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Single-Valued Neutrosophic Graph Structures*

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Abstract

A graph structure is a generalization of undirected graph which is quite useful in studying some structures, including graphs and signed graphs. In this research paper, we apply the concept of single-valued neutrosophic sets to graph structures, and explore some interesting properties of single-valued neutrosophic graph structure. We also discuss the concept of ϕ-complement of single-valued neutrosophic graph structure.

1 Introduction

Fuzzy set theory was introduced by Zadeh [15] to solve problems with uncertainties. At present, in modeling and controlling unsure systems in industry, society and nature, fuzzy sets and fuzzy logic are playing a vital role. In decision making, they can be used as powerfull mathematical tools for approximate reasoning. They play a significant role in complex phenomena which is not easily described by classical mathematics. Atanassov [4] illustrated the extension of fuzzy sets by adding a new component, called intuitionistic fuzzy sets. The intuitionistic fuzzy sets have essentially higher describing possibilities than fuzzy sets. The idea of intuitionistic fuzzy set is more meaningful as well as inventive due to the presence of degree of truth, degree of falsity and the hesitation margin. The hesitation margin of intuitionistic fuzzy set is its indeterminacy value by default. Smarandache [11] submitted the idea of neutrosophic set by combining the non-standard analysis, a tricomponent logic/set/probability theory and philosophy. "It is a branch of philosophy which studies the origin, nature and scope of neutralities as well as their interactions with different ideational spectra" [12]. A neutrosophic set has three components: truth membership, indeterminacy membership and falsity membership, in which each membership value is a real standard or non-standard subset of the nonstandard unit interval $]0-, 1+[$ ([11]). To apply neutrosophic sets in real-life problems more conveniently, Smarandache [11] and Wang et al. [13] defined single-valued neutrosophic sets (SVNSs). Actually, the single valued neutrosophic set was introduced for the first time by Smarandache in 1998 in his book: F. Smarandache, Neutrosophy, Neutrosophic probability, set, and logic, American Res. Press. A SVNS is a generalization of intuitionistic fuzzy sets [4]. In SVNS three components are not dependent and their values are contained in the standard unit interval $[0, 1]$.

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Fuzzy graphs were narrated by Rosenfeld [9] in 1975. Dinesh and Ramakrishnan [6] introduced the notion of a fuzzy graph structure and discussed some related properties. Akram and Akmal [1] introduced the concept of bipolar fuzzy graph structures. Dhavaseelan et al. [5] defined strong neutrosophic graphs. Akram [2] studied single-valued neutrosophic planar graphs. Akram and Shahzadi [3] introduced the notion of neutrosophic soft graphs with applications. In this research paper, we apply the idea of single-valued neutrosophic sets to graph structure, and explore some interesting properties of single-valued neutrosophic graphs. We also discuss the concept of ϕ-complement of single-valued neutrosophic graph structure. Further, we present an application of single-valued neutrosophic graph structures in decision-making.

2 Preliminaries

Sampathkumar [10] introduced the graph structure which is a generalization of undirected graph and is quite useful in studying some structures, including, graphs, signed graphs, labelled graphs and edge colored graphs.

DEFINITION 1 ([10]). A *graph structure* $\check{G} = (V, R_1, \ldots, R_n)$ consists of a non-empty set V together with relations $R_1, R_2, \ldots, R_n$ on V which are mutually disjoint such that each R_i, $1 \leq i \leq n$, is symmetric and irreflexive.

DEFINITION 2 ([11]). A *neutrosophic set* N on a universal set V is an object of the form

$$N = \{(v, T_N(v), I_N(v), F_N(v)) : v \in V\},$$

where $T_N, I_N, F_N : V \rightarrow]0^-, 1^+[$ and $0^- \leq T_N(v) + I_N(v) + F_N(v) \leq 3^+$.

DEFINITION 3 ([13]). A *single-valued neutrosophic (SVN) set* N on a universal set V is an object of the form

$$N = \{(v, T_N(v), I_N(v), F_N(v)) : v \in V\},$$

where $T_N, I_N, F_N : V \rightarrow [0,1]$ and $0 \leq T_N(v) + I_N(v) + F_N(v) \leq 3$.

DEFINITION 4 ([3]). A single-valued neutrosophic graph $G = (X, Y)$ is a pair, where $X : V \rightarrow [0,1]$ is a SVN set on V and $Y : V \times V \rightarrow [0,1]$ is a SVN neutrosophic relation on V such that:

$$\begin{aligned} T_Y(v_1v_2) &\leq \min\{T_X(v_1), T_X(v_2)\}, \\ I_Y(v_1v_2) &\leq \min\{I_X(v_1), I_X(v_2)\}, \\ F_Y(v_1v_2) &\leq \max\{F_X(v_1), F_X(v_2)\}, \end{aligned}$$

for all $v_1, v_2 \in V$. X and Y are said to be SVN vertex set of G and the SVN edge set of G, respectively.

3 Single-Valued Neutrosophic Graph Structures

DEFINITION 1. $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is called a single-valued neutrosophic graph structure (SVNGS) of a graph structure $\check{G} = (S, S_1, S_2..., S_n)$ if

$$Q =< n, T_i(n), I_i(n), F_i(n) >$$

is a single-valued neutrosophic (SVN) set on S and

$$Q_i =< (m, n), T(m, n), I(m, n), F(m, n) >$$

is a single-valued neutrosophic set on S_i such that

$$\begin{aligned} T_i(m,n) &\leq \min\{T(m), T(n)\}, I_i(m,n) \leq \min\{I(m), I(n)\}, F_i(m,n) \\ &\leq \max\{F(m), F(n)\}, \quad \forall m, n \in S. \end{aligned}$$

Note that $T_i(m, n) = 0 = I_i(m, n) = F_i(m, n)$ for all $(m, n) \in S \times S - S_i$ and

$$0 \leq T_i(m, n) + I_i(m, n) + F_i(m, n) \leq 3 \text{ for all } (m, n) \in S_i,$$

where S and S_i $(i = 1, 2, ..., n)$ are underlying vertex and underlying i-edge sets of $\check{G}_n$ respectively.

DEFINITION 2. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS of $\check{G}$. If $\check{H}_n = (Q', Q'_1, Q'_2, ..., Q'_n)$ is a SVNGS of $\check{G}$ such that

$$T'(n) \leq T(n), I'(n) \leq I(n), \quad F'(n) \geq F(n), \quad \forall n \in S,$$

$$T'_i(m, n) \leq T_i(m, n), \quad I'_i(m, n) \leq I_i(m, n) \text{ and } F'_i(m, n) \geq F_i(m, n), \quad \forall m, n \in S_i,$$

where $i = 1, 2, ..., n$. Then $\check{H}_n$ is called a SVN subgraph structure of SVNGS $\check{G}_n$.

DEFINITION 3. A SVNGS $\check{H}_n = (Q', Q'_1, Q'_2, ..., Q'_n)$ is called a SVN induced subgraph structure of $\check{G}_n$ by a subset R of S if

$$T'(n) = T(n), \quad I'(n) = I(n), \quad F'(n) = F(n), \quad \forall n \in R,$$

$$T'_i(m, n) = T_i(m, n), \quad I'_i(m, n) = I_i(m, n) \text{ and } F'_i(m, n) = F_i(m, n), \quad \forall m, n \in R,$$

where $i = 1, 2, ..., n$.

DEFINITION 4. A SVNGS $\check{H}_n = (Q', Q'_1, Q'_2, ..., Q'_n)$ is called a SVN spanning subgraph structure of $\check{G}_n$ if $Q' = Q$ and

$$T'_i(m, n) \leq T_i(m, n), \quad I'_i(m, n) \leq I_i(m, n) \text{ and } F'_i(m, n) \geq F_i(m, n), \quad i = 1, 2, ..., n.$$

EXAMPLE 1. Consider a GS $\check{G} = (S, S_1, S_2)$ and Q, Q_1,Q_2 be SVN subsets of S, S_1, S_2, respectively, such that

$$Q = \{(n_1, 0.5, 0.2, .3), (n_2, 0.7, 0.3, 0.4), (n_3, 0.4, 0.3, 0.5), (n_4, 0.7, 0.3, 0.6)\},$$

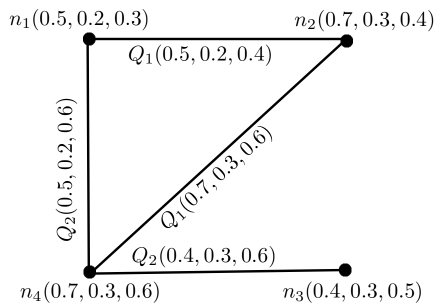

Figure 1: A single-valued neutrosophic graph structure $\check{G}_n$.

$$Q_1 = \{(n_1n_2, 0.5, 0.2, 0.4), (n_2n_4, 0.7, 0.3, 0.6)\},$$
$$Q_2 = \{(n_3n_4, 0.4, 0.3, 0.6), (n_1n_4, 0.5, 0.2, 0.6)\}.$$

Direct calculations show that $\check{G}_n = (Q, Q_1, Q_2)$ is a SVNGS of $\check{G}$ as shown in Figure 1.

EXAMPLE 2. A SVNGS $\check{K}_n = (Q', Q_{11}, Q_{12})$ shown in Figure 2 is a SVN subgraph structure of $\check{G}_n = (Q, Q_1, Q_2)$ shown in Figure 1.

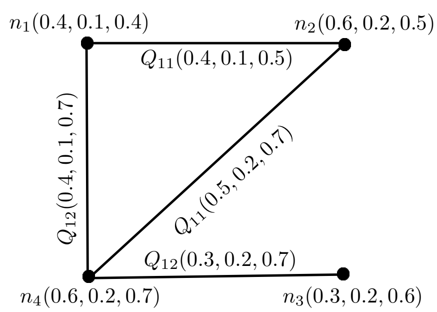

Figure 2: A SVN subgraph structure $\check{K}_n$.

DEFINITION 5. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS of $\check{G}$. Then $mn \in S_i$ is called a SVN Q_i-edge or simply Q_i-edge if $T_i(m, n) > 0$ or $I_i(m, n) > 0$ or $F_i(m, n) > 0$ or all three conditions hold. Consequently, support of Q_i is:

$$\begin{aligned} supp(Q_i) \quad &= \quad \{mn \in Q_i : T_i(m, n) > 0\} \cup \{mn \in Q_i : I_i(m, n) > 0\} \\ &\quad \cup\{mn \in Q_i : F_i(m, n) > 0\}, \quad i = 1, 2, ..., n. \end{aligned}$$

DEFINITION 6. Q_i-path in a SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a sequence of distinct vertices $n_1, n_2, ..., n_m$ (except choice that $n_m = n_1$) in S, such that $n_{j-1}n_j$ is a SVN Q_i-edge for all $j = 2, ..., m$.

DEFINITION 7. A SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is called Q_i-strong for some $i \in \{1, 2, ..., n\}$ if

$$T_i(m, n) = \min\{T(m), T(n)\}, \quad I_i(m, n) = \min\{I(m), I(n)\}$$

and

$$F_i(m, n) = \max\{F(m), F(n)\}, \quad \forall mn \in supp(Q_i).$$

SVNGS $\check{G}_n$ is called strong if it is Q_i-strong for all $i \in \{1, 2, ..., n\}$.

EXAMPLE 3. Consider a SVNGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Figure 3. Then $\check{G}_n$ is a strong SVNGS since it is both Q_1- and Q_2-strong.

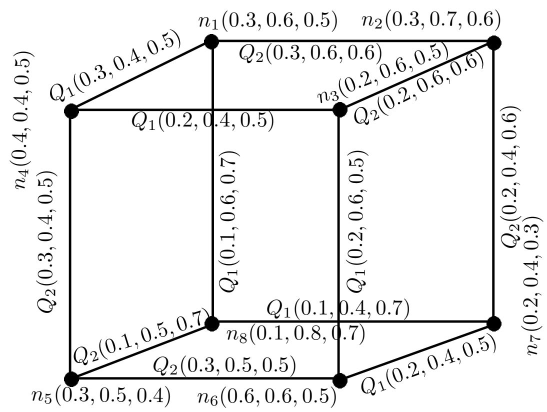

Figure 3: A strong SVNGS $\check{G}_n = (Q, Q_1, Q_2)$.

DEFINITION 8. A SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is called complete or $Q_1Q_2...Q_n$-complete, if $\check{G}_n$ is a strong SVNGS, $supp(Q_i) \neq \phi$ for all $i = 1, 2, ..., n$ and for every pair of vertices $m, n \in S$, mn is a Q_i-*edge* for some i.

EXAMPLE 4. Let $\check{G}_n = (Q, Q_1, Q_2)$ be a SVNGS of graph structure $\check{G} = (S, S_1, S_2)$ such that $S = \{n_1, n_2, n_3\}$, $S_1 = \{n_1n_2\}$ and $S_2 = \{n_2n_3, n_1n_3\}$ as shown in Figure 4. By simple calculations, it can be seen that $\check{G}_n$ is a strong SVNGS. Moreover, $supp(Q_1) \neq \phi$, $supp(Q_2) \neq \phi$ and each pair of vertices in S is either a Q_1-edge or an Q_2-edge. So $\check{G}_n$ is a complete, i.e., Q_1Q_2-complete SVNGS.

DEFINITION 9. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS. Then truth strength, indeterminacy strength and falsity strength of a Q_i-path $P_{Q_i} = n_1, n_2, ..., n_m$ are denoted by $T.P_{Q_i}$, $I.P_{Q_i}$ and $F.P_{Q_i}$, respectively and defined as

$$T.P_{Q_i} = \bigwedge_{j=2}^{m} [T^P_{Q_i}(n_{j-1}n_j)], I.P_{Q_i} = \bigwedge_{j=2}^{m} [I^P_{Q_i}(n_{j-1}n_j)], F.P_{Q_i} = \bigvee_{j=2}^{m} [F^P_{Q_i}(n_{j-1}n_j)].$$

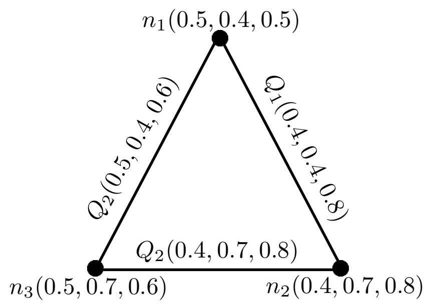

Figure 4: A complete SVNGS.

EXAMPLE 5. Consider a SVNGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Figure 4. We found that $P_{Q_2} = n_2, n_1, n_3$ is a Q_2-path. So $T.P_{Q_2} = 0.4$, $I.P_{Q_2} = 0.4$ and $F.P_{Q_2} = 0.8$.

DEFINITION 10. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS. Then

(i) Q_i-truth strength of connectedness between m and n is defined by $T^{\infty}_{Q_i}(mn) = \bigvee_{j\geq 1}\{T^j_{Q_i}(mn)\}$ such that $T^j_{Q_i}(mn) = (T^{j-1}_{Q_i} \circ T^1_{Q_i})(mn)$ for $j \geq 2$ and

$$T^2_{Q_i}(mn) = (T^1_{Q_i} \circ T^1_{Q_i})(mn) = \bigvee_z (T^1_{Q_i}(mz) \wedge T^1_{Q_i})(zn).$$

(ii) Q_i-indeterminacy strength of connectedness between m and n is defined by $I^{\infty}_{Q_i}(mn) = \bigvee_{j\geq 1}\{I^j_{Q_i}(mn)\}$ such that $I^j_{Q_i}(mn) = (I^{j-1}_{Q_i} \circ I^1_{Q_i})(mn)$ for $j \geq 2$ and

$$I^2_{Q_i}(mn) = (I^1_{Q_i} \circ I^1_{Q_i})(mn) = \bigvee_z (I^1_{Q_i}(mz) \wedge I^1_{Q_i})(zn).$$

(iii) Q_i-Falsity strength of connectedness between m and n is defined by $F^{\infty}_{Q_i}(mn) = \bigwedge_{j\geq 1}\{F^j_{Q_i}(mn)\}$ such that $F^j_{Q_i}(mn) = (F^{j-1}_{Q_i} \circ F^1_{Q_i})(mn)$ for $j \geq 2$ and

$$F^2_{Q_i}(mn) = (F^1_{Q_i} \circ F^1_{Q_i})(mn) = \bigwedge_z (F^1_{Q_i}(mz) \vee F^1_{Q_i})(zn).$$

DEFINITION 11. A SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a Q_i-cycle if

$$(supp(Q), supp(Q_1), supp(Q_2), ..., supp(Q_n)) \text{ is a } Q_i\text{-cycle.}$$

DEFINITION 12. A SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a SVN fuzzy Q_i-cycle (for some i) if $\check{G}_n$ is a Q_i-cycle, no unique Q_i-edge mn is in $\check{G}_n$ such that

$$T_{Q_i}(mn) = \min\{T_{Q_i}(rs) : rs \in S_i = supp(Q_i)\},$$

or

$$I_{Q_i}(mn) = \min\{I_{Q_i}(rs) : rs \in S_i = supp(Q_i)\},$$

or

$$F_{Q_i}(mn) = \max\{F_{Q_i}(rs) : rs \in S_i = supp(Q_i)\}.$$

EXAMPLE 6. Consider a SVNGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Figure 3. Then $\check{G}_n$ is a Q_1-cycle and SVN fuzzy $Q_1 - cycle$, since $(supp(Q), supp(Q_1), supp(Q_2))$ is a Q_1-cycle and there is no unique Q_1-edge satisfying above condition.

DEFINITION 13. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS and p be a vertex in $\check{G}_n$. Let $(Q', Q'_1, Q'_2, ..., Q'_n)$ be a SVNGS induced by $S \setminus \{p\}$ such that, for all $v \neq p, w \neq p$,

$$T_{Q'}(p) = 0 = I_{Q'}(p) = F_{Q'}(p), \;\; T_{Q'_i}(pv) = 0 = I_{Q'_i}(pv) = F_{Q'_i}(pv), \;\; \forall \text{edges } pv \in \check{G}_n,$$

$$T_{Q'}(v) = T_Q(v), I_{Q'}(v) = I_Q(v), F_{Q'}(v) = F_Q(v),$$

$$T_{Q'_i}(vw) = T_{Q_i}(vw), \;\; I_{Q'_i}(vw) = I_{Q_i}(vw) \;\; \text{and} \;\; F_{Q'_i}(vw) = F_{Q_i}(vw).$$

Then p is SVN fuzzy Q_i-cut vertex for any i if

$$T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw), I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw) \;\; \text{and} \; F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw),$$

for some $v, w \in S \setminus \{p\}$. Note that p is a $Q_i - T$ SVN fuzzy cut vertex if $T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw)$, $Q_i - I$ SVN fuzzy cut vertex if $I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw)$ and $Q_i - F$ SVN fuzzy cut vertex if $F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw)$.

EXAMPLE 7. Consider a SVNGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Figure 5 and let $\check{G}'_n = (Q', Q'_1, Q'_2)$ be a SVN subgraph structure of SVNGS $\check{G}_n$ found by deleting vertex n_2. Deleted vertex n_2 is a SVN fuzzy Q_1-I cut vertex since

$$I^{\infty}_{Q_1}(n_2n_5) = 0.4 > 0.3 = I^{\infty}_{Q'_1}(n_2n_5), \quad I^{\infty}_{Q_1}(n_3n_4) = 0.7 = I^{\infty}_{Q'_1}(n_3n_4),$$

and

$$I^{\infty}_{Q_1}(n_3n_5) = 0.4 > 0.3 = I^{\infty}_{Q'_1}(n_3n_5).$$

DEFINITION 14. Suppose $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS and mn be Q_i-*edge*. Let $(Q', Q'_1, Q'_2, ..., Q'_n)$ be a SVN fuzzy spanning subgraph structure of $\check{G}_n$, such that $\forall$ edges $mn \neq rs$,

$$T_{Q'_i}(mn) = 0 = I_{Q'_i}(mn) = F_{Q'_i}(mn), \;\; T_{Q'_i}(rs) = T_{Q_i}(rs),$$

$$I_{Q'_i}(rs) = I_{Q_i}(rs) \;\; \text{and} \;\; F_{Q'_i}(rs) = F_{Q_i}(rs).$$

Then mn is a SVN fuzzy Q_i-bridge if $T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw)$, $I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw)$ and $F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw)$, for some $v, w \in S$. Note that mn is a $Q_i - T$ SVN fuzzy bridge if

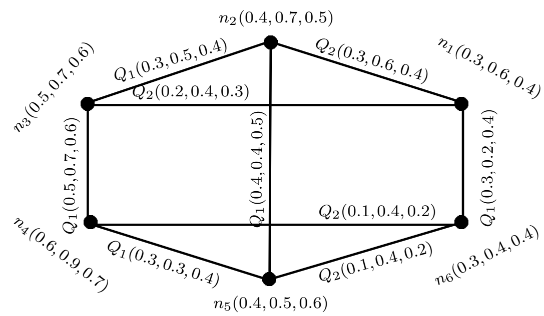

Figure 5: A SVNGS $\check{G}_n = (Q, Q_1, Q_2)$.

$T^{\infty}_{Q_i}(vw) > T^{\infty}_{Q'_i}(vw)$, $Q_i - I$ SVN fuzzy bridge if $I^{\infty}_{Q_i}(vw) > I^{\infty}_{Q'_i}(vw)$ and $Q_i - F$ SVN fuzzy bridge if $F^{\infty}_{Q_i}(vw) > F^{\infty}_{Q'_i}(vw)$.

EXAMPLE 8. Consider the SVNGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Figure 5 and $\check{G}'_n = (Q', Q'_1, Q'_2)$ be a SVN spanning subgraph structure of SVNGS $\check{G}_n$ which is found by deleting Q_1-edge (n_2n_5).Edge (n_2n_5) is a SVN fuzzy Q_1-bridge. Since

$$T^{\infty}_{Q_1}(n_2n_5) = 0.4 > 0.3 = T^{\infty}_{Q'_1}(n_2n_5),$$

$$I^{\infty}_{Q_1}(n_2n_5) = 0.4 > 0.3 = I^{\infty}_{Q'_1}(n_2n_5)$$

and

$$F^{\infty}_{Q_1}(n_2n_5) = 0.5 > 0 = F^{\infty}_{Q'_1}(n_2n_5).$$

DEFINITION 15. A SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a Q_i-tree if

$$(supp(Q), supp(Q_1), supp(Q_2), ..., supp(Q_n))$$

is a Q_i-*tree*. In other words, $\check{G}_n$ is a Q_i-tree if a subgraph of $\check{G}_n$ induced by $supp(Q_i)$ generates a tree.

DEFINITION 16. A SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is a SVN fuzzy Q_i-tree if $\check{G}_n$ has a SVN fuzzy spanning subgraph structure $\check{H}_n = (Q', Q'_1, Q'_2, ..., Q'_n)$ such that $\forall Q_i$-edges mn not in $\check{H}_n$, $\check{H}_n$ is a Q'_i-tree,

$$T_{Q_i}(mn) < T^{\infty}_{Q'_i}(mn), \;\; I_{Q_i}(mn) < I^{\infty}_{Q'_i}(mn) \;\text{ and }\; F_{Q_i}(mn) > F^{\infty}_{Q'_i}(mn).$$

In particular, $\check{G}_n$ is a SVN fuzzy Q_i-T tree if $T_{Q_i}(mn) < T^{\infty}_{Q'_i}(mn)$, a SVN fuzzy Q_i-I tree if $I_{Q_i}(mn) < I^{\infty}_{Q'_i}(mn)$ and a SVN fuzzy Q_i-F tree if $F_{Q_i}(mn) > F^{\infty}_{Q'_i}(mn)$.

EXAMPLE 9. Consider the SVNGS $\check{G}_n = (Q, Q_1, Q_2)$ as shown in Figure 6, which is a Q_2-tree. It is not a Q_1-tree but a SVN fuzzy Q_1-tree since it has a single-valued neutrosophic fuzzy spanning subgraph (Q', Q'_1, Q'_2) as a Q'_1-tree, which is obtained by deleting Q_1-edge n_2n_5 from $\check{G}_n$. Moreover,

$$T_{Q_1}(n_2n_5) = 0.2 < 0.3 = T^{\infty}_{Q'_1}(n_2n_5), \quad I_{Q_1}(n_2n_5) = 0.1 < 0.3 = I^{\infty}_{Q_1{}'}(n_2n_5)$$

and

$$F_{Q_1}(n_2n_5) = 0.6 > 0.5 = F^{\infty}_{Q_1{}'}(n_2n_5).$$

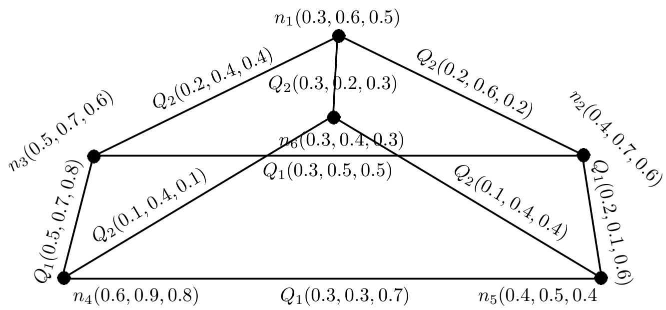

Figure 6: A single-valued neutrosophic fuzzy Q_1-tree.

DEFINITION 17. A SVNGS $\check{G}_{s1} = (Q_1, Q_{11}, Q_{12}, ..., Q_{1n})$ of the graph structure $\check{G}_1 = (S_1, S_{11}, S_{12}, ..., S_{1n})$ is isomorphic to SVNGS $\check{G}_{s2} = (Q_2, Q_{21}, Q_{22}, ..., Q_{2n})$ of the graph structure $\check{G}_2 = (S_2, S_{21}, Q_{22}, ..., S_{2n})$ if we have (f, ϕ) where $f : S_1 \to S_2$ is a bijection and ϕ is a permutation on set $\{1, 2, ..., n\}$ and following relations are satisfied

$$T_{Q_1}(m) = T_{Q_2}(f(m)), \quad I_{Q_1}(m) = I_{Q_2}(f(m)), \quad F_{Q_1}(m) = F_{Q_2}(f(m)),$$

for all $m \in S_1$ and

$$T_{Q_{1i}}(mn) = T_{Q_{2\phi(i)}}(f(m)f(n)), \quad I_{Q_{1i}}(mn) = I_{Q_{2\phi(i)}}(f(m)f(n),$$

$$F_{Q_{1i}}(mn) = F_{Q_{2\phi(i)}}(f(m)f(n)),$$

for all $mn \in S_{1i}$, $i = 1, 2, ..., n$.

EXAMPLE 10. Let $\check{G}_{n1} = (Q, Q_1, Q_2)$ and $\check{G}_{n2} = (Q', Q'_1, Q'_2)$ be two SVNGSs as shown in Figure 7. $\check{G}_{n1}$ is isomorphic $\check{G}_{n2}$ under (f, ϕ) where $f : S \to S'$ is a bijection and ϕ is a permutation on set $\{1, 2\}$ defined as $\phi(1) = 2$, $\phi(2) = 1$ and following relations are satisfied

$$T_Q(n_i) = T_{Q'}(f(n_i)), I_Q(n_i) = I_{Q'}(f(n_i)), \ F_Q(n_i) = F_{Q'}(f(n_i)),$$

for all $n_i \in S$, and

$$T_{Q_i}(n_in_j) = T_{Q'_{\phi(i)}}(f(n_i)f(n_j)), I_{Q_i}(n_in_j) = I_{Q'_{\phi(i)}}(f(n_i)f(n_j)),$$

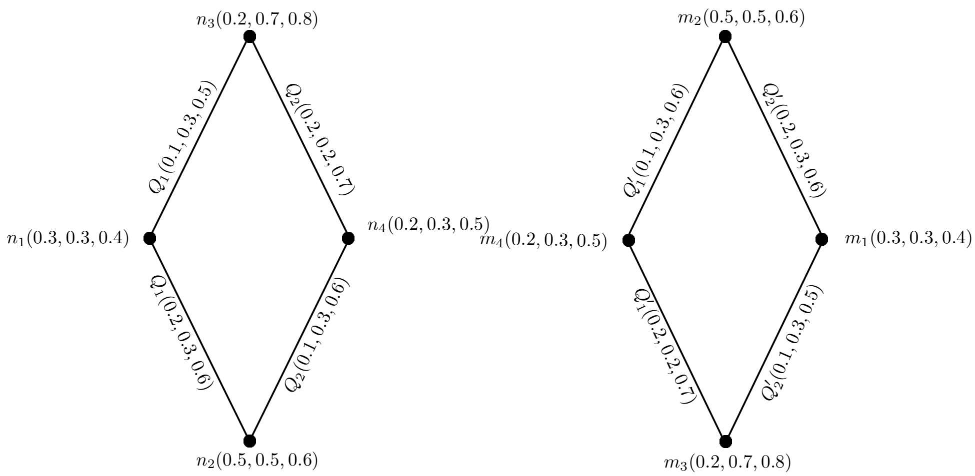

Figure 7: Isomorphic SVN graph structures.

$$F_{Q_i}(n_i n_j) = F_{Q'_{\phi(i)}}(f(n_i)f(n_j)),$$

$\forall n_i n_j \in S_i$ and $i = 1, 2$.

DEFINITION 18. A SVNGS $\check{G}_{s1} = (Q_1, Q_{11}, Q_{12}, ..., Q_{1n})$ of the graph structure $\check{G}_1 = (S_1, S_{11}, S_{12}, ..., S_{1n})$ is identical to SVNGS $\check{G}_{s2} = (Q_2, Q_{21}, Q_{22}, ..., Q_{2n})$ of graph structure $\check{G}_2 = (S_2, S_{21}, Q_{22}, ..., S_{2n})$ if $f : S_1 \to S_2$ is a bijection and following relations are satisfied

$$T_{Q_1}(m) = T_{Q_2}(f(m)), \quad I_{Q_1}(m) = I_{Q_2}(f(m)), \quad F_{Q_1}(m) = F_{Q_2}(f(m)),$$

$\forall m \in S_1$ and

$$T_{Q_{1i}}(mn) = T_{Q_{2i}}(f(m)f(n)), \quad I_{Q_{1i}}(mn) = I_{Q_{2i}}(f(m)f(n)),$$

$$F_{Q_{1i}}(mn) = F_{Q_{2(i)}}(f(m)f(n)),$$

for all $mn \in S_{1i}$ and $i = 1, 2, ..., n$.

EXAMPLE 11. Let $\check{G}_{n1} = (Q, Q_1, Q_2)$ and $\check{G}_{n2} = (Q', Q'_1, Q'_2)$ be two SVNGSs of GSs $\check{G}_1 = (S, S_1, S_2)$ and $\check{G}_2 = (S', S'_1, S'_2)$, respectively, as shown in Figures 8 and 9. SVNGS $\check{G}_{n1}$ is identical to $\check{G}_{n2}$ under $f : S \to S'$ defined as

$$f(n_1) = m_2, \quad f(n_2) = m_1, \quad f(n_3) = m_4, \quad f(n_4) = m_3, \quad f(n_5) = m_5, \quad f(n_6) = m_8,$$

$$f(n_7) = m_7, \quad f(n_8) = m_6, \quad T_Q(n_i) = T_{Q'}(f(n_i)),$$

$$I_Q(n_i) = I_{Q'}(f(n_i)), \quad F_Q(n_i) = F_{Q'}(f(n_i)),$$

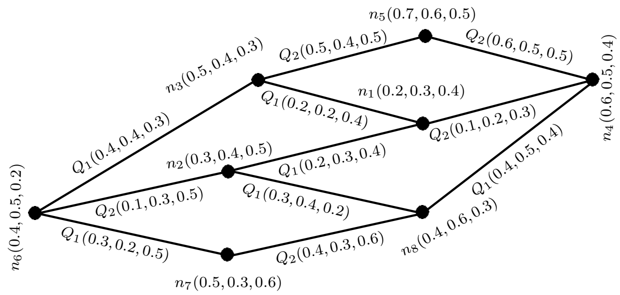

Figure 8: A SVNGS $\check{G}_{n1}$.

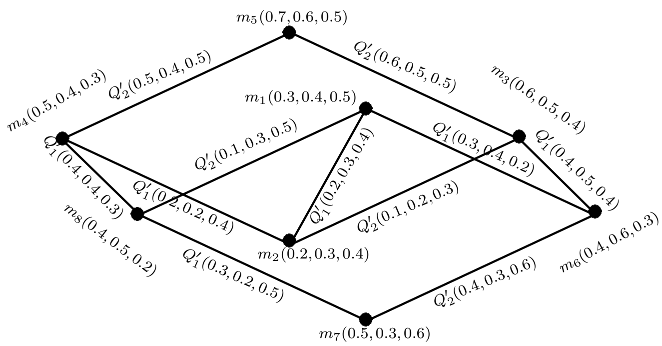

Figure 9: A SVNGS $\check{G}_{n2}$.

for all $n_i \in S$ and

$$T_{Q_i}(n_in_j) = T_{Q'_i}(f(n_i)f(n_j)), \quad I_{Q_i}(n_in_j) = I_{Q'_i}(f(n_i)f(n_j)), \quad F_{Q_i}(n_in_j) = F_{Q'_i}(f(n_i)f(n_j)),$$

for all $n_in_j \in S_i$ and $i = 1, 2$.

DEFINITION 19. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS and ϕ be a permutation on $\{Q_1, Q_2, ..., Q_n\}$ and on $\{1, 2, ..., n\}$ that is $\phi(Q_i) = Q_j$ iff $\phi(i) = j\ \forall i$. If $mn \in Q_i$ for any i and

$$T_{Q_i^\phi}(mn) = T_Q(m) \wedge T_Q(n) - \bigvee_{j \neq i} T_{\phi(Q_j)}(mn), \quad I_{Q_i^\phi}(mn) = I_Q(m) \wedge I_Q(n) - \bigvee_{j \neq i} I_{\phi(Q_j)}(mn),$$

$$F_{Q_i^\phi}(mn) = F_Q(m) \vee F_Q(n) - \bigwedge_{j \neq i} T_{\phi(Q_j)}(mn), \quad i = 1, 2, ..., n,$$

then $mn \in Q_k^\phi$, where k is selected such that

$$T_{Q_k^\phi}(mn) \geq T_{Q_i^\phi}(mn), \quad I_{Q_k^\phi}(mn) \geq I_{Q_i^\phi}(mn) \text{ and } F_{Q_k^\phi}(mn) \geq F_{Q_i^\phi}(mn) \text{ for all } i.$$

And SVNGS $(Q, Q_1^\phi, Q_2^\phi, ..., Q_n^\phi)$ is called ϕ-complement of SVNGS $\check{G}_n$ and denoted by $\check{G}_n^{\phi c}$.

EXAMPLE 12. Let $\check{G}_n = (Q, Q_1, Q_2, Q_3)$ be a SVNGS shown in Figure 10 and ϕ be a permutation on $\{1, 2, 3\}$ defined as: $\phi(1) = 2$, $\phi(2) = 3$, $\phi(3) = 1$. As a result of simple calculations, we see that $n_1n_3 \in Q_3^\phi$, $n_2n_3 \in Q_1^\phi$, $n_1n_2 \in Q_2^\phi$. So, $\check{G}_n^{\phi c} = (Q, Q_1^\phi, Q_2^\phi, Q_3^\phi)$ is ϕ-complement of SVNGS $\check{G}_n$ as showm in Figure 10.

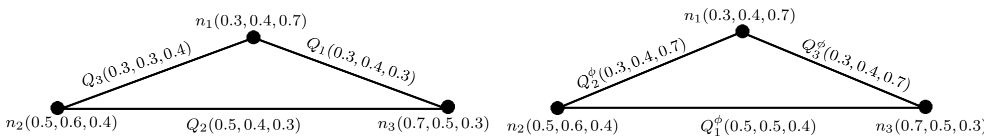

Figure 10: SVNGSs $\check{G}_n$, $\check{G}_n^{\phi c}$.

PROPOSITION 1. A ϕ-complement of a SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is always a strong SVNGS. Moreover, if $\phi(i) = k$, where $i, k \in \{1, 2, ..., n\}$, then all Q_k-edges in SVNGS $(Q, Q_1, Q_2, ..., Q_n)$ become Q_i^ϕ-edges in $(Q, Q_1^\phi, Q_2^\phi, ..., Q_n^\phi)$.

PROOF. According to the definition of ϕ-complement,

$$T_{Q_i^\phi}(mn) = T_Q(m) \wedge T_Q(n) - \bigvee_{j \neq i} T_{\phi(Q_j)}(mn),$$

$$I_{Q_i^\phi}(mn) = I_Q(m) \wedge I_Q(n) - \bigvee_{j \neq i} I_{\phi(Q_j)}(mn),$$

$$F_{Q_i^\phi}(mn) = F_Q(m) \vee F_Q(n) - \bigwedge_{j \neq i} F_{\phi(Q_j)}(mn),$$

for $i \in \{1, 2, ..., n\}$. For expression of truthness in ϕ-complement requirements are shown as: Since

$$T_Q(m) \wedge T_Q(n) \geq 0, \bigvee_{j \neq i} T_{\phi(Q_j)}(mn) \geq 0 \;\text{ and }\; T_{Q_i}(mn) \leq T_Q(m) \wedge T_Q(n), \;\; \forall Q_i,$$

we see that

$$\bigvee_{j \neq i} T_{\phi(Q_j)}(mn) \leq T_Q(m) \wedge T_Q(n),$$

which implies that

$$T_Q(m) \wedge T_Q(n) - \bigvee_{j \neq i} T_{\phi(Q_j)}(mn) \geq 0.$$

Therefore, $T_{Q_i^\phi}(mn) \geq 0\ \forall i$. Moreover, $T_{Q_i^\phi}(mn)$ achieves its maximum value when $\bigvee_{j\neq i} T_{\phi(Q_j)}(mn)$ is zero. It is obvious that when $\phi(Q_i) = Q_k$ and mn is a Q_k-edge then $\bigvee_{j\neq i} T_{\phi(Q_j)}(mn)$ gets zero value. So

$$T_{Q_i^\phi}(mn) = T_Q(m) \wedge T_Q(n),\ \ for\ (mn) \in Q_k,\ \ \phi(Q_i) = Q_k.$$

Similarly, we have

$$I_{Q_i^\phi}(mn) = I_Q(m) \wedge I_Q(n),\ \ for\ (mn) \in Q_k,\ \ \phi(Q_i) = Q_k.$$

In the similar way for expression of falsity in ϕ-complement requirements are shown as: Since

$$F_Q(m) \vee F_Q(n) \geq 0,\quad \bigwedge_{j\neq i} F_{\phi(Q_j)}(mn) \geq 0\ \text{ and }\ F_{Q_i}(mn) \leq F_Q(m) \vee F_Q(n) \forall Q_i,$$

we see that

$$\bigwedge_{j\neq i} F_{\phi(Q_j)}(mn) \leq F_Q(m) \vee F_Q(n),$$

which implies that

$$F_Q(m) \vee F_Q(n) - \bigwedge_{j\neq i} F_{\phi(Q_j)}(mn) \geq 0.$$

Therefore, $F_{Q_i^\phi}(mn)$ is non-negative for all i. Moreover, $F_{Q_i^\phi}(mn)$ attains its maximum value when $\bigwedge_{j\neq i} F_{\phi(Q_j)}(mn)$ becomes zero. It is clear that when $\phi(Q_i) = Q_k$ and mn is a Q_k-edge then $\bigwedge_{j\neq i} F_{\phi(Q_j)}(mn)$ gets zero value. So

$$F_{Q_i^\phi}(mn) = F_Q(m) \vee F_Q(n)\ \text{ for } (mn) \in Q_k,\ \phi(Q_i) = Q_k.$$

This completes the proof.

DEFINITION 20. Let $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS and ϕ a permutation on $\{1, 2, ..., n\}$. Then

(i) If $\check{G}_n$ is isomorphic to $\check{G}_n^{\phi c}$, then $\check{G}_n$ is said to be self-complementary.

(ii) If $\check{G}_n$ is identical to $\check{G}_n^{\phi c}$, then $\check{G}_n$ is said to be strong self-complementary.

DEFINITION 21. Suppose $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ be a SVNGS. Then

(i) If $\check{G}_n$ is isomorphic to $\check{G}_n^{\phi c}$, for all permutations ϕ on $\{1, 2, ..., n\}$, then $\check{G}_n$ is totally self-complementary.

(ii) If $\check{G}_n$ is identical to $\check{G}_n^{\phi c}$, for all permutations ϕ on $\{1, 2, ..., n\}$, then $\check{G}_n$ is totally strong self-complementary.

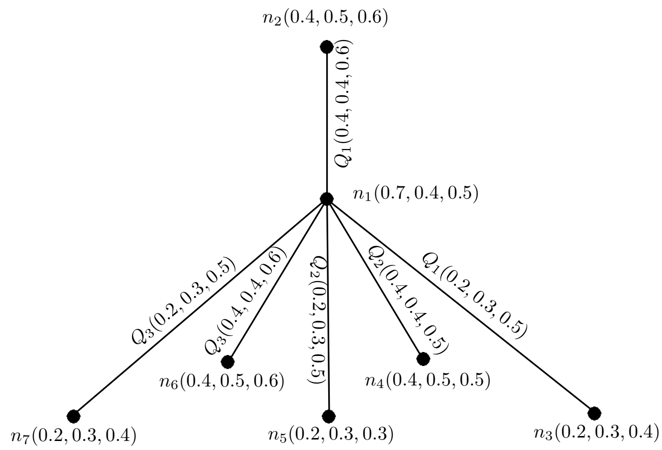

Figure 11: A totally strong self-complementary SVNGS.

REMARK 1. All strong SVNGSs are self-complementary or totally self-complementary SVNGSs.

EXAMPLE 13. A SVNGS $\check{G}_n = (Q, Q_1, Q_2, Q_3)$ in Figure 11 is a totally strong self-complementary SVNGS.

THEOREM 1. A SVNGS is totally self-complementary if and only if it is strong SVNGS.

PROOF. Consider a strong SVNGS $\check{G}_n$ and a permutation ϕ on $\{1, 2, ..., n\}$. By Proposition 1, ϕ-complement of a SVNGS $\check{G}_n = (Q, Q_1, Q_2, ..., Q_n)$ is always a strong SVNGS. Moreover, if $\phi(i) = k$, where $i, k \in \{1, 2, ..., n\}$, then all Q_k-edges in SVNGS $(Q, Q_1, Q_2, ..., Q_n)$ become Q_i^ϕ-edges in $(Q, Q_1^\phi, Q_2^\phi, ..., Q_n^\phi)$. This leads

$$T_{Q_k}(mn) = T_Q(m) \wedge T_Q(n) = T_{Q_i^\phi}(mn), \quad I_{Q_k}(mn) = I_Q(m) \wedge I_Q(n) = I_{Q_i^\phi}(mn)$$

and

$$F_{Q_k}(mn) = F_Q(m) \vee F_Q(n) = F_{Q_i^\phi}(mn).$$

Hence under the mapping(identity mapping) $f : S \to S$, $\check{G}_n$ and $\check{G}_n^\phi$ are isomorphic such that

$$T_Q(m) = T_Q(f(m)), \quad I_Q(m) = I_Q(f(m)), \quad F_Q(m) = F_Q(f(m)),$$

$$T_{Q_k}(mn) = T_{Q_i^\phi}(f(m)f(n)) = T_{Q_i^\phi}(mn), \quad I_{Q_k}(mn) = I_{Q_i^\phi}(f(m)f(n)) = I_{Q_i^\phi}(mn),$$

$$F_{Q_k}(mn) = F_{Q_i^\phi}(f(m)f(n)) = F_{Q_i^\phi}(mn),$$

for all $mn \in S_k$, $\phi^{-1}(k) = ii$ and $k = 1, 2, ..., n$. This is satisfied for every permutation ϕ on $\{1, 2, ..., n\}$. Hence $\check{G}_n$ is totally self-complementary SVNGS. Conversely, let for every permutation ϕ on $\{1,2,...,n\}$, $\check{G}_n$ and $\check{G}_n^{\phi}$ are isomorphic. Then according to the definition of isomorphism of SVNGSs and ϕ-complement of SVNGS,

$$T_{Q_k}(mn) = T_{Q_i^{\phi}}(f(m)f(n)) = T_Q(f(m)) \wedge T_Q(f(n)) = T_Q(m) \wedge T_Q(n),$$

$$I_{Q_k}(mn) = I_{Q_i^{\phi}}(f(m)f(n)) = I_Q(f(m)) \wedge I_Q(f(n)) = T_Q(m) \wedge I_Q(n),$$

$$F_{Q_k}(mn) = F_{Q_i^{\phi}}(f(m)f(n)) = F_Q(f(m)) \vee T_Q(f(n)) = F_Q(m) \wedge T_Q(n),$$

for all $mn \in S_k$ and $k = 1,2,...,n$. Hence $\check{G}_n$ is strong SVNGS.

REMARK 2. Every self-complementary SVNGS is totally self-complementary.

THEOREM 2. If $\check{G} = (S, S_1, S_2, ..., S_n)$ is a totally strong self-complementary GS and $Q = (T_Q, I_Q, F_Q)$ is a SVN subset of S where T_Q, I_Q, F_Q are constant valued functions then a strong SVNGS of $\check{G}$ with SVN vertex set Q is always a totally strong self-complementary SVNGS.

PROOF. Consider three constants p, q, $r \in [0,1]$, such that $T_Q(m) = p, I_Q(m) = q, F_Q(m) = r$ $\forall m \in S$ Since $\check{G}$ is totally self-complementary strong GS, so there is a bijection $f : S \to S$ for any permutation ϕ^{-1} on $\{1,2,...,n\}$, such that for any S_k-edge (mn), $(f(m)f(n))$ [a S_i-edge in $\check{G}$] is a S_k-edge in $\check{G}^{\phi^{-1}c}$. Hence for every Q_k-edge (mn), $(f(m)f(n))$ [a Q_i-edge in $\check{G}_n$] is a Q_k^{ϕ}-edge in ${\check{G}_n}^{\phi^{-1}c}$. Moreover $\check{G}_n$ is strong SVNGS, so

$$T_Q(m) = p = T_Q(f(m)),\ \ I_Q(m) = q = I_Q(f(m)),\ \ F_Q(m) = r = F_Q(f(m)),\ \ \forall m \in S,$$

$$T_{Q_k}(mn) = T_Q(m) \wedge T_Q(n) = T_Q(f(m)) \wedge T_Q(f(n)) = T_{Q_i^{\phi}}(f(m)f(n)),$$

$$I_{Q_k}(mn) = I_Q(m) \wedge I_Q(n) = I_Q(f(m)) \wedge I_Q(f(n)) = I_{Q_i^{\phi}}(f(m)f(n)),$$

$$F_{Q_k}(mn) = F_Q(m) \vee I_Q(n) = F_Q(f(m)) \vee F_Q(f(n)) = F_{Q_i^{\phi}}(f(m)f(n)),$$

for all $mn \in S_i$ and $i = 1,2,...,n$. This shows $\check{G}_n$ is seif-complementary strong SVNGS. Every permutation ϕ, ϕ^{-1} on $\{1,2,...,n\}$ satisfies above expressions, thus $\check{G}_n$ is totally strong self-complementary SVNGS.

REMARK 3. Converse of Theorem 2 may not be true, for example a SVNGS shown in Figure 11 is a totally strong self-complementary, it is strong and its underlying GS is a totally strong self-complementary but T_Q, I_Q, F_Q are not constant functions.

Table 1: SVN set Q of eight countries

Country	T	I	F
Bangladesh	0.8	0.7	0.6
Malaysia	0.7	0.7	0.8
Singapore	0.9	0.5	0.5
United Arab Emirates	1.0	0.5	0.6
Pakistan	0.9	0.5	0.5
India	0.8	0.7	0.7
Kenya	0.7	0.6	0.7
Italy	0.9	0.6	0.5

Table 2: SVN set of crimes between Pakistan and other countries during maritime trade

Type of crime	(P, UAE)	(P, B)	(P, M)	(P, S)
Human trafficking	(0.7 , 0.4, 0.5)	(0.8, 0.3, 0.4)	(0.7, 0.4, 0.2)	(0.6, 0.4, 0.2)
Illegal Carrying of Weapons	(0.6, 0.3, 0.6)	(0.7, 0.3, 0.4)	(0.4, 0.5, 0.5)	(0.4, 0.3, 0.5)
Black money transfer	(0.6, 0.3, 0.2)	(0.7, 0.5, 0.4)	(0.2, 0.4, 0.3)	(0.9, 0.2, 0.2)
Smuggling of precious metals	(0.8, 0.3, 0.2)	(0.6, 0.3, 0.3)	(0.2, 0.4, 0.3)	(0.8, 0.5, 0.5)
Drug trafficking	(0.7, 0.3, 0.3)	(0.5, 0.4, 0.3)	(0.6, 0.5, 0.6)	(0.8, 0.4, 0.3)
Smuggling of rare plants and animals	(0.3, 0.5, 0.5)	(0.4, 0.3, 0.4)	(0.4, 0.4, 0.5)	(0.2, 0.3, 0.3)

4 Application

Detection of crucial crimes during maritime trade: Waters are very important for trade in whole World but crimes through waters are increasing day by day. Crimes held during maritime trade are in abundance but some are very crucial including human trafficking, illegal carrying of weapons, black money transfer, smuggling of precious metals, drug trafficking and smuggling of rare plants and animals. Using SVNGS, we can easily investigate the fact that between any two countries which maritime crime is chronic and increasing rapidly with time. Moreover, we can decide which country is most sensitive for particular type of maritime crimes. We consider a set S consisting of eight countries.
S={Bangladesh, Malaysia, Singapore, United Arab Emirates, Pakistan, India, Kenya, Italy}. Let Q be the SVN set on S, defined in Table 1.

In Table 1, T depicts the importance of that particular country in the World due to its geographic position, F indicates the degree of its non-importance in the World, and I expresses, to which extent it is undecided/indeterminate to be beneficial for the world, geographically.
Let Bangladesh = B, Malaysia = M, Singapore = S, United Arab Emirates = UAE, Pakistan = P , India = I, Kenya = K, Italy = IT.
In Tables 2–7, we have shown the values of T, I, and F of different crimes for each pair of countries. Many relations on set S can be defined, let we define six relations on S as:
S_1 = Human trafficking, S_2 = Illegal carrying of weapons, S_3 = Black money transfer, S_4 = Smuggling of precious metals, S_5 = Drug trafficking, S_6 = Smuggling of rare

Table 3: SVN set of crimes between UAE and other countries during maritime trade

Type of crime	(UAE, B)	(UAE, M)	(UAE, S)	(UAE, I)
Human trafficking	(0.7, 0.3, 0.4)	(0.6, 0.2, 0.5)	(0.3, 0.2, 0.5)	(0.6, 0.4, 0.2)
Illegal carrying of weapons	(0.5, 0.2, 0.2)	(0.5, 0.3, 0.2)	(0.4, 0.3, 0.5)	(0.4, 0.3, 0.5)
Black money transfer	(0.6, 0.3, 0.3)	(0.6, 0.2, 0.3)	(0.6, 0.2, 0.3)	(0.6, 0.4, 0.5)
Smuggling of precious metals	(0.6, 0.2, 0.2)	(0.6, 0.3, 0.3)	(0.6, 0.3, 0.3)	(0.8, 0.3, 0.2)
Drug trafficking	(0.6, 0.2, 0.2)	(0.5, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.7, 0.4, 0.3)
Smuggling of rare plants and animals	(0.3, 0.4, 0.4)	(0.4, 0.3, 0.4)	(0.4, 0.2, 0.5)	(0.3, 0.3, 0.3)

Table 4: SVN set of crimes between Bangladesh and other countries during maritime trade

Type of crime	(B, M)	(B, S)	(B, I)	(B, K)
Human trafficking	(0.6, 0.3, 0.4)	(0.8, 0.3, 0.2)	(0.5, 0.2, 0.5)	(0.6, 0.4, 0.5)
Illegal carrying of weapons	(0.5, 0.2, 0.5)	(0.5, 0.3, 0.2)	(0.7, 0.3, 0.5)	(0.4, 0.3, 0.5)
Black money transfer	(0.4, 0.2, 0.2)	(0.7, 0.4, 0.3)	(0.1, 0.1, 0.2)	(0.1, 0.3, 0.4)
Smuggling of precious metals	(0.4, 0.2, 0.2)	(0.6, 0.3, 0.3)	(0.2, 0.3, 0.3)	(0.2, 0.2, 0.4)
Drug trafficking	(0.6, 0.2, 0.2)	(0.5, 0.4, 0.3)	(0.6, 0.3, 0.5)	(0.5, 0.4, 0.4)
Smuggling of rare plants and animals	(0.2, 0.3, 0.3)	(0.3, 0.2, 0.3)	(0.2, 0.1, 0.4)	(0.5, 0.2, 0.2)

Table 5: SVN set of crimes between Malaysia and other countries during maritime trade

Type of crime	(M, S)	(M, I)	(M, K)	(M, IT)
Human trafficking	(0.5, 0.3, 0.4)	(0.6, 0.2, 0.3)	(0.3, 0.2, 0.5)	(0.6, 0.4, 0.5)
Illegal carrying of weapons	(0.6, 0.2, 0.2)	(0.5, 0.3, 0.2)	(0.4, 0.3, 0.5)	(0.4, 0.3, 0.5)
Black money transfer	(0.6, 0.3, 0.3)	(0.2, 0.2, 0.3)	(0.2, 0.2, 0.3)	(0.2, 0.4, 0.5)
Smuggling of precious metals	(0.6, 0.2, 0.2)	(0.6, 0.3, 0.3)	(0.2, 0.3, 0.3)	(0.2, 0.2, 0.6)
Drug trafficking	(0.5, 0.2, 0.2)	(0.5, 0.4, 0.3)	(0.4, 0.3, 0.6)	(0.7, 0.4, 0.2)
Smuggling of rare plants and animals	(0.3, 0.4, 0.4)	(0.4, 0.3, 0.4)	(0.6, 0.2, 0.2)	(0.5, 0.3, 0.3)

Table 6: SVN set of crimes between Singapore and other countries during maritime trade

Type of crime	(S, I)	(S, K)	(S, IT)	(P, I)
Human trafficking	(0.5, 0.3, 0.4)	(0.3, 0.2, 0.5)	(0.3, 0.2, 0.5)	(0.6, 0.4, 0.6)
Illegal carrying of weapons	(0.7, 0.4, 0.5)	(0.5, 0.3, 0.2)	(0.4, 0.3, 0.5)	(0.8, 0.2, 0.4)
Black money transfer	(0.5, 0.3, 0.4)	(0.6, 0.2, 0.3)	(0.6, 0.2, 0.3)	(0.7, 0.4, 0.5)
Smuggling of precious metals	(0.8, 0.3, 0.7)	(0.6, 0.3, 0.3)	(0.6, 0.3, 0.3)	(0.6, 0.2, 0.4)
Drug trafficking	(0.7, 0.3, 0.4)	(0.5, 0.4, 0.3)	(0.6, 0.3, 0.2)	(0.8, 0.4, 0.4)
Smuggling of rare plants and animals	(0.7, 0.5, 0.6)	(0.4, 0.3, 0.4)	(0.6, 0.2, 0.5)	(0.7, 0.3, 0.3)

Table 7: SVN set of crimes between Italy and other countries during maritime trade

Type of crime	(IT, P)	(IT, UAE)	(IT, B)	(IT, I)
Human trafficking	(0.5, 0.3, 0.4)	(0.3, 0.2, 0.5)	(0.3, 0.2, 0.5)	(0.6, 0.4, 0.6)
Illegal carrying of weapons	(0.8, 0.3, 0.3)	(0.6, 0.3, 0.2)	(0.4, 0.3, 0.5)	(0.7, 0.3, 0.5)
Black money transfer	(0.6, 0.3, 0.3)	(0.5, 0.2, 0.3)	(0.2, 0.2, 0.3)	(0.5, 0.4, 0.5)
Smuggling of precious metals	(0.7, 0.3, 0.3)	(0.6, 0.3, 0.3)	(0.2, 0.3, 0.3)	(0.7, 0.3, 0.6)
Drug trafficking	(0.9, 0.3, 0.3)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.8, 0.3, 0.3)
Smuggling of rare plants and animals	(0.3, 0.4, 0.4)	(0.4, 0.3, 0.4)	(0.6, 0.2, 0.5)	(0.7, 0.3, 0.3)

Table 8: SVN set of crimes between Kenya and other countries during maritime trade

Type of crime	(K, P)	(K, UAE)	(K, I)	(K, IT)
Human trafficking	(0.5, 0.3, 0.4)	(0.6, 0.2, 0.5)	(0.5, 0.2, 0.5)	(0.6, 0.4, 0.5)
Illegal carrying of weapons	(0.6, 0.2, 0.5)	(0.5, 0.3, 0.4)	(0.5, 0.3, 0.5)	(0.4, 0.3, 0.5)
Black money transfer	(0.5, 0.3, 0.3)	(0.5, 0.2, 0.3)	(0.5, 0.2, 0.3)	(0.5, 0.4, 0.5)
Smuggling of precious metals	(0.4, 0.2, 0.2)	(0.6, 0.3, 0.3)	(0.6, 0.3, 0.3)	(0.4, 0.2, 0.4)
Drug trafficking	(0.7, 0.2, 0.2)	(0.5, 0.4, 0.3)	(0.5, 0.3, 0.5)	(0.8, 0.4, 0.2)
Smuggling of rare plants and animals	(0.3, 0.4, 0.4)	(0.7, 0.3, 0.4)	(0.6, 0.2, 0.4)	(0.7, 0.3, 0.3)

plants and animals, such that $(S, S_1, S_2, S_3, S_4, S_5, S_6)$ is a graph structure. An element in a relation detects that kind of crime during maritime trade between those two countries.
As $(S, S_1, S_2, S_3, S_4, S_5, S_6)$ is a graph structure, an element will not be in more than one relations, so it can appear just once. Therefore, we will consider it an element of that relation for which its percentage of truth is high, and percentage of both falsity and indeterminacy is low as compared to other relations.
According to given data, we write the elements in relations with their truth, falsity and indeterminacy values, resulting sets are SVN sets on S_1, S_2, S_3, S_4, S_5, S_6, respectively. We can name these sets as Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, respectively. Let $S_1 = \{(Bangladesh, Pakistan), (Malaysia, Pakistan), (Bangladesh, Singapore)\}$,
S_2=$\{(Pakistan, India)\}$,
S_3=$\{(Singapore, Pakistan)\}$,
S_4=$\{(India, Singapore)$, $(UnitedArabEmirates, India)\}$,
$S_5 = \{(Italy, Pakistan), (India, Italy)\}$,
$S_6 = \{(Kenya, Singapore)\}$.
And corresponding SVN sets are:
Q_1=$\{((B, P), 0.8, 0.2, 0.2)$,$((M, P), 0.7, 0.4, 0.2)$,$((B, S), 0.8, 0.3, 0.2)\}$,
Q_2=$\{((P, I), 0.8, 0.2, 0.4)\}$,
Q_3=$\{((S, P), 0.9, 0.2, 0.2), \}$,
Q_4=$\{((I, S), 0.8, 0.3, 0.4)$,$((UAE, I), 0.8, 0.3, 0.2)\}$,
Q_5=$\{((IT, P), 0.9, 0.3, 0.3)$,$((I, IT), 0.8, 0.3, 0.3)\}$,
$Q_6 = \{((K, S), 0.7, 0.2, 0.4)\}$.
Clearly, $(Q, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6)$ is a SVNGS as shown in Fig. 12.

In SVNGS shown in Fig. 12 every edge detects most frequent crime between adjacent countries during maritime trade. For instance: most frequent maritime crime between Pakistan and Singapore is black money transfer, its strength is 90%, weakness is 20% , and indeterminacy is 20% . We can also note that for relation human trafficking, vertex Pakistan has highest vertex degree, it means Pakistan is most sensitive country for human trafficking. Moreover, according to our SVNGS most frequent crime is human trafficking. It means that navy and maritime forces of these eight countries should take action to control human trafficking.

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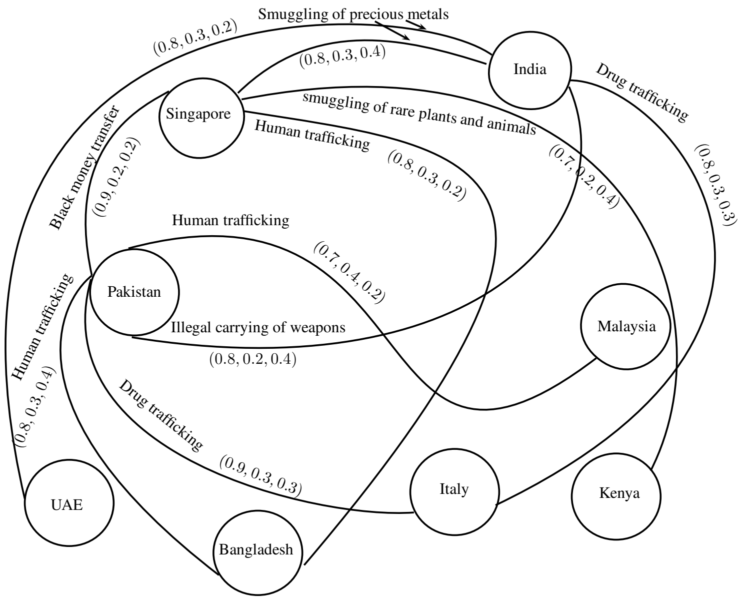

Figure 12: SVNGS showing most crucial maritime crime between any two countries

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