

Note On The Stability Property Of The Trivial Equilibrium Point Of A Stage Structured Mosquito Population Model*

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Abstract

We revisit the stability property of a stage structured mosquito population model which was proposed by Jinping Fan and Hui Wan. By constructing some suitable Lyapunov function, we show that the conditions which ensure the local stability of the trivial equilibrium point is enough to ensure its global stability.

1 Introduction

During the last decades, many scholars investigated the dynamic behaviors of the ecosystem, see [1]–[13] and the references cited therein. Also, many scholars studied the dynamic behaviors of the stage structured ecosystem, see [1]–[9], [12] and the references cited therein.

Recently, Fan and Wan [8] proposed the following stage structured mosquito population model

$$\begin{aligned}\frac{dJ}{dt} &= \frac{b_v N}{1+N} - \alpha J - d_0 J - d_1 J^2, \\ \frac{dN}{dt} &= \alpha J - \mu_v N,\end{aligned}\tag{1}$$

with the initial conditions $J(0) > 0, N(0) > 0$. For more background on establishing the model, one can refer to [8] for more detail. System (1) always admits the trivial equilibrium point $E_0(0, 0)$, concerned with the stability property of this equilibrium, the authors obtained the following result.

THEOREM A. Assume that $R_0 = \frac{b_v \alpha}{\mu_v (\alpha + d_0)} < 1$. Then $E_0(0, 0)$ is locally asymptotically stable.

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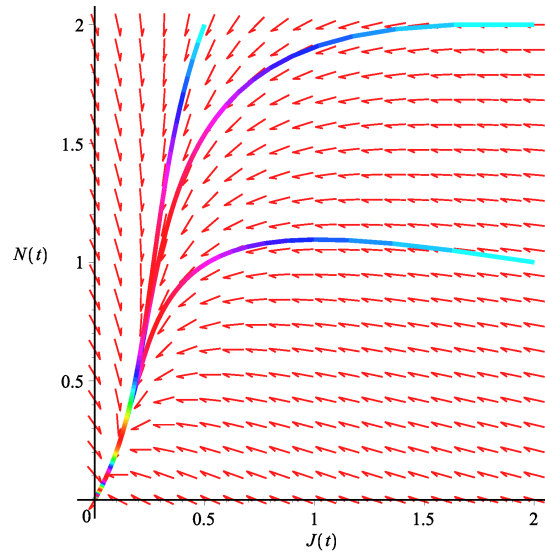


Figure 1: Dynamics behaviors of of system (2), the initial conditions $(J(0), N(0)) = (2, 2)$, $(2, 1)$ and $(0.5, 2)$, respectively.

Also, if $R_0 > 1$, then the system admits a unique positive equilibrium $E_1(J^*, N^*)$, concerned with the stability property of the positive equilibrium, by applying the Bendixson-Dulac principle, the authors obtained

THEOREM B. If $R_0 > 1$, then the system admits a unique positive equilibrium $E_1(J^*, N^*)$, which is globally asymptotically stable.

Now, an interesting issue may be raised: Is it possible for us to obtain a set of sufficient conditions which ensure the global asymptotic stability of the trivial equilibrium?

Let's consider the following example.

EXAMPLE 1. Consider the following system

$$\begin{aligned} \frac{dJ}{dt} &= \frac{N}{1+N} - J - J - J^2, \\ \frac{dN}{dt} &= J - N. \end{aligned} \tag{2}$$

Here, we assume that $b_v = \alpha = d_0 = d_1 = \mu_v = 1$. Then $R_0 = \frac{b_v \alpha}{\mu_v (\alpha + d_0)} = \frac{1}{2} < 1$. Then from Theorem A, $E_0(0, 0)$ is locally asymptotically stable, however, numeric simulation (Fig.1) shows that in this case, $E_0(0, 0)$ is globally asymptotically stable.

Example 1 motivated us to propose the following conjecture.

CONJECTURE. The condition $R_0 = \frac{b_v \alpha}{\mu_v(\alpha + d_0)} < 1$ is enough to ensure the global asymptotic stability of the trivial point $E_0(0, 0)$.

The aim of this paper is to give an answer to the above conjecture, more precisely, we have the following result.

THEOREM 1. Assume that $R_0 = \frac{b_v \alpha}{\mu_v(\alpha + d_0)} < 1$. Then the trivial equilibrium point $E_0(0, 0)$ is globally asymptotically stable.

2 Proof of Theorem 1

PROOF. We will prove this by constructing some suitable Lyapunov function. Let's define a Lyapunov function

$$V(J, N) = K_1 J + K_2 N,$$

where K_1, K_2 are some positive constants determined later. One could easily see that the function V is zero at the boundary equilibrium $E_0(0, 0)$ and is positive for all other positive values of J and N . The time derivative of V along the trajectories of (1) is

$$\begin{aligned} D^+V(t) &= K_1 \left(\frac{b_v N}{1+N} - \alpha J - d_0 J - d_1 J^2 \right) + K_2 \left(\alpha J - \mu_v N \right) \\ &\leq \left(K_1 b_v - K_2 \mu_v \right) N + \left(K_2 \alpha - K_1(\alpha + d_0) \right) J - K_1 d_1 J^2. \end{aligned}$$

Now let's take $K_1 = \frac{\alpha}{\alpha + d_0}$, $K_2 = 1$, it then follows from (2) that

$$D^+V(t) = \left(\frac{\alpha b_v}{\alpha + d_0} - \mu_v \right) N - \frac{\alpha d_1}{\alpha + d_0} J^2 < 0$$

strictly for all $J, N > 0$ except the trivial equilibrium $E_0(0, 0)$, where $D^+V(t) = 0$. Thus, $V(J, N)$ satisfies Lyapunov's asymptotic stability theorem ([13]), and the boundary equilibrium $E_1(0, 0)$ of system (1) is globally asymptotically stable. This ends the proof of Theorem 1.

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