Applied Mathematics E-Notes, 20(2020), 108-114 © ISSN 1607-2510
Available free at mirror sites of http://www.math.nthu.edu.tw/∼amen/

On The Stability Of Positive Weak Solution For (p, q)-Laplacian Nonlinear System*

Salah Abdelnaby Khafagy†

Received 9 March 2019

Abstract

In this paper, we study the stability and instability of positive weak solution for the (p, q)-Laplacian nonlinear system

$$\left.\begin{array}{ll} -\Delta_p u + \lambda_p |u|^{p-2}u = a(x)f(u)g(v) & \text{in } \Omega, \\ -\Delta_q v + \lambda_q |v|^{q-2}v = b(x)h(u)k(v) & \text{in } \Omega, \\ Bu = 0 = Bv & \text{on } \partial\Omega. \end{array}\right\}$$

where Δ_p with $p > 1$ denotes the p-Laplacian defined by $\Delta_p u \equiv div[|\nabla u|^{p-2}\nabla u]$, λ_p, λ_q are positive parameters, $a(x), b(x) : \Omega \to R$ are continuous functions, $f, g, h, k : [0, \infty) \times [0, \infty) \to R$ are c^1 functions and $\Omega \subset R^n$ is a bounded domain with smooth boundary $Bu = rm(x)u + (1 - r)\frac{\partial u}{\partial n}$ where $r \in [0, 1]$, $m : \partial\Omega \to R^+$ with $m = 1$ when $r = 1$. We provide a simple proof to establish that every positive weak solution for the given system is stable (unstable) under certian conditions.

1 Introduction

Nonlinear boundary value problems with p-Laplacian operator arise in a variety of physical phenomena, such as: reaction-diffusion problems, non-Newtonian fluids, petroleum extraction, flow through porous media, etc. Consequently, the study of such problems and their far reaching generalizations have attracted several mathematicians in recent years.

Many authors are interested in the study of the stability and instability of nonnegative solutions of linear [4] , semilinear (see [10, 19, 20, 23]), semiposiotne (see [7, 8, 22]) and nonlinear (see [1, 3, 7, 18]) systems, due to the great number of applications in reaction-diffusion problems, in fluid mechanics, in Newtonian fluids, glaciology, population dynamics, etc.; see [5, 6, 9, 11, 14, 15, 16] and references therein. Also, in the recent past, many authors devoted their attention to study the singular p-Laplacian nonlinear systems (see [12, 13, 21]).

In this paper we consider the stability and instability of positive weak solution for the (p, q)-Laplacian nonlinear system

$$\left.\begin{array}{ll} -\Delta_p u + \lambda_p |u|^{p-2}u = a(x)f(u)g(v) & \text{in } \Omega, \\ -\Delta_q v + \lambda_q |v|^{q-2}v = b(x)h(u)k(v) & \text{in } \Omega, \\ Bu = 0 = Bv & \text{on } \partial\Omega. \end{array}\right\} \quad (1)$$

where Δ_p with $p > 1$ denotes the p-Laplacian defined by $\Delta_p u \equiv \mathrm{div}[|\nabla u|^{p-2}\nabla u]$, λ_p, λ_q are positive parameters, $a(x), b(x) : \Omega \to R$ are continuous functions satisfying either $a(x), b(x) > 0$ or $a(x), b(x) < 0$ for all $x \in \Omega$, $f, g, h, k : [0, \infty) \times [0, \infty) \to R$ are c^1 functions and $\Omega \subset R^n$ is a bounded domain with smooth boundary $Bu = rm(x)u + (1 - r)\frac{\partial u}{\partial n}$ where $r \in [0, 1]$, $m : \partial\Omega \to R^+$ with $m = 1$ when $r = 1$. We provide a simple proof to establish that every positive solution is stable (unstable) under certain conditions on the functions $a(x)$, $b(x)$, $f(u)$, $g(v)$, $h(u)$ and $k(v)$.

Tertikas in [22] proved the stability and instability results of positive solutions for the semilinear system

$$-\Delta u = \lambda f(u) \text{ in } \Omega,\ Bu = 0 \text{ on } \partial\Omega,$$

*Mathematics Subject Classifications: 34D20,35D30,35J92.

†Department of Mathematics, College of Science in Zulfi, Majmaah University, 11952, Saudi Arabia. E-mail: s.khafagy@mu.edu.sa; Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt.

under various choices of the function f. In [7], the authors studied the uniqueness and stability of nonnegative solutions for classes of nonlinear elliptic Dirichlet problems in a ball, when the nonlinearity is monotone, negative at the origin, and either concave or convex.

Khafagy in [17] studied the stability and instability for the nonlinear system

$$\left.\begin{array}{ll} -\Delta_{P,p}u + a(x)|u|^{p-2}u = \lambda b(x)u^{\alpha} & \text{in } \Omega, \\ Bu = 0 & \text{on } \partial\Omega. \end{array}\right\} \tag{2}$$

where $\Delta_{P,p}$ with $p > 1$ and $P = P(x)$ is a weight function, denotes the weighted p-Laplacian defined by $\Delta_{P,p}u \equiv div[P(x)|\nabla u|^{p-2}\nabla u]$, $a(x)$ is a weight function, the continuous function $b(x) : \Omega \to R$ satisfies either $b(x) > 0$ or $b(x) < 0$ for all $x \in \Omega$, λ is a positive parameter, $0 < \alpha < p-1$ and $\Omega \subset R^N$ is a bounded domain with smooth boundary $Bu = \delta h(x)u + (1-\delta)\frac{\partial u}{\partial n}$ where $\delta \in [0,1]$, $h : \partial\Omega \to R^+$ with $h = 1$ when $\delta = 1$. He proved that if $0 < \alpha < p-1$ and $b(x) > 0 (< 0)$ for all $x \in \Omega$, then every positive weak solution u of (2) is linearly stable (unstable) respectively.

Finally, let us explain the plan of the paper. In section 2, we study the stability and instability of the positive weak solution of (1). In section 3, we introduce some applications regarding the stability properties of the positive weak solution of some special cases of system (1).

We recall that, if (u,v) is any positive weak solution of (1), then the linearized equation about (u,v) is given by

$$\left.\begin{array}{c} -(p-1)[\ \text{div}[|\nabla u|^{p-2}\nabla w] - \lambda_p|u|^{p-2}w] - a(x)f_u(u)g(v)w \\ -a(x)f(u)g_v(v)z = \mu w,\ \text{ in } \Omega, \\ -(q-1)[\ \text{div}[|\nabla v|^{q-2}\nabla z] - \lambda_q|v|^{q-2}z] - b(x)h_u(u)k(v)w \\ -b(x)h(u)k_v(v)z = \mu z,\ \text{ in } \Omega, \\ Bw = 0 = Bz, \qquad \text{on } \partial\Omega, \end{array}\right\} \tag{3}$$

where $f_u\ (u)$ denotes the derivative of $f(u)$ with respect to u, μ is the eigenvalue corresponding to the eigenfunction (ϕ,ψ).

Definition 1 *We call a solution (u,v) of (1) a linearly stable solution if all eigenvalues of (3) are strictly positive (which can be implied if the principal eigenvalue $\mu_1 > 0$). Otherwise (u,v) is linearly unstable.*

2 Main Results

In this section, we assume the following hypotheses

(H_1) $f(u)/u^{p-1}$ is strictly increasing (decreasing), i.e., $uf_u(u) - (p-1)f(u) > 0 (< 0)$.

(H_2) $k(v)/v^{q-1}$ is strictly increasing (decreasing), i.e., $vk_v(v) - (q-1)k(v) > 0 (< 0)$.

$(H3)$ $h(u) > 0$, $\forall u > 0$ and $g(v) > 0\ \forall v > 0$.

(H_4) $f(u)g_v(v)$ and $h_u(u)k(v)$ have the same sign, i.e., $f(u)g_v(v), h_u(u)k(v) > 0\ (< 0)$.

(H_5) $a(x)$ and $b(x)$ have the same sign, i.e., $a(x)$, $b(x) > 0\ (< 0)$.

We shall prove the stability and instability of the positive weak solution (u,v) of (1) under the above conditions. Our main results are the following theorems.

Theorem 1 *If $f(u)/u^{p-1}$ and $k(v)/v^{q-1}$ are strictly increasing, $f(u)g_v(v)$, $h_u(u)k(v) > 0$ and $a(x), b(x) > 0$, then every positive weak solution (u,v) of (1) is linearly unstable.*

Proof. Let (u_0, v_0) be any positive weak solution of (1). Then the linearized equation about (u_0, v_0) is

$$\left.\begin{array}{c} -(p-1)[\ \mathrm{div}[|\nabla u_0|^{p-2}\nabla w] - \lambda_p |u_0|^{p-2} w] - a(x) f_u(u_0) g(v_0) w \\ +a(x) f(u_0) g_v(v_0) z = \mu w, \text{in } \Omega, \\ -(q-1)[\ \mathrm{div}[|\nabla v_0|^{q-2}\nabla z] - \lambda_q |v_0|^{q-2} z] - b(x) h_u(u_0) k(v_0) w \\ +b(x) h(u_0) k_v(v_0) z = \mu z, \text{in } \Omega, \\ Bw = 0 = Bz, \qquad \text{on } \partial\Omega. \end{array}\right\} \tag{4}$$

Let μ_1 be the first eigenvalue of (4) and let (ϕ, ψ), $(\phi, \psi \geq 0)$ be the corresponding eigenfunction. Multiplying the first equation of (1) by $(p-1)\phi$ and integrating over Ω, we have

$$(p-1)[\int_\Omega -\ \mathrm{div}[|\nabla u_0|^{p-2}\nabla u_0] + \lambda_p \int_\Omega |u_0|^{p-2} u_0 - \int_\Omega a(x) f(u_0) g(v_0)]\phi dx = 0. \tag{5}$$

Also, multiplying the second equation of (1) by $(q-1)\psi$ and integrating over Ω, we have

$$(q-1)[\int_\Omega -\ \mathrm{div}[|\nabla v_0|^{q-2}\nabla v_0] + \lambda_q \int_\Omega |v_0|^{q-2} v_0 - \lambda \int_\Omega b(x) h(u_0) k(v_0)]\psi dx = 0. \tag{6}$$

On the other hand, multiplying the first equation of (4) by u_0 and integrating over Ω, we have

$$\begin{aligned} \mu_1 \int_\Omega u_0 \phi dx &= -(p-1)[\int_\Omega \ \mathrm{div}[|\nabla u_0|^{p-2}\nabla \phi] u_0 - \lambda_p \int_\Omega |u_0|^{p-2} u_0 \phi] dx \\ &\quad - \int_\Omega b(x) h_u(u_0) k(v_0) v_0 \phi dx - \int_\Omega b(x) h(u_0) k_v(v_0) v_0 \psi dx. \end{aligned} \tag{7}$$

Also, multiplying the second equation of (4) by v_0 and integrating over Ω, we have

$$\begin{aligned} \mu_1 \int_\Omega v_0 \psi dx &= -(q-1)[\int_\Omega \ \mathrm{div}[|\nabla v_0|^{q-2}\nabla \psi] v_0 - \lambda_q \int_\Omega |v_0|^{q-2} v_0 \psi] dx \\ &\quad - \int_\Omega b(x) h_u(u_0) k(v_0) v_0 \phi dx - \int_\Omega b(x) h(u_0) k_v(v_0) v_0 \psi dx. \end{aligned} \tag{8}$$

Now, by combining (5) and (7), we have

$$\begin{aligned} -\mu_1 \int_\Omega u_0 \phi dx &= (p-1)[\int_\Omega \ \mathrm{div}[|\nabla u_0|^{p-2}\nabla \phi] u_0 - \int_\Omega \ \mathrm{div}[|\nabla u_0|^{p-2}\nabla u_0]\phi] dx \\ &\quad + \int_\Omega a(x)[u_0 f_u(u_0) - (p-1) f(u_0)] g(v_0) \phi dx \\ &\quad + \int_\Omega a(x) u_0 f(u_0) g_v(v_0) \psi dx. \end{aligned} \tag{9}$$

Applying Green's first identity on the first term of the R.H.S of (9), we have

$$\int_\Omega [\ \mathrm{div}[|\nabla u_0|^{p-2}\nabla \phi] u_0 - \ \mathrm{div}[|\nabla u_0|^{p-2}\nabla u_0]\phi] dx = \int_{\partial\Omega} |\nabla u_0|^{p-2}[u_0 \frac{\partial \phi}{\partial n} - \phi \frac{\partial u_0}{\partial n}] ds. \tag{10}$$

From (10) in (9), we have

$$\begin{aligned}
-\mu_1\int\limits_\Omega u_0\phi dx &= (p-1)\int\limits_{\partial\Omega}|\nabla u_0|^{p-2}[u_0\frac{\partial\phi}{\partial n}-\phi\frac{\partial u_0}{\partial n}]ds\\
&+\int\limits_\Omega a(x)[u_0f_u(u_0)-(p-1)f(u_0)]g(v_0)\phi dx\\
&+\int\limits_\Omega a(x)u_0f(u_0)g_v(v_0)\psi dx. \qquad (11)
\end{aligned}$$

Also, (6) and (8) give

$$\begin{aligned}
-\mu_1\int\limits_\Omega v_0\psi dx &= (q-1)[\int\limits_\Omega \operatorname{div}[|\nabla v_0|^{q-2}\nabla\psi]v_0dx-\int\limits_\Omega \operatorname{div}[|\nabla v_0|^{q-2}\nabla v_0]\psi dx]\\
&+\int\limits_\Omega b(x)[v_0k_v(v_0)-(q-1)k(v_0)]h(u_0)\psi dx\\
&+\int\limits_\Omega b(x)v_0h_u(u_0)k(v_0)\phi dx. \qquad (12)
\end{aligned}$$

Applying Green's first identity on the first term of the R.H.S. of (12), we have

$$\int\limits_\Omega[\,div[|\nabla v_0|^{q-2}\nabla\psi]v_0-\operatorname{div}[|\nabla v_0|^{q-2}\nabla v_0]\psi]dx=\int\limits_{\partial\Omega}|\nabla v_0|^{q-2}[v_0\frac{\partial\psi}{\partial n}-\psi\frac{\partial v_0}{\partial n}]ds. \qquad (13)$$

From (13) in (12), we have

$$\begin{aligned}
-\mu_1\int\limits_\Omega v_0\psi dx &= (q-1)\int\limits_{\partial\Omega}|\nabla v_0|^{q-2}[v_0\frac{\partial\psi}{\partial n}-\psi\frac{\partial v_0}{\partial n}]ds\\
&+\int\limits_\Omega b(x)[v_0k_v(v_0)-(q-1)k(v_0)]h(u_0)\psi dx\\
&+\int\limits_\Omega b(x)v_0h_u(u_0)k(v_0)\phi dx. \qquad (14)
\end{aligned}$$

Adding (11) and (14), we have

$$\begin{aligned}
-\mu_1\int\limits_\Omega[u_0\phi+v_0\psi]dx &= (p-1)\int\limits_{\partial\Omega}|\nabla u_0|^{p-2}[u_0\frac{\partial\phi}{\partial n}-\phi\frac{\partial u_0}{\partial n}]ds\\
&+(q-1)\int\limits_{\partial\Omega}|\nabla v_0|^{q-2}[v_0\frac{\partial\psi}{\partial n}-\psi\frac{\partial v_0}{\partial n}]ds\\
&+\int\limits_\Omega a(x)[u_0f_u(u_0)-(p-1)f(u_0)]g(v_0)\phi dx\\
&+\int\limits_\Omega b(x)[v_0k_v(v_0)-(q-1)k(v_0)]h(u_0)\psi dx\\
&+\int\limits_\Omega a(x)u_0f(u_0)g_v(v_0)\psi dx\\
&+\int\limits_\Omega b(x)v_0h_u(u_0)k(v_0)\phi dx. \qquad (15)
\end{aligned}$$

Now, when $r=1$, we have $Bu_0 = u_0 = 0$ and $Bv_0 = v_0 = 0$ for $s \in \partial\Omega$ and also we have $\phi = \psi = 0$ for $s \in \partial\Omega$. Then

$$\int_{\partial\Omega} |\nabla u_0|^{p-2}[u_0 \frac{\partial \phi(s)}{\partial n} - \phi \frac{\partial u_0(s)}{\partial n}]ds = \int_{\partial\Omega} |\nabla v_0|^{q-2}[v_0 \frac{\partial \psi(s)}{\partial n} - \psi \frac{\partial v_0(s)}{\partial n}]ds = 0 \quad (16)$$

Also, when $r \neq 1$, we have

$$\frac{\partial u_0}{\partial n} = -\frac{rmu_0}{1-r}, \quad \frac{\partial \phi}{\partial n} = -\frac{rm\phi}{1-r},$$

and

$$\frac{\partial v_0}{\partial n} = -\frac{rmv_0}{1-r}, \quad \frac{\partial \psi}{\partial n} = -\frac{rm\psi}{1-r},$$

which implies again the result given by (16). Hence (15) becomes

$$\begin{aligned} -\mu_1 \int_\Omega [u_0\phi + v_0\psi]dx &= \int_\Omega a(x)[u_0 f_u(u_0) - (p-1)f(u_0)]g(v_0)\phi dx \\ &+ \int_\Omega b(x)[v_0 k_v(v_0) - (q-1)k(v_0)]h(u_0)\psi dx \\ &+ \int_\Omega a(x)u_0 f(u_0) g_v(v_0)\psi dx \\ &+ \int_\Omega b(x) v_0 h_u(u_0) k(v_0)\phi dx. \end{aligned} \quad (17)$$

Since $f(u)/u^{p-1}$ and $k(v)/v^{q-1}$are strictly increasing, we have from C_1 that

$$[u_0 f_u(u_0) - (p-1)f(u_0)] > 0 \ \text{ and } \ [v_0 k_v(v_0) - (q-1)k(v_0)] > 0. \quad (18)$$

Using equation (18), hypothesis H_3, the fact that $f(u)g_v(v)$, $h_u(u)k(v) > 0$, $a(x) > 0$ and $b(x) > 0$ for all $x \in \Omega$, (17) becomes

$$-\mu_1 \int_\Omega [u_0\phi + v_0\psi]dx > 0.$$

So $\mu_1 < 0$ and the result follows. This completes the proof. ■

Theorem 2 *If $f(u)/u^{p-1}$ and $k(v)/v^{q-1}$are strictly increasing, $f(u)g_v(v)$, $h_u(u)k(v) > 0$, and $a(x), b(x) < 0$ for all $x \in \Omega$, then every positive weak solution (u, v) of (1) is linearly stable.*

Proof. The proof is similar to that of Theorem 1. We obtain

$$-\mu_1 \int_\Omega [u_0\phi + v_0\psi]dx > 0,$$

and so $\mu_1 < 0$ and the result follows. ■

Theorem 3 *If $f(u)/u^{p-1}$ and $k(v)/v^{q-1}$are strictly decreasing, $f(u)g_v(v), h_u(u)k(v) < 0$, and $a(x), b(x) > 0$ for all $x \in \Omega$, then every positive weak solution (u, v) of (1) is linearly stable.*

Proof. The proof proceeds in the same way as for previous Theorems and we can easily obtain that

$$-\mu_1 \int_\Omega [u_0\phi + v_0\psi]dx < 0.$$

Then $\mu_1 > 0$ and the result follows. ■

Remark 1 *In (1), when $f(u) = u^\beta$, $g(v) = v^\gamma$, $h(u) = u^r$, $k(v) = v^\delta$, $a(x) = b(x) = \lambda$ where $\lambda, \beta, \gamma, \delta, r$ are positive constants, $\beta > p-1$ and $\delta > q-1$, we have some results in [2].*

3 Applications

Here we introduce some examples.

Example 1 *Consider the Reaction-Diffusion system with unequal diffusion coefficients involving the Laplacian*

$$\left.\begin{array}{ll} -\Delta u = \lambda f(u)g(v) & in\ \Omega, \\ -\Delta v = \mu h(u)k(v) & in\ \Omega, \\ Bu = 0 = Bv & on\ \ \partial\Omega. \end{array}\right\} \tag{19}$$

where λ, μ *are positive parameters,* f, k *are strictly increasing (decreasing) functions,* $h(u) > 0$, $\forall u > 0$, $g(v) > 0$ $\forall v > 0$ *and* $f(u)g_v(v), h_u(u)k(v) > 0(< 0)$ *. Hence according to Theorems 1 and 3 in the case* $p = q = 2$ *and* $\lambda_p = \lambda_q = 0$*, any positive weak solution* (u, v) *of (19) is linearly unstable (stable) respectively.*

Example 2 *Consider the Reaction-Diffusion system with unequal diffusion coefficients involving the* (p, q)*-Laplacian*

$$\left.\begin{array}{ll} -\Delta_p u = \lambda\ u^\alpha v^\beta & in\ \Omega, \\ -\Delta_q v = \mu\ u^\gamma v^\delta & in\ \Omega, \\ Bu = 0 = Bv & on\ \ \partial\Omega. \end{array}\right\} \tag{20}$$

where $\lambda, \mu, \alpha, \beta, \gamma, \delta$ *are positive constants,* $\alpha > p - 1$ *and* $\delta > q - 1$*. Hence according to Theorem 1, any positive weak solution* (u, v) *of (20) is linearly unstable.*

Acknowledgment. The author would like to thank the Deanship of Scientific Research at Majmaah University for supporting this work under Project Number No. 134-1440.

References

[1] G. Afrouzi and S. Rasouli, Stability properties of non-negative solutions to a non-autonomous p-Laplacian equation, Chaos Solitons and Fractals, 29(2006), 1095–1099.

[2] G. Afrouzi and S. Rasouli, On critical exponent for instability of positive solutions to reactin-diffusion system involving the (p, q)-Laplacian, Int. J. Nonlinear Sci., 2(2006), 61–64.

[3] G. Afrouzi and Z. Sadeeghi, On the stability of nonnegative solutions to classes of p-Laplacian systems, World Journal of Modelling and Simulation, 4(2008), 149–152.

[4] G. Afrouzi and Z. Sadeeghi, Stability results for a class of elliptic problems, Int. J. Nonlinear Sci., 6(2008), 114–117.

[5] K. Ali, and A. Ghanmi, Nehari manifold and multiplicity result for elliptic equation involving p-Laplacian problems, Bol. Soc. Paran. Mat., 36(2018), 197–208.

[6] C. Atkinson and K. Ali, Some boundary value problems for the Bingham model, J. Non-Newton. Fluid Mech., 41(1992), 339–363.

[7] I. Ali, A. Castro and R. Shivaji, Uniqueness and stability of nonnegative solutions for semipositone problems in a ball, Proc. Amer. Math. Soc., 117(1993), 775–782.

[8] K. Brown and R. Shivaji, Instability of nonnegative solutions for a class of semipositone problems. Proc. Amer. Math. Soc., 112(1991), 121–124.

[9] A. Ghanmi, Multiplicity of nontrivial solutions of a class of fractional p-Laplacian problem, Zeitschrift fur Analysis und ihre Anwendung, 34(2015), 309–319.

[10] L. Karatson and P. Simon, On the stability properties of nonnegative solutions of semilinear problems with convex or concave nonlinearity, J. Comput. Appl. Math., 131(2001), 497–501.

[11] S. Khafagy, Existence results for (p, q)-Laplacian nonlinear system, Appl. Math. E-Notes, 17(2017), 242–250.

[12] S. Khafagy, Maximum principle and existence of weak solutions for nonlinear system involving singular p-Laplacian operators, J. Partial Differ. Equ., 29(2016), 89–101.

[13] S. Khafagy, On positive weak solutions for nonlinear elliptic system involving singular p-Laplacian operator, J. Math. Anal., 7(2016), 10–17.

[14] S. Khafagy, On positive weak solutions for a class of nonlinear system, Ital. J. Pure Appl. Math., 40(2018), 149–156.

[15] S. Khafagy, On positive weak solutions for a class of weighted (p, q)-Laplacian nonlinear system, Rom. J. Math. Comput. Sci., 7(2017), 86–92.

[16] S. Khafagy, On positive weak solutions for a nonlinear system involving weighted (p, q)-Laplacian operators, J. Math. Anal., 9(2018), 86–96.

[17] S. Khafagy, On the stabiblity of positive weak solution for weighted p-Laplacian nonlinear system, New Zealand J. Math., 45(2015), 39–43.

[18] S. Khafagy and H. Serag, Stability results of positive weak solution for singular p-Laplacian nonlinear system, J. Appl. Math. Inform., 36(2018), 173–179.

[19] P. Korman and J. Shi, Instability and exact multiplicity of solutions of semilinear equations, Electron. J. Differ. Equ. Conf., 5(2000), 311–322.

[20] C. Maya and R. Shivaji, Instability of nonnegative solutions for a class of semilinear elliptic boundary value problems, J. Comput. Appl. Math., 88(1998), 125–128.

[21] S. Rasouli, On the Existence of positive solutions for a class of nonlinear elliptic system with multiple parameters and singular weights, Commun. Korean Math. Soc. 27 (3) (2012), 557–564.

[22] A. Tertikas, Stability and instability of positive solutions of semilinear problems. Proc. Amer. Math. Soc., 114(1992), 1035–1040.

[23] I. Voros, Stability properties of non-negative solutions of semilinear symmetric cooperative systems, Electron. J. Differential Equations 2004, No. 105, 6 pp.