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Dynamic Behaviors Of A Stage Structure Single Species Model With Cannibalism*

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Abstract

A stage structure single species model with cannibalism takes the form

$$\begin{aligned}\frac{dx}{dt} &= \alpha y - \gamma x - \Omega x - \theta xy,\\ \frac{dy}{dt} &= \Omega x - \beta y\end{aligned}$$

is revisited in this paper, where $\alpha, \gamma, \Omega, \theta$ and β are all positive constants. We first show by numeric simulation that one of the main result of Shujing Gao is incorrect. Then by constructing some suitable Lyapunov function, sufficient conditions which ensure the globally asymptotically stability of the boundary equilibrium of above system is obtained.

1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following stage structure single species model with cannibalism

$$\begin{aligned}\tfrac{dx}{dt} &= \alpha y - \gamma x - \Omega x - \theta xy,\\ \tfrac{dy}{dt} &= \Omega x - \beta y,\end{aligned} \tag{1}$$

where α, γ, Ω, θ and β are all positive constants, $x(t)$ is the density of the immature species at time t, $y(t)$ is the density of the mature species at time t, respectively.

The dynamic behaviors of the stage structured ecosystem has recently been studied by many scholars, see [1]–[22] and the references cited therein. Also, topics such as the extinction, persistent and stability of the ecosystem are extensively studied in [1]–[30].

Gao [30] proposed the stage structured system (1). Concerned with the stability property of the nonnegative equilibrium of system (1), the author obtained the following results (Theorem 1.2, 1.3, 2.1 and 2.2, respectively).

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THEOREM A. Assume that

$$\alpha > \beta \tag{2}$$

holds. Then the boundary equilibrium $E_1(0,0)$ is unstable. Assume that

$$\alpha < \beta \tag{3}$$

holds. Then the boundary equilibrium $E_1(0,0)$ is locally asymptotically stable.

THEOREM B. Assume that

$$\alpha\Omega - \beta\gamma - \beta\Omega > 0 \tag{4}$$

holds. Then the positive equilibrium $E_2(x^*, y^*)$ is locally asymptotically stable, here

$$x^* = \frac{\alpha\Omega - \beta\gamma - \beta\Omega}{\theta\Omega},\ y^* = \frac{\alpha\Omega - \beta\gamma - \beta\Omega}{\beta\theta}.$$

THEOREM C. Assume that

$$\alpha > \beta, \alpha\Omega - \beta\gamma - \beta\Omega > 0 \tag{5}$$

hold. Then the positive equilibrium $E_2(x^*, y^*)$ is globally asymptotically stable.

THEOREM D. Assume that

$$\alpha < \beta, \alpha < \Omega + 2\gamma, \alpha + \Omega < 2\beta \tag{6}$$

hold. Then the boundary equilibrium $E_1(0,0)$ is globally asymptotically stable.

Now let's consider the following two examples.

EXAMPLE 1.1. Consider the following system

$$\begin{aligned} \tfrac{dx}{dt} &= 2y - 2x - x - xy, \\ \tfrac{dy}{dt} &= x - y. \end{aligned} \tag{7}$$

Here, we assume that $\alpha = 2$, $\gamma = 2$, $\Omega = 1$, $\theta = 1$ and $\beta = 1$. Then $\alpha = 2 > 1 = \beta$ holds. That is, the condition (2) of Theorem A holds, hence, $E_1(0,0)$ should be unstable, however, numeric simulation (Fig. 1) shows that in this case, $E_1(0,0)$ is globally asymptotically stable.

Above example shows that although the condition (2) of Theorem A holds, the result of Theorem A may still not hold.

EXAMPLE 1.2. Consider the following system

$$\begin{aligned} \tfrac{dx}{dt} &= y - x - x - xy, \\ \tfrac{dy}{dt} &= x - 2y. \end{aligned} \tag{8}$$

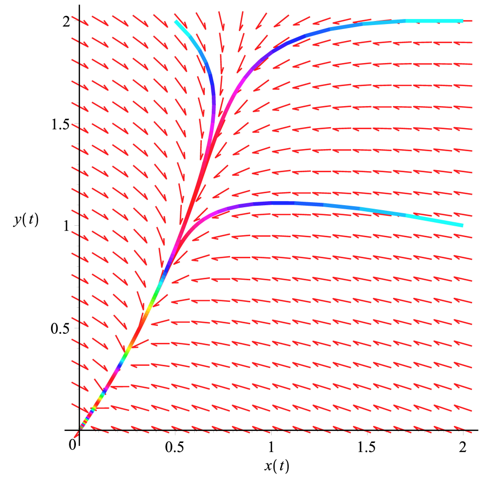

Figure 1: Dynamic behaviors of the system (7) the initial condition $(x(0), y(0)) = (2, 2)$, $(2, 1)$ and $(0.5, 2)$, respectively.

Here, we assume that $\alpha = 1$, $\gamma = 1$, $\Omega = 1$, $\theta = 1$ and $\beta = 2$. Then $\alpha = 1 < 2 = \beta$ and $\alpha = 1 < 3 = \Omega + 2\gamma$ hold. That is, the first and second inequalities in (6) does not hold, hence, one could only obtain the local stability property of $E_1(0, 0)$ from Theorem A, and could not draw any conclusion about the global asymptotic stability property of this equilibrium, however, numeric simulation (Fig.2) shows that in this case, $E_1(0, 0)$ is globally asymptotically stable.

Above example shows that although the condition (6) of Theorem D does not hold, the result of Theorem D may still hold.

Above two examples show that one needs to revisit the stability property of the system (1).

The aim of this paper is to obtain a set of sufficient condition which ensure the local and global asymptotically stable of the nonnegative equilibrium $E_1(0, 0)$.

2 Main Results

For the stability property of the non-negative equilibrium $E_1(0, 0)$, we have the following result.

THEOREM 2.1. Assume that

$$\alpha\Omega - \beta\gamma - \beta\Omega < 0 \tag{9}$$

holds. Then the nonnegative equilibrium $E_1(0, 0)$ of system (1) is locally asymptotically stable and globally asymptotically stable.

PROOF. To end the proof of Theorem 2.1, it is enough to show that $E_1(0, 0)$ is globally asymptotically stable under the assumption (9).

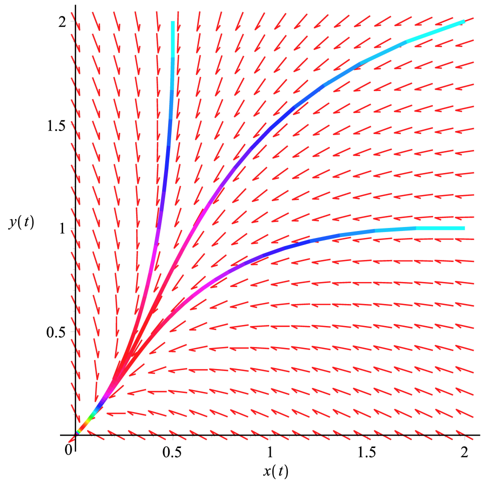

Figure 2: Dynamic behaviors of the system (8) the initial condition $(x(0), y(0)) = (2, 2)$, $(2, 1)$ and $(0.5, 2)$, respectively.

We will prove this assertion by constructing the suitable Lyapunov function.

Now let's consider the Lyapunov function

$$V(x, y) = K_1 x + K_2 y,$$

where K_1 and K_2 are some constants determined later. One could easily see that the function V is zero at the boundary equilibrium $E_1(0,0)$ and is positive for all other positive values of x, y. The time derivative of V along the trajectories of (1) is

$$\begin{aligned} & D^+V(t) \\ = \quad & K_1\Big(\alpha y - \gamma x - \Omega x - \theta xy\Big) + K_2\Big(\Omega x - \beta y\Big) \\ = \quad & \Big(K_1\alpha - K_2\beta\Big)y + \Big(K_2\Omega - K_1(\gamma + \Omega)\Big)x - K_1\theta xy. \end{aligned}$$

Let's take $K_1 = \beta, K_2 = \alpha$. Then

$$D^+V(t) = \Big(\alpha\Omega - \beta(\gamma + \Omega)\Big)x - \beta\theta xy.$$

It then follows from (9) that $D^+V(t) < 0$ strictly for all $x, y > 0$ except the boundary equilibrium $E_1(0,0)$, where $D^+V(t) = 0$. Thus, $V(x, y)$ satisfies Lyapunov's asymptotic stability theorem ([22]), and the boundary equilibrium $E_1(0,0)$ of system (1) is globally asymptotically stable.

This ends the proof of Theorem 2.1.

REMARK 2.1. In Theorem C, to ensure the second inequality of (5) holds, it is natural to require $\alpha > \beta$. Hence, we need not write out this inequality.

For the global asymptotically stability of the positive equilibrium $E_2(x^*, y^*)$, from Theorem C we have the following result.

THEOREM 2.2. Once system (1) admits a positive equilibrium $E_2(x^*, y^*)$, it is globally asymptotically stable.

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