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Asymptotic Bounds For Solutions Of A Periodic Reaction Diffusion System*

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Abstract

The purpose of this paper is to investigate the asymptotic behavior of nonnegative solutions of a periodic reaction diffusion system. By De Giorgi iteration technique, we obtain the a priori upper bound of nonnegative periodic solutions of the considered periodic system. We then establish the existence of the maximum periodic solution and asymptotic bounds of nonnegative solutions of the initial boundary value problem.

1 Introduction

In this paper, we consider the following periodic reaction diffusion system

$$\frac{\partial u}{\partial t} = \operatorname{div}(|\nabla u|^{p_1-2}\nabla u) + b_1 u^{\alpha_1} v^{\beta_1}, \qquad (x,t) \in \Omega \times \mathbb{R}^+, \tag{1}$$

$$\frac{\partial v}{\partial t} = \operatorname{div}(|\nabla v|^{p_2-2}\nabla u) + b_2 u^{\alpha_2} v^{\beta_2}, \qquad (x,t) \in \Omega \times \mathbb{R}^+, \tag{2}$$

$$u(x,t) = v(x,t) = 0, \qquad (x,t) \in \partial\Omega \times \mathbb{R}^+, \tag{3}$$

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \qquad x \in \Omega, \tag{4}$$

where $p_1,\ p_2 > 2$, $\alpha_1,\ \alpha_2,\ \beta_1,\ \beta_2 \geq 1$, $\alpha_1 < p_1 - 1$, $\beta_2 < p_2 - 1$, $\beta_1 < p_1 - \alpha_1 - 1$, $\alpha_2 < p_2 - \beta_2 - 1$, $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary, $b_1 = b_1(x,t)$ and $b_2 = b_2(x,t)$ are nonnegative continuous functions and T-periodic ($T > 0$) with respect to t, and u_0, v_0 are nonnegative bounded smooth functions.

In recent years, periodic reaction diffusion equations and systems are of particular interests since they can take into account periodic fluctuations occurring in the phenomena appearing in the models, and have been extensively studied by many researchers (see e.g. [1]–[5]). The models for the evolution of the biological species living in the periodic environment are often described by coupled systems of periodic nonlinear diffusion equations, and therefore it is important to study the existence and asymptotic behavior of solutions of these systems. To our knowledge, however, it seems that there are few papers that deal with the asymptotic behavior of periodic-parabolic systems with degeneracy.

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This work is an extension of [6]. We establish the existence of nontrivial nonnegative periodic solutions of the problem (1)-(4) and their asymptotic behavior. Since (1) and (2) have periodic sources, it is not appropriate to consider the steady state approach and we shall seek some new approaches. Our idea is to consider all nonnegative periodic solutions, which will be showed to have a priori upper bound C_0 according to the maximum norm. Then by monotonicity method we show the existence of the maximum periodic solution and asymptotic bounds of nonnegative solutions of the initial boundary value problem.

2 Preliminaries

Since (1) and (2) are degenerate whenever $|\nabla u| = |\nabla v| = 0$, we focus our main efforts on the discussion of weak solutions.

DEFINITION 1. A vector-valued function (u, v) is said to be a weak upper-solution to the problem (1)-(4) in $Q_\tau = \Omega \times (0, \tau)$ with $\tau > 0$, if

$$u \in L^{p_1}(0, \tau; W^{1,p_1}(\Omega)) \cap L^\infty(Q_\tau), \quad v \in L^{p_2}(0, \tau; W^{1,p_2}(\Omega)) \cap L^\infty(Q_\tau),$$

and for any nonnegative function $\varphi \in C^1(\overline{Q}_\tau)$ with $\varphi|_{\partial\Omega\times[0,\tau)} = 0$, we have

$$\begin{cases} \displaystyle\int_\Omega u(x,\tau)\varphi(x,\tau)dx - \int_\Omega u_0(x)\varphi(x,0)dx - \iint_{Q_\tau} u\frac{\partial\varphi}{\partial t}dxdt \\ \displaystyle + \iint_{Q_\tau} |\nabla u|^{p_1-2}\nabla u\nabla\varphi dxdt \geq \iint_{Q_\tau} b_1 u^{\alpha_1} v^{\beta_1}\varphi dxdt, \\ \displaystyle\int_\Omega v(x,\tau)\varphi(x,\tau)dx - \int_\Omega v_0(x)\varphi(x,0)dx - \iint_{Q_\tau} v\frac{\partial\varphi}{\partial t}dxdt \\ \displaystyle + \iint_{Q_\tau} |\nabla v|^{p_2-2}\nabla v\nabla\varphi dxdt \geq \iint_{Q_\tau} b_2 u^{\alpha_2} v^{\beta_2}\varphi dxdt, \\ u(x,t) \geq 0, \quad v(x,t) \geq 0, \qquad (x,t) \in \partial\Omega \times (0,\tau), \\ u(x,0) \geq u_0(x), \quad v(x,0) \geq v_0(x), \qquad x \in \Omega. \end{cases}$$

Replacing "$\geq$" by "$\leq$" in the above inequalities, it follows the definition of a weak lower-solution. Furthermore, if (u, v) is a weak upper-solution as well as a weak lower-solution, then we call it a weak solution of the problem (1)-(4).

DEFINITION 2. A vector-valued function (u, v) is said to be a T-periodic solution of the problem (1)-(3), if it is a solution such that $u(\cdot, 0) = u(\cdot, T)$, $v(\cdot, 0) = v(\cdot, T)$ in Ω. A vector-valued function $(\overline{u}, \overline{v})$ is said to be a T-periodic upper-solution of the problem (1)-(3), if it is an upper-solution such that $\overline{u}(\cdot, 0) \geq \overline{u}(\cdot, T)$, $\overline{v}(\cdot, 0) \geq \overline{v}(\cdot, T)$ in Ω. A vector-valued function $(\underline{u}, \underline{v})$ is said to be a T-periodic lower-solution of the problem (1)-(3), if it is a lower-solution such that $\underline{u}(\cdot, 0) \leq \underline{u}(\cdot, T)$, $\underline{v}(\cdot, 0) \leq \underline{v}(\cdot, T)$ in Ω. A pair of T-periodic upper-solution $(\overline{u}, \overline{v})$ and T-periodic lower-solution $(\underline{u}, \underline{v})$ are said to be ordered if $\overline{u} \geq \underline{u}$, $\overline{v} \geq \underline{v}$ in $\overline{Q}_T = \overline{\Omega} \times (0, T)$.

LEMMA 1 ([6]). Let $(\underline{u}, \underline{v})$ be a lower-solution of the problem (1)-(4) with the initial value $(\underline{u}_0, \underline{v}_0)$, and $(\overline{u}, \overline{v})$ an upper-solution of the problem (1)-(4) with the initial value $(\overline{u}_0, \overline{v}_0)$. Then $\underline{u} \leq \overline{u}$, $\underline{v} \leq \overline{v}$ *a.e.* in Q_T if $\underline{u}_0 \leq \overline{u}_0$, $\underline{v}_0 \leq \overline{v}_0$ *a.e.* in Ω .

LEMMA 2. (see [6]) For any nonnegative bounded initial value, the problem (1)-(4) admits a global nonnegative solution, and the problem (1)-(3) admits a nontrivial nonnegative periodic solution.

The main results of this paper is the following theorem.

THEOREM 1. The problem (1)-(3) admits a maximal periodic solution (U, V). Moreover, if (u, v) is the solution of the initial boundary value problem (1)-(4) with nonnegative initial value (u_0, v_0), then for any $\varepsilon > 0$, there exists t_1 depending on u_0 and ε, t_2 depending on v_0 and ε, such that

$$0 \le u \le U + \varepsilon, \quad \text{for} \quad x \in \Omega, \quad t \ge t_1,$$

$$0 \le v \le V + \varepsilon, \quad \text{for} \quad x \in \Omega, \quad t \ge t_2.$$

3 Proofs

In this section, we prove the main results of this paper. Firstly, we establish some important estimates on nonnegative periodic solutions of the problem (1)-(3).

LEMMA 3. Let (u, v) be a nonnegative periodic solution of the problem (1)-(3). Then there exists positive constants r and s such that

$$\frac{\alpha_2}{p_2 - \beta_2 - 1} < \frac{p_1 + r - 1}{p_2 + s - 1} < \frac{p_1 - \alpha_1 - 1}{\beta_1}$$

and

$$\|u\|_{L^r(Q_T)} \le C, \quad \|v\|_{L^s(Q_T)} \le C. \tag{5}$$

In addition, we have

$$\iint_{Q_T} |\nabla u|^{p_1} dxdt \le C, \quad \iint_{Q_T} |\nabla v|^{p_2} dxdt \le C, \tag{6}$$

where C is a positive constant depending on p_1, p_2, α_1, β_2, r, s and $|\Omega|$.

PROOF. For $r > 1$, multiplying (1) by u^{r-1} and integrating over Q_T, we deduce

$$\iint_{Q_T} \frac{\partial u}{\partial t} u^{r-1} dxdt + \iint_{Q_T} |\nabla u|^{p_1 - 2} \nabla u \nabla u^{r-1} dxdt = \iint_{Q_T} b_1 u^{\alpha_1 + r - 1} v^{\beta_1} dxdt.$$

By the periodic boundary value condition, we see that the first term of the left hand side of the above equality vanishes. That is

$$(r-1) \left(\frac{p_1}{p_1 + r - 2} \right)^{p_1} \iint_{Q_T} |\nabla u^{\frac{p_1 + r - 2}{p_1}}|^{p_1} dxdt = \iint_{Q_T} b_1 u^{\alpha_1 + r - 1} v^{\beta_1} dxdt.$$

Then we have

$$\iint_{Q_T} |\nabla u^{\frac{p_1 + r - 2}{p_1}}|^{p_1} \le \frac{B_1}{r-1} \left(\frac{p_1 + r - 2}{p_1} \right)^{p_1} \iint_{Q_T} u^{\alpha_1 + r - 1} v^{\beta_1} dxdt,$$

where B_1 denotes the maximum of $b_1(x,t)$ in Q_T. By using Poincaré's inequality, we obtain

$$\begin{aligned}\iint_{Q_T} u^{p_1+r-2}dxdt \leq & C_{p_1}\iint_{Q_T} |\nabla u^{\frac{p_1+r-2}{p_1}}|^{p_1}dxdt \\ \leq & \frac{C_{p_1}B_1}{r-1}\left(\frac{p_1+r-2}{p_1}\right)^{p_1}\iint_{Q_T} u^{\alpha_1+r-1}v^{\beta_1}dxdt,\end{aligned} \tag{7}$$

where C_{p_1} is a constant depending only on $|\Omega|$ and N, and it becomes very large when the measure of the domain Ω becomes small. Notice that $\beta_1 < p_1-\alpha_1-1$ implies $\alpha_1 < p_1-1$, then $\alpha_1+r-1 < p_1+r-2$. According to Young's inequality, we have

$$u^{\alpha_1+r-1}v^{\beta_1} \leq \varepsilon_1 u^{p_1+r-2} + C(\varepsilon_1)v^{\frac{\beta_1(p_1+r-2)}{p_1-\alpha_1-1}},$$

where $\varepsilon_1 > 0$ and $C(\varepsilon_1)$ are constants of Young's inequality. Take

$$\varepsilon_1 = \frac{1}{2}\frac{r-1}{C_{p_1}B_1}\left(\frac{p_1}{p_1+r-2}\right)^{p_1},$$

from (7) we have

$$\iint_{Q_T} u^{p_1+r-2}dxdt \leq \frac{1}{2}\iint_{Q_T} u^{p_1+r-2}dxdt + C_1\iint_{Q_T} v^{\frac{\beta_1(p_1+r-2)}{p_1-\alpha_1-1}}dxdt.$$

That is

$$\iint_{Q_T} u^{p_1+r-2}dxdt \leq C\iint_{Q_T} v^{\frac{\beta_1(p_1+r-2)}{p_1-\alpha_1-1}}dxdt. \tag{8}$$

Also, we can get an similar estimate on v^s for $s > 1$, and hence

$$\begin{aligned}&\iint_{Q_T} u^{p_1+r-2}dxdt + \iint_{Q_T} v^{p_2+s-2}dxdt \\ \leq & C\iint_{Q_T} v^{\frac{\beta_1(p_1+r-2)}{p_1-\alpha_1-1}}dxdt + C\iint_{Q_T} u^{\frac{\alpha_2(p_2+s-2)}{p_2-\beta_2-1}}dxdt.\end{aligned} \tag{9}$$

Since $\beta_1 < p_1-\alpha_1-1$, $\alpha_2 < p_2-\beta_2-1$, there must exist $r \geq \max\{2(\alpha_1+1), 2\alpha_2\}$ and $s \geq \max\{2(\beta_2+2), 2\beta_1\}$ such that

$$\frac{\beta_1}{p_1-\alpha_1-1} < \frac{p_2+s-2}{p_1+r-2} < \frac{p_2-\beta_2-1}{\alpha_2}.$$

By Young's inequality, we have

$$\begin{aligned}\iint_{Q_T} u^{\frac{\alpha_2(p_2+s-2)}{p_2-\beta_2-1}}dxdt &\leq \varepsilon_2\iint_{Q_T} u^{p_1+r-2}dxdt + C(\varepsilon_2)|Q_T|, \\ \iint_{Q_T} v^{\frac{\beta_1(p_1+r-2)}{p_1-\alpha_1-1}}dxdt &\leq \varepsilon_3\iint_{Q_T} v^{p_2+s-2}dxdt + C(\varepsilon_3)|Q_T|.\end{aligned}$$

Take $\varepsilon_2 = \frac{1}{2C_2}$, $\varepsilon_3 = \frac{1}{2C_1}$, it follows from (9) that

$$\iint_{Q_T} u^{p_1+r-2} dxdt + \iint_{Q_T} v^{p_2+s-2} dxdt \le C.$$

Thus we complete the proof of inequality (5).

Now we show the proof of (6). Multiplying (1) by u and integrating over Q_T, by the periodic boundary value condition and Hölder's inequality, we have

$$\begin{aligned}\iint_{Q_T} |\nabla u|^{p_1} dxdt \le& C \iint_{Q_T} u^{\alpha_1+1} v^{\beta_1} dxdt \\ \le& C \left(\iint_{Q_T} u^{2(\alpha_1+1)} dxdt\right)^{1/2} \left(\iint_{Q_T} v^{2\beta_1} dxdt\right)^{1/2}.\end{aligned}$$

Due to $r \ge \max\{2(\alpha_1+1), 2\beta_2\}$, $s \ge \max\{2(\beta_2+1), 2\alpha_1\}$, the first inequality in (6) follows from (5) immediately. The same is true for the second inequality. The proof is completed.

In the following we show the uniform upper bound of the maximum norm of nonnegative periodic solutions.

LEMMA 4. Let (u, v) be a nonnegative periodic solution of (1)-(3). Then there is a positive constant C_0 such that

$$\|u\|_{L^\infty(Q_T)} \le C_0, \quad \|v\|_{L^\infty(Q_T)} \le C_0. \tag{10}$$

PROOF. Denote $s_+ = \max\{s, 0\}$ and take k be a determined positive constant. Multiplying (1) by $(u-k)_+$ and integrating over Q_T, we have

$$\begin{aligned}&\frac{1}{2}\iint_{Q_T} \frac{\partial}{\partial t}(u-k)_+^2 dxdt + \iint_{Q_T} |\nabla (u-k)_+|^{p_1} dxdt \\ \le& C \iint_{Q_T} u^{\alpha_1} v^{\beta_1} (u-k)_+ dxdt.\end{aligned} \tag{11}$$

Denote $\mu(k) = \mathrm{mes}\{(x,t) \in Q_T : u(x,t) > k\}$. Combine Lemma 3 (with r and s large enough) with Young's and Hölder's inequalities, we have

$$\begin{aligned}&\frac{1}{2}\iint_{Q_T} \frac{\partial}{\partial t}(u-k)_+^2 dxdt + \iint_{Q_T} |\nabla (u-k)_+|^{p_1} dxdt \\ \le& C \left(\iint_{Q_T} (u^{\alpha_1} v^{\beta_1})^{\xi'} dxdt\right)^{\xi'} \left(\iint_{Q_T} (u-k)_+^{\xi} dxdt\right)^{1/\xi} \\ \le& C \left(\iint_{Q_T} (u-k)_+^{\xi\eta} dxdt\right)^{1/\xi\eta} \mu(k)^{(1-\frac{1}{\eta})\frac{1}{\xi}},\end{aligned} \tag{12}$$

where constants $\xi, \eta > 1$ are to be determined. By Nirenberg-Gagliardo's inequality and Lemma 3, we have

$$\left(\iint_{Q_T} (u-k)_+^{\xi\eta} dxdt\right)^{1/\xi\eta} \le C \left(\iint_{Q_T} |\nabla (u-k)_+|^{p_1} dxdt\right)^{\theta/p_1}, \tag{13}$$

where
$$\theta = \left(1 - \frac{1}{\xi\eta}\right)\left(\frac{1}{N} - \frac{1}{p_1} + 1\right)^{-1}.$$
Combining (11) with (12) and (13), we have
$$\iint_{Q_T} |\nabla(u-k)_+|^{p_1} dxdt \leq C \left(\iint_{Q_T} |\nabla(u-k)_+|^{p_1} dxdt\right)^{\frac{\theta}{p_1}} \mu(k)^{(1-\frac{1}{\eta})\frac{1}{\xi}}. \tag{14}$$
Set
$$w(k) = \iint_{Q_T} |\nabla(u-k)_+|^{p_1} dxdt,$$
from (14) we obtain
$$w(k) \leq C\mu(k)^{\frac{p_1}{p_1-\theta}(1-\frac{1}{\eta})\frac{1}{\xi}}. \tag{15}$$
Take $k_h = M(2-2^{-h})$, $h = 0, 1, \ldots$, and $M > 0$ to be determined. It follows from (13) that
$$(k_{h+1} - k_h)^{\xi\eta}\mu(k_{h+1}) \leq \iint_{Q_T} (u-k_h)_+^{\xi\eta} dxdt \leq Cw(k_h)^{\frac{\xi\eta\theta}{p_1}}.$$
Combining the above inequality with (15) we obtain
$$\mu(k_{h+1}) \leq C4^{h\xi\eta}\mu(k_h)^{\frac{\theta(\eta-1)}{p_1-\theta}} = Cb^h\mu(k_h)^\gamma,$$
where $b = 4^{\xi\eta}$ and $\gamma = \frac{(\eta-1)(\xi\eta-1)N}{\xi\eta N(p_1-2)+\xi\eta p_1+N}$. For any $\xi > 1$, take η be a positive constant satisfying
$$\eta > \max\{p_1, \frac{\xi p_1 + N}{\xi N} + p_1 - 1\},$$
then we have $\gamma > 1$. By Lemma 3, we can select M large enough such that
$$\mu(k_0) = \mu(M) \leq C^{-\frac{1}{\gamma-1}} b^{-\frac{1}{(\gamma-1)^2}}.$$
According to Lemma 5.6 in [7, p. 95], we have $\mu(k_h) \to 0$, as $h \to +\infty$, which implies $u(x,t) \leq 2M$ in Q_T.

Similarly, we can get the uniform upper bound estimate for $\|v\|_{L^\infty(Q_T)}$. The proof is completed.

PROOF OF THEOREM 1. Firstly, we establish the existence of the maximal periodic solution of the periodic boundary value problem. Define a Poincaré map $P = (P_1, P_2) : C(\overline{\Omega}) \times C(\overline{\Omega}) \to C(\overline{\Omega}) \times C(\overline{\Omega})$ with $P(u_0(x), v_0(x)) = (u(x,T), v(x,T))$, where $(u(x,t), v(x,t))$ is the solution of the initial boundary value problem (1)-(4) with initial value $(u_0(x), v_0(x))$. A similar argument as [6] shows that the map P is well defined.

Let λ_i $(i = 1, 2)$ be the first eigenvalue of the problem
$$\begin{cases} -\mathrm{div}(|\nabla\varphi|^{p_i-2}\nabla\varphi) = \mu|\varphi|^{p_i-2}\varphi, & x \in \Omega' \\ \varphi = 0, & x \in \partial\Omega', \end{cases}$$

and φ_i be the corresponding positive eigenfunction, where $\Omega' \supset\supset \Omega$. It is easy to see if $x \in \overline{\Omega}$, then $\varphi_1(x) > 0$ and $\varphi_2(x) > 0$, that is $\min\limits_{\overline{\Omega}} \varphi_1(x) > 0$ and $\min\limits_{\overline{\Omega}} \varphi_2(x) > 0$. Let $(u_n(x,t), v_n(x,t))$ be the solution of the problem (1)-(4) with initial value

$$(u_0(x), v_0(x)) = (\overline{u}(x), \overline{v}(x)) = (K_1\varphi_1(x), K_2\varphi_2(x)),$$

where $K_1, K_2 > 0$ are taken as [6] such that $(\overline{u}(x), \overline{v}(x))$ be a T-periodic upper-solution. Then we have $(u_n(x,T), v_n(x,T)) = P^n(\overline{u}(x), \overline{v}(x))$ and

$$u_{n+1}(x,t) \le u_n(x,t) \le \overline{u}(x), \quad v_{n+1}(x,t) \le v_n(x,t) \le \overline{v}(x),$$

by comparison principle. By a rather standard argument, we conclude that there exist $u^*(x)$, $v^*(x) \in C(\overline{\Omega})$ and a subsequence of $\{P^n(\overline{u}(x), \overline{v}(x))\}$, denoted by itself for simplicity, such that

$$(u^*(x), v^*(x)) = \lim_{n\to\infty} P^n(\overline{u}(x), \overline{v}(x)).$$

Similar to the proof in [6], we can prove that $(U(x,t), V(x,t))$, which is the even extension of the solution of the initial boundary value problem (1)-(4) with initial value $(u^*(x), v^*(x))$, is a periodic solution of (1)-(3). Moreover, by Lemma 4, we see that any nonnegative periodic solution $(u(x,t), v(x,t))$ of (1)-(3) must satisfy

$$\|u(x,t)\|_{L^\infty(Q_T)} \le C_0, \quad \|v(x,t)\|_{L^\infty(Q_T)} \le C_0.$$

Therefore, if we take K_1, K_2 also satisfy

$$K_1 \ge \frac{C_0}{\min\limits_{x\in\overline{\Omega}} \varphi_1(x)}, \quad K_2 \ge \frac{C_0}{\min\limits_{x\in\overline{\Omega}} \varphi_2(x)},$$

by the comparison principle and $u^*(x) \ge u(x,0), v^*(x) \ge v(x,0)$, we obtain $U(x,t) \ge u(x,t)$, $V(x,t) \ge v(x,t)$, which means that $(U(x,t), V(x,t))$ is the maximal periodic solution of (1)-(3).

Let $(u(x,t), v(x,t))$ be the solution of the initial boundary problem (1)-(4) with given nonnegative initial value $(u_0(x), v_0(x))$, $(\omega_1(x,t), \omega_2(x,t))$ be the solution of (1)-(4) with initial value $(\omega_1(x,0), \omega_2(x,0)) = (R_1\varphi_1(x), R_2\varphi_2(x))$, where R_1, R_2 are positive constants satisfying the same conditions as K_1, K_2 and also

$$R_1 \ge \frac{\|u_0\|_{L^\infty}}{\min\limits_{x\in\overline{\Omega}} \varphi_1(x)}, \quad R_2 \ge \frac{\|v_0\|_{L^\infty}}{\min\limits_{x\in\overline{\Omega}} \varphi_2(x)}.$$

Then we have

$$u(x, t+kT) \le w_1(x, t+kT), \quad v(x, t+kT) \le w_2(x, t+kT)$$

for any $(x,t) \in Q_T$, $k = 0, 1, 2, \ldots$. A similar argument as [1] shows that

$$(\omega_1^*(x,t), \omega_2^*(x,t)) = (\lim_{k\to\infty} \omega_1(x, t+kT), \lim_{k\to\infty} \omega_2(x, t+kT))$$

exists and $(\omega_1^*(x,t), \omega_2^*(x,t))$ is a nontrivial nonnegative periodic solution of (1)-(3). Therefore, for any $\varepsilon > 0$, there exists k_0 such that

$$u(x, t+kT) \le \omega_1^*(x,t) + \varepsilon \le U(x,t) + \varepsilon,$$
$$v(x, t+kT) \le \omega_2^*(x,t) + \varepsilon \le V(x,t) + \varepsilon$$

for $k \ge k_0$ and $(x,t) \in \overline{Q}_T$. Provided that the periodicities of $\omega_1^*(x,t)$, $\omega_2^*(x,t)$, $U(x,t)$ and $V(x,t)$ are taken into account, then the conclusion follows immediately.

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