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On The Growth Of An E-Valued Meromorphic Function And Its Derivative*

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Abstract

In this article, the relative growth of an E-valued meromorphic function and its derivative is studied and we obtain the bound for $\frac{T(r,f')}{T(r,f)}$ for an E-valued meromorphic function of finite order. We also extend the related results of S. K. Singh and H. S. Gopalakrishna [4] to E-valued meromorphic function. Our results are significant and much stronger than the result obtained by Z. Wu and Y. Chen [5].

1 Introduction

In 1982, H. J. W. Ziegler [6] successively extended the classical Nevanlinna theory of meromorphic functions to vector valued meromorphic functions in finite dimensional spaces. Later in 1996, C. G. Hu and C. C. Yang [3] established the Nevanlinna's theory in an infinite dimensional Hilbert space. C. G. Hu [2] assumed, E is an infinite dimensional Banach space with a Schauder basis $\{e_j\}$, $j = 1, 2, \ldots$ and was able to present the statement of first and second fundamental theorem of Nevanlinna and Nevanlinna's deficiency relation in E. In 2006, C. G. Hu and Q. J. Hu [1] successively proved the generalized Poisson-Jensen-Nevanlinna formula, first and second fundamental theorem of Nevanlinna for E-valued meromorphic functions.

2 Basic Notions of Nevanlinna Theory in Infinite Dimensional Banach Space

Assume that E is a infinite dimensional complex Banach space with a Schauder basis $\{e_j\}_{j=1}^{\infty}$ and $\mathbb{C}$ is a complex plane. Let $D = C_r = \{z : |z| < r\}$. An E-valued meromorphic function $f(z)$ in a domain $D \subset \mathbb{C}$ can be written as

$$f(z) = \sum_{j=1}^{\infty} f_j(z)e_j = (f_1(z), f_2(z), \ldots, f_j(z), \ldots),$$

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where each $f_j(z)$ is a complex-valued meromorphic functions in D. We now introduce the generalized quantities of the Nevanlinna theory (see [1]): For any $a \in E \cup \{\infty\}$, $n(r,a,f) = n(r,a)$ denotes the number of a-points of f in $|z| \le r$, counted with multiplicities and $n(r,\infty,f) = n(r,f)$ denote the number of poles of f in $|z| \le r$. Then we have the counting function of finite or infinite a-points as

$$N(r,a) \equiv N(r,a,f) = n(0,a)\log r + \int_0^r \frac{n(t,a)-n(0,a)}{t}dt,$$

$$N(r,f) \equiv N(r,\infty,f) = n(0,f)\log r + \int_0^r \frac{n(t,f)-n(0,f)}{t}dt,$$

$$m(r,f) \equiv m(r,\infty,f) = \frac{1}{2\pi}\int_0^{2\pi} \log^+ \left\|f(re^{i\phi})\right\| d\phi,$$

$$m(r,a) \equiv m(r,a,f) = \frac{1}{2\pi}\int_0^{2\pi} \log^+ \frac{1}{\|f(re^{i\phi})-a\|}d\phi, (a \neq \infty),$$

and

$$T(r,f) = m(r,f) + N(r,f),$$

where $\log^+ x = \max\{\log x, 0\}$. The volume function associated with E-valued meromorphic function f is given by

$$V(r,a,f) = \frac{1}{2\pi}\int_{C_r} \log\left|\frac{r}{\xi}\right| \Delta \log\|f(\xi)-a\|\, d\sigma \wedge d\tau, \qquad a \in E$$

and the curvature function is given by

$$V(r,0,f') = G(r,f) = \int_0^r \frac{dt}{2\pi t}\int_{C_t} \Delta \log\|f'(\xi)\|\, d\sigma \wedge d\tau.$$

The order ρ of an E-valued meromorphic function f is defined by

$$\rho = \limsup_{r\to\infty} \frac{\log T(r,f)}{\log r}$$

and the lower order λ of f is defined by

$$\lambda = \liminf_{r\to\infty} \frac{\log T(r,f)}{\log r}.$$

We now define the following deficiencies as in [2]: For any $a \in E \cup \{\infty\}$, the number

$$\delta(a) = \delta(a,f) = \liminf_{r\to+\infty} \frac{m(r,a)}{T(r,f)} = 1 - \limsup_{r\to+\infty} \frac{V(r,a)+N(r,a)}{T(r,f)}$$

is called the deficiency of the point a, a point a with $\delta(a) > 0$ is called deficient.

The quantity

$$\theta(a) = \theta(a,f) = \liminf_{r\to+\infty} \frac{N(r,a)-\overline{N}(r,a)}{T(r,f)}$$

is called the index of multiplicity of a, and

$$\begin{aligned}\Theta(a)=\Theta(a,f) &= \liminf_{r\to+\infty}\frac{m(r,a)+N(r,a)-\overline{N}(r,a)}{T(r,f)}\\ &= 1-\limsup_{r\to+\infty}\frac{V(r,a)+\overline{N}(r,a)}{T(r,f)}.\end{aligned}$$

In particular, we have

$$\delta(\infty)=\liminf_{r\to+\infty}\frac{m(r,f)}{T(r,f)}=1-\limsup_{r\to+\infty}\frac{N(r,f)}{T(r,f)}\quad\text{since } V(r,\infty)=0,$$

$$\theta(\infty)=\liminf_{r\to+\infty}\frac{N(r,f)-\overline{N}(r,f)}{T(r,f)},$$

$$\Theta(\infty)=1-\limsup_{r\to+\infty}\frac{\overline{N}(r,f)}{T(r,f)}.$$

The quantity

$$\delta_G=\delta_G(f)=\liminf_{r\to+\infty}\frac{G(r,f)}{T(r,f)}$$

is called the Ricci Index of $f(z)$.

The function f is called admissible if $\frac{S(r_\gamma)}{T(r_\gamma,f)}\to 0$ for a sequence $r_\gamma\to+\infty$.

THEOREM 1 ([1]). (E-valued Nevanlinna's first fundamental theorem) Let $f(z)$ be an E-valued meromorphic mapping in C_R. Then for $0<r<R$, $a\in E$, $f(z)\neq a$,

$$T(r,f)=V(r,a)+N(r,a)+m(r,a)+\log\|c_q(a)\|+\epsilon(r,a).$$

Here $\epsilon(r,a)$ is a function such that $|\epsilon(r,a)|\le\log^+\|a\|+\log 2$, $\epsilon(r,0)\equiv 0$, and $c_q(a)\in E$ is the co-efficient of the first term in the Laurent series at the point a.

THEOREM 2 ([1]). (E-valued Nevanlinna's second fundamental theorem) Let $f(z)$ be a non-constant E-valued meromorphic mapping of compact projection in C_R and $a^{[k]}\in E\cup\{\infty\}$ $(k=1,2,\ldots,q)$ be $q\ge 3$ distinct finite or infinite points. Then

$$\sum_{k=1}^{q} m\left(r,a^{[k]}\right)+G(r,f)\le T(r,f)-N_1(r)+S(r),$$

where $N_1(r)=N(r,0,f')+2N(r,f)-N(r,f')$ and

$$G(r,f)=\int_0^r\frac{dt}{2\pi t}\int_{C_t}\Delta\log\|f'(\xi)\|\,d\sigma\wedge d\tau.$$

If $R=+\infty$, then $S(r)$ satisfies $S(r)=O\{\log T(r,f)\}+O(\log r)$ as $r\to+\infty$ without exception if $f(z)$ has finite order and otherwise as $r\to+\infty$ outside a set J of exceptional intervals of finite measure $\int_J dr<+\infty$. If $0<R<+\infty$, then

$$S(r)=O\left\{\log^+ T(r,f)\right\}+O\left\{\log\frac{1}{R-r}\right\}$$

holds as $r \to R$ without exception if f has finite order

$$\rho = \limsup_{r\to R} \frac{\log T(r,f)}{\log(1/R-r)},$$

and otherwise as $r \to R$ outside of a set J exceptional intervals such that $\int_J d\frac{1}{R-r} < +\infty$. In all cases, the exceptional set J is independent of the choice of the finite points $a^{[k]} \in E$ and of their number.

THEOREM 3 ([2]). (E-valued Nevanlinna deficiency relation) Let $f(z)$ be an E-valued meromorphic function and admissible with the property of compact projection. Then the set $\{a \in E \cup \{\infty\} : \Theta(a) > 0\}$ is at most countable and summing over all such points

$$\sum_a [\delta(a) + \theta(a)] + \delta_G \leq \sum_a \Theta(a) + \delta_G \leq 2.$$

THEOREM 4 (Lemma 3.1(A) of [1]) Let $f(z)$ be an E-valued meromorphic function with the property of compact projection, and let

$$\begin{aligned} S_1(r) &= \frac{1}{2\pi}\int_0^{2\pi} \log^+ \frac{\left\|f'(re^{i\phi})\right\|}{\left\|f(re^{i\phi})\right\|} d\phi + \frac{1}{2\pi}\int_0^{2\pi} \log^+ \left[F(re^{i\phi})\left\|f(re^{i\phi})\right\|\right] d\phi \\ &\quad + p\log^+ \frac{2p}{\delta} - \log\left\|c'_{l'}\right\|. \end{aligned}$$

Then

$$G(r) + \sum_{k=1}^{p+1} m(r, a^{[k]}) + N_1(r) \leq 2T(r,f) + S_1(r),$$

where $N_1(r) = N(r,0,f') + 2N(r,f) - N(r,f')$ is the generalized counting function of multiple points, $a^{[\nu]} = (a_1^{[\nu]}, \ldots, a_j^{[\nu]}, \ldots)(p \geq 2) \in E$ are distinct finite points, and

$$F(z) = \sum_{\nu=1}^{p} \frac{1}{\left\|f(z) - a^{[\nu]}\right\|}.$$

3 Main Results

S. K. Singh and H. S. Gopalkrishna [4] proved the following result:

THEOREM 5. If f is a non-constant meromorphic function of order ρ then

$$\liminf_{r\to\infty} \frac{T(r,f')}{T(r,f)} \geq \sum_{a\in\mathbb{C}} \Theta(a,f)$$

where $r \to \infty$ without restriction if ρ is finite and $r \to \infty$ outside an exceptional set of finite measure if $\rho = +\infty$.

In [5], Z. Wu and Y. Chen proved the following result.

THEOREM 6**.** Let $f(z)$ be an admissible E-valued meromorphic function of compact projection in $\mathbb{C}$ of finite order and assume $\sum_a \delta(a) = 2$. Then

$$\lim_{r\to+\infty} \frac{T(r,f')}{T(r,f)} = 2 - \delta(\infty).$$

Now in this article, we obtain a THEOREM 5 for E-valued meromorphic function $f(z)$ in modified form and also extend the related results of S. K. Singh and H. S. Gopalakrishna [4]. THEOREM 6 is also proved as a consequence of our main result. We prove the following main results.

THEOREM 7**.** Let $f(z)$ be an admissible and non-constant E-valued meromorphic function of finite order ρ with compact projection then

$$\sum_{a\in E} \Theta(a,f) + \delta_G \leq \liminf_{r\to+\infty} \frac{T(r,f')}{T(r,f)},$$

where $r \to +\infty$ without restriction if ρ is finite and $r \to +\infty$ outside an exceptional set of finite measure if $\rho = +\infty$.

To prove THEOREM 7, we first prove the following Lemma, which plays an prominent role in the proof of the THEOREM 7**.**

LEMMA 1. Let $f(z)$ be a non-constant E-valued meromorphic function with the property of compact projection in C_r and

$$a^{[\gamma]} = \left(a_1^{[\gamma]}, a_2^{[\gamma]}, \ldots, a_j^{[\gamma]}, \ldots\right) (p \geq 2) \in E$$

are finite or infinite distinct points then

$$\sum_{\mu=1}^{p} m(r, a^{[\mu]}, f) + N\left(r, \frac{1}{f'}\right) + G(r,f) \leq T(r,f') + S(r,f),$$

where

$$S(r,f) = \frac{1}{2\pi}\int_0^{2\pi} \log^+ \left\{F(re^{i\phi}) \left\|f'^{i\phi})\right\|\right\} d\phi - \log\left\|c_p'\right\| + p\log^+ \frac{2p}{\delta}$$

and

$$F(z) = \sum_{\nu=1}^{p} \frac{1}{\left\|f(z) - a^{[\nu]}\right\|}.$$

PROOF. Following the proof of Lemma 3.1(A) in [1], we obtain the required result.

PROOF OF THEOREM 7. Let $\{a^{[\mu]}\}$, $\mu = 1, 2, \ldots, \infty$ be an infinite sequence of distinct elements of E, which includes every $a \in E$ for which $\Theta(a, f) > 0$. Then

$$\sum_{\mu=1}^{\infty} \Theta\left(a^{[\mu]}, f\right) = \sum_{a \in E} \Theta(a, f). \tag{1}$$

We have

$$\sum_{\mu=1}^{p} m(r, a^{[\mu]}, f) + G(r, f) \leq T(r, f') - N\left(r, \frac{1}{f'}\right) + S(r, f).$$

Adding $\sum_{\mu=1}^{p} N\left(r, a^{[\mu]}, f\right)$ to both sides, we obtain

$$\begin{aligned}\sum_{\mu=1}^{p} T(r, a^{[\mu]}, f) + G(r, f) &\leq T(r, f') + \sum_{\mu=1}^{p} N\left(r, a^{[\mu]}, f\right) - N\left(r, \frac{1}{f'}\right) + S(r, f) \\ &= T(r, f') + \sum_{\mu=1}^{p} \overline{N}\left(r, a^{[\mu]}, f\right) - N_0\left(r, \frac{1}{f'}\right) + S(r, f),\end{aligned}$$

where $N_0\left(r, \frac{1}{f'}\right)$ is formed with the zeros of f' which are not zeros of any of $f - a^{[\mu]}$ $(\mu = 1, 2, \ldots, p)$. Since $N_0\left(r, \frac{1}{f'}\right) \geq 0$, we have

$$\sum_{\mu=1}^{p} T(r, a^{[\mu]}, f) \leq T(r, f') + \sum_{\mu=1}^{p} \overline{N}\left(r, a^{[\mu]}, f\right) - G(r, f) + S(r, f).$$

By an E-valued Nevanlinna's first fundamental theorem, we have

$$T(r, a, f) = T(r, f) - V(r, a, f) + O(1).$$

Using this in the above equation, we obtain

$$\sum_{\mu=1}^{p} \left[T(r, f) - V(r, a^{[\mu]}, f) + O(1)\right] \leq T(r, f') + \sum_{\mu=1}^{p} \overline{N}\left(r, a^{[\mu]}, f\right) - G(r, f) + S(r, f).$$

We further obtain

$$pT(r, f) \leq T(r, f') + \sum_{\mu=1}^{p} \left[\overline{N}\left(r, a^{[\mu]}, f\right) + V(r, a^{[\mu]}, f)\right] - G(r, f) + S(r, f).$$

Then

$$\begin{aligned}p \quad &\leq \quad \liminf_{r\to+\infty} \frac{T(r, f')}{T(r, f)} + \sum_{\mu=1}^{p} \limsup_{r\to+\infty} \frac{\overline{N}\left(r, a^{[\mu]}, f\right) + V(r, a^{[\mu]}, f)}{T(r, f)} - \liminf_{r\to+\infty} \frac{G(r, f)}{T(r, f)} \\ &\quad + \limsup_{r\to+\infty} \frac{S(r, f)}{T(r, f)}.\end{aligned}$$

It follows that

$$p \le \liminf_{r\to+\infty} \frac{T(r,f')}{T(r,f)} + \sum_{\mu=1}^{p} \left[1 - \Theta(a^{[\mu]}, f)\right] - \delta_G(f).$$

So

$$\sum_{\mu=1}^{p} \Theta(a^{[\mu]}, f) + \delta_G(f) \le \liminf_{r\to+\infty} \frac{T(r,f')}{T(r,f)}.$$

Letting $p \to \infty$ and using (1), we get

$$\sum_{a\in E} \Theta(a,f) + \delta_G(f) \le \liminf_{r\to+\infty} \frac{T(r,f')}{T(r,f)}. \qquad (2)$$

COROLLARY 1. Let $f(z)$ be a admissible E-valued meromorphic function of finite order ρ with the property of compact projection such that

$$\sum_{a\in \overline{E}} \Theta(a,f) + \delta_G = 2, \qquad \overline{E} = E \cup \{\infty\}.$$

Then

(i)

$$\lim_{r\to+\infty} \frac{T(r,f')}{T(r,f)} = 2 - \Theta(\infty, f).$$

(ii)

$$\begin{aligned} 1 - \Theta(a,f) + \delta_G &\le \liminf_{r\to+\infty} \frac{V(r,a) + \overline{N}(r,a)}{T(r,f)} \le \limsup_{r\to+\infty} \frac{V(r,a) + \overline{N}(r,a)}{T(r,f)} \\ &= 1 - \Theta(a,f). \end{aligned}$$

PROOF. Given that

$$\sum_{a\in \overline{E}} \Theta(a,f) + \delta_G = 2,$$

we have

$$\sum_{a\in E} \Theta(a,f) + \Theta(\infty,f) + \delta_G = 2.$$

It follows that

$$\sum_{a\in E} \Theta(a,f) + \delta_G = 2 - \Theta(\infty,f).$$

Using (2), we write

$$\liminf_{r\to+\infty} \frac{T(r,f')}{T(r,f)} \ge \sum_{a\in E} \Theta(a,f) + \delta_G = 2 - \Theta(\infty,f).$$

On the other hand, we know that

$$\begin{aligned} T(r, f') &= m(r, f') + N(r, f') = m(r, \frac{f'}{f}) + m(r, f) + N(r, f') \\ &\leq T(r, f) + \overline{N}(r, f) + S(r, f) \end{aligned}$$

and

$$\limsup_{r\to+\infty} \frac{T(r, f')}{T(r, f)} \leq 1 + \limsup_{r\to+\infty} \frac{\overline{N}(r, f)}{T(r, f)}.$$

So

$$\limsup_{r\to+\infty} \frac{T(r, f')}{T(r, f)} \leq 2 - \Theta(\infty, f).$$

Thus

$$\lim_{r\to+\infty} \frac{T(r, f')}{T(r, f)} = 2 - \Theta(\infty, f).$$

(ii) Let $a \in E \cup \{\infty\}$ and $\left\{a^{[k]}\right\}, k = 1, 2, \ldots \infty$ be an infinite sequence of distinct elements of $E \cup \{\infty\}$ which includes every $b \in E \cup \{\infty\}$ such that $b \neq a$ and $\Theta(b, f) \neq 0$. Then

$$\sum_{k=1}^{\infty} \Theta\left(a^{[k]}, f\right) = \sum_{b\in\overline{E}, b\neq a} \Theta(b, f) = 2 - \Theta(a, f). \tag{3}$$

By E-valued Nevanlinna's second fundamental theorem, we have

$$\begin{aligned} (q-2)T(r, f) + G(r, f) \leq & \sum_{k=1}^{q-1} \left[V(r, a^{[k]}, f) + \overline{N}(r, a^{[k]}, f)\right] \\ & + \left[V(r, a, f) + \overline{N}(r, a, f)\right] + S(r, f), \end{aligned}$$

$$\begin{aligned} (q-2)T(r, f) \leq & \sum_{k=1}^{q-1} \left[V(r, a^{[k]}, f) + \overline{N}(r, a^{[k]}, f)\right] + \left[V(r, a, f) + \overline{N}(r, a, f)\right] \\ & -G(r, f) + S(r, f), \end{aligned}$$

$$\begin{aligned} (q-2)T(r, f) \leq & \sum_{k=1}^{q-1} \frac{\left[V(r, a^{[k]}, f) + \overline{N}(r, a^{[k]}, f)\right]}{T(r, f)} + \frac{\left[V(r, a, f) + \overline{N}(r, a, f)\right]}{T(r, f)} \\ & -\frac{G(r, f)}{T(r, f)} + \frac{S(r, f)}{T(r, f)}, \end{aligned}$$

$$\begin{aligned} (q-2) \leq & \sum_{k=1}^{q-1} \limsup_{r\to+\infty} \frac{\left[V(r, a^{[k]}, f) + \overline{N}(r, a^{[k]}, f)\right]}{T(r, f)} \\ & + \liminf_{r\to+\infty} \frac{\left[V(r, a, f) + \overline{N}(r, a, f)\right]}{T(r, f)} - \liminf_{r\to+\infty} \frac{G(r, f)}{T(r, f)} + \limsup_{r\to+\infty} \frac{S(r, f)}{T(r, f)}, \end{aligned}$$

$$(q-2) \leq \liminf_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)} + \sum_{k=1}^{q-1}[1-\Theta\left(a^{[k]},f\right)] - \delta_G,$$

$$(q-2)+\delta_G \leq \liminf_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)} + (q-1) - \sum_{k=1}^{q-1}\Theta\left(a^{[k]},f\right),$$

$$\delta_G - 1 \leq \liminf_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)} - \sum_{k=1}^{q-1}\Theta\left(a^{[k]},f\right).$$

So

$$\liminf_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)} \geq \sum_{k=1}^{q-1}\Theta\left(a^{[k]},f\right) + \delta_G - 1.$$

Let $q \to \infty$ and using (3), we get

$$\begin{aligned}\liminf_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)} &\geq \sum_{k=1}^{\infty}\Theta\left(a^{[k]},f\right) + \delta_G - 1\\ &= 2-\Theta(a,f)+\delta_G-1 = 1-\Theta(a,f)+\delta_G.\end{aligned}$$

On the other hand, by the definition of $\Theta(a,f)$, we have

$$\limsup_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)} = 1-\Theta(a,f).$$

Thus

$$\begin{aligned}1-\Theta(a,f)+\delta_G &\leq \liminf_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)}\\ &\leq \limsup_{r\to+\infty} \frac{\left[V(r,a,f)+\overline{N}(r,a,f)\right]}{T(r,f)} = 1-\Theta(a,f).\end{aligned}$$

COROLLARY 2 Let $f(z)$ be a admissible E-valued meromorphic function of finite order ρ with the property of compact projection such that

$$\sum_{a\in\overline{E}}\delta(a,f)+\delta_G = 2.$$

Then

$$\lim_{r\to+\infty}\frac{T(r,f')}{T(r,f)} = 2-\delta(\infty,f).$$

PROOF. We know that $\delta(a,f)\leq\Theta(a,f)$, $\forall a\in E\cup\{\infty\}=\overline{E}$ and

$$\sum\delta(a,f)+\delta_G \leq \sum\Theta(a,f)+\delta_G \leq 2.$$

Given $\sum \delta(a,f)+\delta_G=2$. Then $\sum \Theta(a,f)+\delta_G=2$. We observe that

$$\sum \delta(a,f)+\delta_G=\sum \Theta(a,f)+\delta_G=2.$$

Then

$$\sum_{a\in\overline{E}} \delta(a,f)=\sum_{a\in\overline{E}} \Theta(a,f).$$

So

$$\delta(a,f)=\Theta(a,f) \qquad \forall a\in\overline{E}.$$

By using Corollary 1(i), we have

$$\lim_{r\to+\infty}\frac{T(r,f')}{T(r,f)}=2-\Theta(\infty,f)=2-\delta(\infty,f).$$

So

$$\lim_{r\to+\infty}\frac{T(r,f')}{T(r,f)}=2-\delta(\infty,f)$$

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