Applied Mathematics E-Notes, 20(2020), 70-78 © ISSN 1607-2510
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Some Generalized Fixed-Point Theorems on Complex Valued S-Metric Spaces*

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Received 27 February 2019

Abstract

In this paper, we define new contractive conditions on a complex valued S-metric space. These contractive conditions generalize the classical Rhoades' contractive condition, Nemytskii-Edelstein contractive condition and Ćirić's contractive condition. Also we prove some fixed-point theorems using these contractive conditions on a complex valued S-metric space.

1 Introduction and Mathematical Preliminaries

It is a very famous problem studying the existence and uniqueness fixed-point theorems for a self-mapping on various metric spaces. Recently, new generalized metric spaces such as S-metric, G-metric, b-metric spaces have been presented and some fixed-point theorems have been proved for self-mappings on these generalized metric spaces (for example, see [1, 2, 4, 8, 9, 10, 12, 13, 14, 15, 16, 17, 19]). In 2012, Sedghi et al. defined the notion of an S-metric space and proved some fixed-point theorems such as the Banach's contraction principle and the Nemytskii-Edelstein fixed-point theorem on an S-metric space [15]. In 2014, Sedghi and Dung proved new generalized fixed-point theorems such as the Ćirić's fixed-point result on an S-metric space [16]. The present authors obtained the generalizations of the Banach's contraction principle and the Rhoades' condition on an S-metric space (see [12, 13] for more details).

In 2011, Azam et al. introduced the notion of a complex valued metric space [3]. In 2013, Verma and Pathak defined the concept of property $(E.A)$ on a complex valued metric space to obtain some common fixed-point results for two pairs of weakly compatible mappings, satisfying a contractive condition "max" type [18]. More recent studies in this context can be found in [5, 6]. In 2014, Mlaiki presented the notion of a complex valued S-metric space as a generalization of a complex valued metric space [7]. Also the present authors proved new fixed-point theorems on a complex valued S-metric space (see [11] for more details).

At first, we recall some known definitions and lemmas before stating our aims. Let $\mathbb{C}$ be the set of complex numbers and z_1, $z_2 \in \mathbb{C}$. The partial order $\precsim$ is defined on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leq Re(z_2),\ Im(z_1) \leq Im(z_2)$$

and

$$z_1 \prec z_2 \text{ if and only if } Re(z_1) < Re(z_2),\ Im(z_1) < Im(z_2).$$

Also we write $z_1 \precnsim z_2$ if one of the following conditions holds:

1. $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$,
2. $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$,
3. $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$.

*Mathematics Subject Classifications: 47H10, 54H25.

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Note that

$$0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|$$

and

$$z_1 \precsim z_2,\ z_2 \prec z_3 \Rightarrow z_1 \prec z_3.$$

Definition 1 **([7])** *Let X be a nonempty set. A complex valued S-metric on X is a function $S : X \times X \times X \to \mathbb{C}$ that satisfies the following conditions for all $z, w, q, t \in X$:*

($\mathcal{C}$**S1**) $0 \precsim S(z, w, q)$,

($\mathcal{C}$**S2**) $S(z, w, q) = 0$ *if and only if* $z = w = q$,

($\mathcal{C}$**S3**) $S(z, w, q) \precsim S(z, z, t) + S(w, w, t) + S(q, q, t)$.

The pair (X, S) is called a complex valued S-metric space.

Definition 2 **([7])** *Let (X, S) be a complex valued S-metric space.*

1. *A sequence $\{z_n\}$ in X converges to z if and only if for all ε such that $0 \prec \varepsilon \in \mathbb{C}$ there exists a natural number n_0 such that for all $n \geq n_0$, we have $S(z_n, z_n, z) \prec \varepsilon$ and it is denoted by $\lim_{n\to\infty} z_n = z$.*

2. *A sequence $\{z_n\}$ in X is called a Cauchy sequence if for all ε such that $0 \prec \varepsilon \in \mathbb{C}$ there exists a natural number n_0 such that for all $n, m \geq n_0$, we have $S(z_n, z_n, z_m) \prec \varepsilon$.*

3. *A complex valued S-metric space (X, S) is called complete if every Cauchy sequence is convergent.*

Lemma 1 **([7])** *Let (X, S) be a complex valued S-metric space and $\{z_n\}$ a sequence in X. Then $\{z_n\}$ converges to z if and only if $|S(z_n, z_n, z)| \to 0$ as $n \to \infty$.*

Lemma 2 **([7])** *Let (X, S) be a complex valued S-metric space and $\{z_n\}$ a sequence in X. Then $\{z_n\}$ is a Cauchy sequence if and only if $|S(z_n, z_n, z_{n+m})| \to 0$ as $n \to \infty$.*

Lemma 3 **([7])** *If (X, S) be a complex valued S-metric space, then $S(z, z, w) = S(w, w, z)$ for all $z, w \in X$.*

Definition 3 **([18])** *The "max" function is defined for the partial order relation $\precsim$ as follow:*

1. $\max\{z_1, z_2\} = z_2 \Leftrightarrow z_1 \precsim z_2$.

2. $z_1 \precsim \max\{z_2, z_3\} \Rightarrow z_1 \precsim z_2$ *or* $z_1 \precsim z_3$.

3. $\max\{z_1, z_2\} = z_2 \Leftrightarrow z_1 \precsim z_2$ *or* $|z_1| < |z_2|$.

Lemma 4 **([18])** *Let $z_1, z_2, z_3, \ldots \in \mathbb{C}$ and the partial order relation $\precsim$ be defined on $\mathbb{C}$. Then the following statements are satisfied:*

1. *If* $z_1 \precsim \max\{z_2, z_3\}$ *then* $z_1 \precsim z_2$ *if* $z_3 \precsim z_2$,

2. *If* $z_1 \precsim \max\{z_2, z_3, z_4\}$ *then* $z_1 \precsim z_2$ *if* $\max\{z_3, z_4\} \precsim z_2$,

3. *If* $z_1 \precsim \max\{z_2, z_3, z_4, z_5\}$ *then* $z_1 \precsim z_2$ *if* $\max\{z_3, z_4, z_5\} \precsim z_2$, *and so on.*

Motivated by the above studies, we define some new contractive conditions on a complex valued S-metric space. These contractive conditions generalize the classical Rhoades' contractive condition, Nemytskii-Edelstein contractive condition and Ćirić's contractive condition on a complex valued S-metric space. We investigate the relationships among these contractive conditions with counterexamples. Also we prove some fixed-point theorems as generalizations of the classical fixed-point theorems (for example, Nemytskii-Edelstein fixed-point theorem and Ćirić's fixed-point result) on a complex valued S-metric space.

2 Some Fixed-Point Results on Complex Valued S-Metric Spaces

At first, we define the Rhoades' condition on a complex valued S-metric space.

Definition 4 *Let (X,S) be a complex valued S-metric space and T a self-mapping of X. We define*

$$S(Tz,Tz,Tw) \prec \max\left\{S(z,z,w), S(Tz,Tz,z), S(Tw,Tw,w), S(Tw,Tw,z), S(Tz,Tz,w)\right\}, \tag{1}$$

for all $z,w \in X$ with $z \neq w$.

Now we introduce the notion of diameter on a complex valued S-metric space and present a generalization of the condition (1).

Definition 5 *Let (X,S) be a complex valued S-metric space and A a nonempty subset of X. Then we define*

$$diam\left\{A\right\} = \sup\left\{|S(z,z,w)| : z,w \in A\right\},$$

which is called the diameter of A. If A is a bounded set, then we will write $diam\{A\} < \infty$.

Definition 6 *Let (X,S) be a complex valued S-metric space, T a self-mapping of X and $U_z = \{T^n z : n \in \mathbb{N}\}$, $diam\{U_z\} < \infty$ and $diam\{U_w\} < \infty$. We define*

$$|S(Tz,Tz,Tw)| < diam\left\{U_z \cup U_w\right\}, \tag{2}$$

for all $z,w \in X$ with $z \neq w$.

In the following proposition, we give the relationship between the conditions (1) and (2).

Proposition 1 *Let (X,S) be a complex valued S-metric space and T a self-mapping of X. If T satisfies the condition (1), then T satisfies the condition (2).*

Proof. Suppose that the condition (1) is satisfied by T. Then we get

$$S(Tz,Tz,Tw) \prec \max\left\{S(z,z,w), S(Tz,Tz,z), S(Tw,Tw,w), S(Tw,Tw,z), S(Tz,Tz,w)\right\} = \alpha$$

and so we obtain

$$|S(Tz,Tz,Tw)| < |\alpha| < diam\left\{U_z \cup U_w\right\}.$$

Hence the condition (2) is satisfied. ■

In the following example, we see that the converse of Proposition 1 is not always true.

Example 1 *Let $X = (0,1)$ with the complex valued S-metric defined as*

$$S(z,w,q) = 5e^{ik}\left(|z-q| + |z+q-2w|\right) \quad \left(k \in \left[0, \frac{\pi}{2}\right]\right),$$

for all $z,w,q \in X$. Let us define the function $T : X \to X$ as

$$Tz = \begin{cases} z & \text{if } z \in (0,1),\ z \neq \frac{1}{2},\ z \neq \frac{1}{3}, \\ \frac{1}{3} & \text{if } z = \frac{1}{2}, \\ \frac{1}{2} & \text{if } z = \frac{1}{3}, \end{cases}$$

for all $z \in X$. Then T is a self-mapping on the complex valued S-metric space (X,S). For $z = \frac{1}{4}, w = \frac{1}{5} \in X$ we have

$$S(Tz,Tz,Tw) = \frac{e^{ik}}{2},\quad S(Tw,Tw,w) = 0,\quad S(z,z,w) = \frac{e^{ik}}{2},$$

$$S(Tw,Tw,z)=\frac{e^{ik}}{2},\ \ S(Tz,Tz,z)=0,\ \ S(Tz,Tz,w)=\frac{e^{ik}}{2}$$

and so we get

$$S(Tz,Tz,Tw)=\frac{e^{ik}}{2}\prec\max\left\{\frac{e^{ik}}{2},0,0,\frac{e^{ik}}{2},\frac{e^{ik}}{2}\right\},$$

which implies

$$|S(Tz,Tz,Tw)|=\frac{1}{2}<\left|\frac{e^{ik}}{2}\right|=\frac{1}{2}.$$

Therefore T does not satisfy the condition (1). It can be easily seen that T satisfies the condition (2) since $\sup(0,1)=1$.

We call the complex valued S-metric space X as compact if every sequence in X has a convergent subsequence.

Let (X,S) and (Y,S^*) be two complex valued S-metric spaces and $T:X\to Y$ be a function. Then T is continuous at $x\in X$ if and only if $Tx_n\to Tx$ whenever $x_n\to x$. In the following theorem, we obtain a fixed point theorem for a self-mapping satisfying the condition (2) on a compact complex valued S-metric space.

Theorem 1 *Let (X,S) be a compact complex valued S-metric space and T a continuous self-mapping of X satisfying the condition (2). Then T has a unique fixed point.*

Proof. There exists a compact subset Y of X such that $TX\subset Y$ since T is a continuous self-mapping and X is compact. Hence we get $TY\subset Y$ and $Z=\bigcap\limits_{n=1}^{\infty}T^nY$ is a nonempty compact subset of X. We show that Z is a singleton consisting of the unique fixed point z_0 of T. Suppose that Z is not a singleton. Then we get $diam\{Z\}>0$. Since Z is compact subset, there exist $z,w\in Z$ with $|S(z,z,w)|=diam\{Z\}$. Also there exist $z_0,w_0\in Z$ with $Tz_0=z$ and $Tw_0=w$ since T maps Z onto itself. From the condition (2), we obtain

$$diam\{Z\}=|S(z,z,w)|=|S(Tz_0,Tz_0,Tw_0)|<diam\{Z\},$$

which is a contradiction. Therefore, T has a unique fixed point. ■

By Proposition 1, we deduce the following corollary.

Corollary 1 *Let (X,S) be a compact complex valued S-metric space and T a continuous self-mapping of X satisfying the condition (1). Then T has a unique fixed point.*

In the following proposition, we see that a complex valued S-metric function is continuous.

Proposition 2 *Let (X,S) be a complex valued S-metric space and $\{z_n\}$, $\{w_n\}$ be two sequences. If $\{z_n\}\to z$ and $\{w_n\}\to w$, then $S(z_n,z_n,w_n)\to S(z,z,w)$.*

Proof. Assume that $\{z_n\}\to z$ and $\{w_n\}\to w$. Then there exist $n_1,n_2\in\mathbb{N}$ such that

$$S(z_n,z_n,z)\prec\frac{\varepsilon}{4}\text{ for each }n\geq n_1$$

and

$$S(w_n,w_n,w)\prec\frac{\varepsilon}{4}\text{ for each }n\geq n_2.$$

If we take $n_0=\max\{n_1,n_2\}$ then using the condition $(\mathcal{C}S3)$ and Lemma 3, we get

$$S(z_n,z_n,w_n)\precsim 2S(z_n,z_n,z)+2S(w_n,w_n,w)+S(z,z,w)\prec\varepsilon+S(z,z,w)$$

and so

$$S(z_n, z_n, w_n) - S(z, z, w) \prec \varepsilon. \tag{3}$$

Also we have

$$\begin{aligned} S(z, z, w) &\precsim 2S(z, z, z_n) + 2S(w, w, w_n) + S(z_n, z_n, w_n) \\ &\prec \varepsilon + S(z_n, z_n, w_n) \end{aligned}$$

and so

$$S(z, z, w) - S(z_n, z_n, w_n) \prec \varepsilon. \tag{4}$$

From the inequalities (3) and (4), we obtain

$$|S(z_n, z_n, w_n) - S(z, z, w)| < \varepsilon,$$

that is, $S(z_n, z_n, w_n) \to S(z, z, w)$. Consequently, the complex valued S-metric function is continuous. ∎

Now we introduce the Nemytskii-Edelstein condition on a complex valued S-metric space.

Definition 7 *Let (X, S) be a complex valued S-metric space and T be a self-mapping of X. We define*

$$S(Tz, Tz, Tw) \prec S(z, z, w), \tag{5}$$

for all $z, w \in X$ with $z \neq w$.

In the following proposition, we give the relationship between the condition (1) and the condition (5).

Proposition 3 *Let (X, S) be a complex valued S-metric space and T a self-mapping of X. If T satisfies the condition (5), then T satisfies the condition (1).*

Proof. The proof can be easily seen from Definitions 4 and 7. ∎

Using Propositions 1 and 3, we deduce the following corollary.

Corollary 2 *Let (X, S) be a complex valued S-metric space and T a self-mapping of X. If T satisfies the condition (5), then T satisfies the condition (2).*

In the following example, we see that the converses of Proposition 3 and Corollary 2 are not always true.

Example 2 *Let $X = [0, 1]$ with complex valued S-metric given in Example 1. Let us define the function $T : X \to X$ as*

$$Tz = \begin{cases} z + \frac{4}{5} & \text{if } z \in \left[0, \frac{1}{5}\right), \\ 1 & \text{if } z \in \left[\frac{1}{5}, 1\right], \end{cases}$$

for all $z \in X$. Then T is a self-mapping on the complex valued S-metric space (X, S). For $z = \frac{1}{6}, w = \frac{1}{7} \in X$ we have

$$S(Tz, Tz, Tw) = \frac{5}{21}e^{ik}, \quad S(z, z, w) = \frac{5}{21}e^{ik}$$

and so we get

$$S(Tz, Tz, Tw) = \frac{5}{21}e^{ik} \prec S(z, z, w) = \frac{5}{21}e^{ik},$$

which implies

$$|S(Tz, Tz, Tw)| = \frac{5}{21} < |S(z, z, w)| = \frac{5}{21}.$$

Therefore T does not satisfy the condition (5). It can be easily seen that T satisfies the conditions (1) and (2).

We prove the classical Nemytskii-Edelstein fixed-point theorem on a compact complex valued S-metric space.

Theorem 2 *Let (X, S) be a compact complex valued S-metric space and T a self-mapping of X satisfying the condition (5). Then T has a unique fixed point.*

Proof. Let us define the function $\psi : X \to [0, 1)$ as

$$\psi(z) = |S(z, z, Tz)| .$$

The function ψ takes on its minimum value since (X, S) is a compact complex valued S-metric space. That is, there exists $z_0 \in X$ such that

$$|S(z_0, z_0, Tz_0)| < |S(z, z, Tz)| ,$$

for all $z \in X$. Now we prove that z_0 is a fixed point of T. Suppose that z_0 is not fixed point of T, that is, $Tz_0 \neq z_0$. Using the condition (5), we get

$$S(Tz_0, Tz_0, TTz_0) \prec S(z_0, z_0, Tz_0)$$

and so

$$|S(Tz_0, Tz_0, TTz_0)| < |S(z_0, z_0, Tz_0)| ,$$

which contradicts the minimality of $|S(z_0, z_0, Tz_0)|$ among all $|S(z, z, Tz)|$. Therefore, z_0 is a fixed point of T. We now show that the fixed point z_0 is unique. Assume that w_0 is another fixed point of T, that is, $Tw_0 = w_0$ and $z_0 \neq w_0$. Using the condition (5), we obtain

$$S(z_0, z_0, w_0) = S(Tz_0, Tz_0, Tw_0) \prec S(z_0, z_0, w_0)$$

and so

$$|S(z_0, z_0, w_0)| < |S(z_0, z_0, w_0)| ,$$

which implies $z_0 = w_0$. Consequently, z_0 is a unique fixed point of T. ■

Remark 1 *We can deduce the following results for a continuous self-mapping on a compact complex valued S-metric space.*

1. *Corollary 1 is a generalization of Theorem 2.*
2. *Theorem 1 is another generalization of Theorem 2 by Proposition 1.*
3. *If we consider Example 2 then T has a unique fixed point $z = 1$ since the conditions (1) and (2) are satisfied.*
4. *If we take the metric function as $S : X \times X \times X \to [0, \infty)$ in Theorem 2 then we get Theorem 3.3 given in [15].*

Finally we introduce the Ćirić's condition on a complex valued S-metric space.

Definition 8 *Let (X, S) be a complex valued S-metric space and T a self-mapping of X. We define*

$$S(Tz, Tz, Tw) \precsim h \max \{S(z, z, w), S(Tz, Tz, z), S(Tw, Tw, w), S(Tw, Tw, z), S(Tz, Tz, w)\} , \tag{6}$$

for all $z, w \in X$ and some $h \in \left[0, \frac{1}{3}\right)$.

In the following proposition, we give the relationship between the condition (1) and (6).

Proposition 4 *Let (X, S) be a complex valued S-metric space and T a self-mapping of X. If T satisfies the condition (6), then T satisfies the condition (1).*

Proof. The proof can be easily seen from Definitions 4 and 8. ■

Using Propositions 1 and 4, we deduce the following corollary.

Corollary 3 *Let (X, S) be a complex valued S-metric space and T a self-mapping of X. If T satisfies the condition (6), then T satisfies the condition (2).*

We note that the self-mapping T defined in Example 2 satisfies the conditions (1) and (2) but does not satisfy the condition (6).

We prove the Ćirić's fixed-point result on a complete complex valued S-metric space.

Theorem 3 *Let (X, S) be a complete complex valued S-metric space and T a self-mapping of X satisfying the condition (6). Then T has a unique fixed point.*

Proof. Let $z_0 \in X$ and the sequence $\{z_n\}$ be defined as follows:

$$Tz_n = z_{n+1},\ n = 0, 1, 2, \ldots.$$

Assume that $z_n \neq z_{n+1}$ for all n. By the condition (6) and Lemma 3, we get

$$\begin{aligned} & S(z_n, z_n, z_{n+1}) \\ = \ & S(Tz_{n-1}, Tz_{n-1}, Tz_n) \\ \precsim \ & h \max \{S(z_{n-1}, z_{n-1}, z_n), S(z_n, z_n, z_{n-1}), S(z_{n+1}, z_{n+1}, z_n), S(z_{n+1}, z_{n+1}, z_{n-1}), S(z_n, z_n, z_n)\} \\ = \ & h \max \{S(z_{n-1}, z_{n-1}, z_n), S(z_{n+1}, z_{n+1}, z_n), S(z_{n+1}, z_{n+1}, z_{n-1})\} \\ = \ & h\alpha \end{aligned}$$

and so

$$|S(z_n, z_n, z_{n+1})| \leq h\,|\alpha| \leq 2h\,|S(z_{n+1}, z_{n+1}, z_n)| + h\,|S(z_{n-1}, z_{n-1}, z_n)|,$$

which implies

$$|S(z_n, z_n, z_{n+1})| \leq \frac{h}{1-2h}\,|S(z_{n-1}, z_{n-1}, z_n)|. \tag{7}$$

Let $a = \frac{h}{1-2h}$. Then we have $a < 1$ since $3h < 1$. We note that $1 - 2h \neq 0$ since $0 \leq h < \frac{1}{3}$. Using mathematical induction and the inequality (7), we obtain

$$|S(z_n, z_n, z_{n+1})| \leq a^n\,|S(z_0, z_0, z_1)|. \tag{8}$$

We now prove that the sequence $\{z_n\}$ is Cauchy. For all $n, m \in \mathbb{N}$, $n < m$, using the inequality (8) and the condition $(\mathcal{C}S3)$, we get

$$|S(z_n, z_n, z_m)| \leq \frac{a^n}{1-a}\,|S(z_0, z_0, z_1)|.$$

Hence $|S(z_n, z_n, z_m)| \to 0$ as $n, m \to \infty$. Therefore $\{z_n\}$ is a Cauchy sequence. Using the completeness hypothesis, there exists $z \in X$ such that $\{z_n\} \to z$.

Now we show that z is a fixed point of T. On the contrary, assume that z is not a fixed point of T, that is, $Tz \neq z$. Then using the condition (6), we obtain

$$\begin{aligned} S(z_n, z_n, z) \ &= \ S(Tz_{n-1}, Tz_{n-1}, Tz) \\ &\precsim \ h \max \{S(z_{n-1}, z_{n-1}, z), S(z_n, z_n, z_{n-1}), S(Tz, Tz, z), S(Tz, Tz, z_{n-1}), S(z_n, z_n, z)\} \end{aligned}$$

and so taking the limit for $n \to \infty$ we have

$$S(z, z, Tz) \precsim hS(Tz, Tz, z)$$

and by Lemma 3, we obtain

$$|S(z,z,Tz)| = |S(Tz,Tz,z)| \leq h\,|S(Tz,Tz,z)|\,,$$

which implies $Tz = z$, that is, z is a fixed point of T. We prove that z is the unique fixed point of T. Assume that w is another fixed point of T such that $z \neq w$. Using the condition (6), we get

$$\begin{aligned} S(z,z,w) &= S(Tz,Tz,Tw) \\ &\precsim h\max\{S(z,z,w), S(z,z,z), S(w,w,w), S(w,w,z), S(z,z,w)\} \end{aligned}$$

and so by Lemma 3, we find

$$|S(z,z,w)| \leq |S(z,z,w)|\,,$$

which implies $z = w$ since $h \in \left[0, \frac{1}{3}\right)$. Consequently, z is the unique fixed point of T. ■

Remark 2 *We can deduce the following results for a continuous self-mapping on a compact complete complex valued S-metric space.*

1. Corollary 1 is a generalization of Theorem 3.
2. Theorem 1 is another generalization of Theorem 3 by Proposition 1.
3. If we take the metric function as $S : X \times X \times X \to [0,\infty)$ in Theorem 3, then we get Corollary 2.21 given in [16].

Acknowledgment. The authors would like to thank the anonymous referee for his/her comments that helped us improve this article.

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