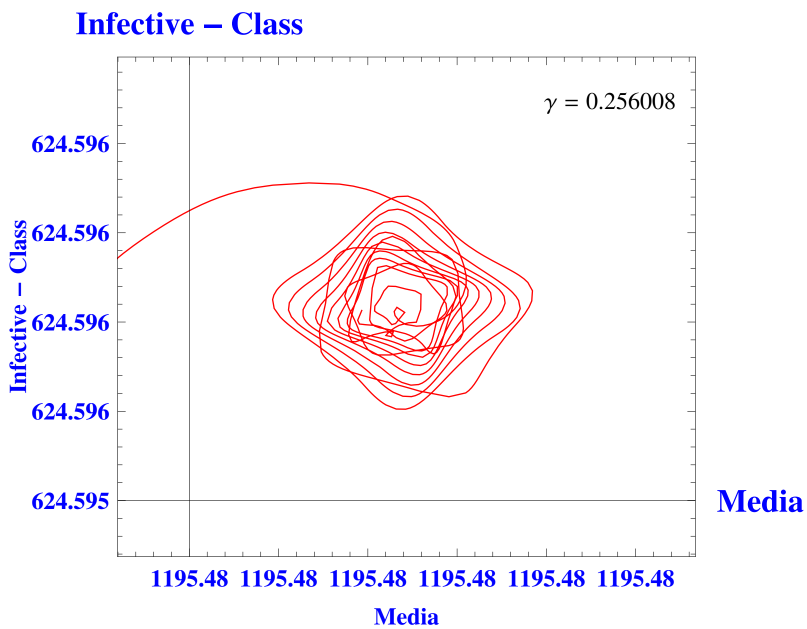

Figure 9: Phase portrait corresponding to stability of equilibrium point E_2 in $I - M$ plane.

6 Discussions of Results

We note that one of the eigenvalues of variational matrix corresponding to the point E_1 is positive and every eigenvalues of variational matrix corresponding to the point E_2 are negative. Hence the equilibrium point E_1 is unstable and E_2 is asymptotically stable for these values of constants.

Further, to check the stability of the solution of the system we find the maximum Lyapunov exponent [1], which characterizes the separation of two infinitely close trajectories of the system and is defined as

$$\lambda = \lim_{t\to\infty} \lim_{|\delta \mathbf{Z}_0|\to 0} \frac{1}{t} \ln \frac{|\delta \mathbf{Z}(t)|}{|\delta \mathbf{Z}_0|} \lambda = \lim_{t\to\infty} \lim_{|\delta \mathbf{Z}_0|\to 0} \frac{1}{t} \ln \frac{|\delta \mathbf{Z}(t)|}{|\delta \mathbf{Z}_0|}$$

where $\delta\mathbf{Z}(t)$ is the distance between any two trajectories at time t and $\delta\mathbf{Z}_0$ is the initial separation vector. Taking the parameters as $\beta = 0.345; a = 0.05; \mu = 0.45; \lambda_0 = 5.382; b = 0.32; r = 0.35; \alpha = 0.4; \mu_0 = 0.550; \gamma = 0.016; z = 119.27; v = 13.1; x = 68.51; y = 12.96;$ we plot the Lyapunov characteristic exponent as shown in Figure 10. Clearly the maximal Lyapunov exponent is negative which implies that the solution of the system is asymptotically stable.

Figures for different values of the variables were plotted using Mathematica to draw a conclusion regarding the model considered in (1). We tried to draw figures when only one variable was varied whereas the values of the other variables were kept fixed. There was a very minimal change in the figures if each variable, except when γ was varied. The value of γ was varied for a very wide range keeping the value of other variables fixed.

It is evident from the Figures 1, 3 and 5 that, when $\gamma \leq 0.016$ that is, when an awareness program is transmitted by media for a short period of time, the society becomes well aware about the programs. Whereas, when $0.016 < \gamma \leq 0.245$, that is, when an awareness program is transmitted by media for a longer period of time, it is evident from the Figures 2, 4 and 6 that these awareness programs have a negative impact on the society. On further increasing γ the situation turns out to be chaotic slowly but certainly as is shown in Figures 7, 8 and 9.

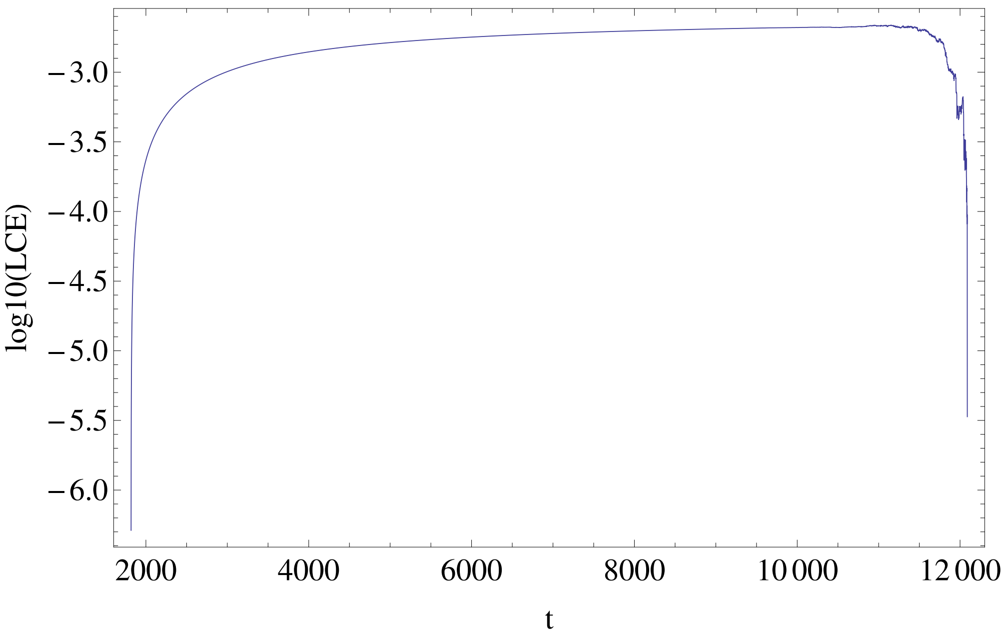

Figure 10: Lyapunov Characteristic Exponent for the system.

7 Conclusion

It is very evident from the figures that awareness programs run by either the print or the electronic media help the society in getting aware to a certain extent provided the program has been presented correctly for a short period of time or for only that period of time which may be sufficient for it to reach to every sector of the society. However, if the awareness program is transmitted by media for a very long period of time it gives a negative impact on the society as it develops a curiosity in the young generation to know about its whereabouts and they indulge into the act which sometimes becomes fatal.

A similar situation happened in the case of Blue Whale Game. The government was interested in making the society aware about the challenge that is given through the Blue Whale game. As such the media started the awareness campaign through the print as well as electronic media. But the way in which it was transmitted for a very long period of time again and again, it motivated kids as well as young generation to understand the challenge more closely and in turn it turned to be fatal for many of them.

Since media is one of the foremost way for helping an awareness program to reach to every sector of the society, it becomes their responsibility to telecast the program firstly correctly and secondly to only a reasonable period of time so that it only creates awareness among the society about the positive aspect of the respective awareness program.

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